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Plenary Lectures

Recovery of Data Matrices from Incomplete and Corrupted Entries: Theory and Algorithms

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A problem of considerable interest concerns the recovery of a data matrix from a sampling of its entries. In partially filled out surveys, for instance, we would like to infer the many missing entries. In the area of recommender systems, users submit ratings on a subset of entries in a database, and the vendor provides recommendations based on the user's preferences. Because users only rate a few items, we would like to infer their preference for unrated items (this is the famous Netflix problem). Formally, suppose that we observe a few entries selected uniformly at random from a matrix. Can we complete the matrix and recover the entries that we have not seen?

This talk discusses two surprising phenomena. The first is that one can recover low-rank matrices exactly from what appear to be highly incomplete sets of sampled entries; that is, from a minimally sampled set of entries. Further, perfect recovery is possible by solving a simple convex optimization program, namely, a convenient semidefinite program. The second is that exact recovery via convex programming is further possible even in situations where a positive fraction of the observed entries are corrupted in an almost arbitrary fashion. In passing, this suggests the possibility of a principled approach to principal component analysis that is robust vis a vis outliers and corrupted data. We discuss algorithms for solving these optimization problems emphasizing the suitability of our methods for large scale problems. Finally, we present applications in the area of video surveillance, where our methodology allows for the detection of objects in a cluttered background, and in the area of face recognition, where it offers a principled way of removing shadows and specularities in images of faces.

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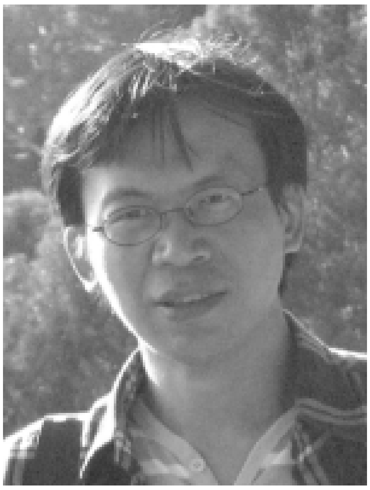

Minimizing Solutions of the Planar n-Body Problem in Certain Topological Classes

Kuo-Chang Chen
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The n-body problem concerns the motion of n celestial bodies moving in space in accordance with Newton's law of universal gravitation. In recent years variational methods have been successfully applied to the n-body problem to construct miscellaneous solutions with some equal masses. For most choices of mass, a special class of solutions called retrograde solutions can be also constructed by variational methods. In this talk I will briefly outline the proof and discuss generalizations of this result to the n-body problem with some topological constraints.

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Metastable Lifetimes in Random Dynamical Systems

Barbara Gentz
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The overdamped motion of a Brownian particle in a potential landscape with multiple wells is known to exhibit metastability, i.e., for weak noise, transitions between potential minima occur only on exponentially long time scales. The small-noise asymptotics of transition times is usually described by the Eyring–Kramers formula. However, this formula breaks down in the presence of non-quadratic saddles or wells.

We will show how recent results by Bovier, Eckhoff, Gayrard and Klein, who gave the first rigorous proof of the Eyring–Kramers formula, can be extended to degenerate landscapes. We shall conclude by further extending these results to a Ginzburg–Landau partial differential equation in a bounded interval of critical length, perturbed by weak space–time white noise.

This is joint work with Florent Barret (Palaiseau), Nils Berglund (Orléans), and Bastien Fernandez (CPT–CNRS, Marseille).

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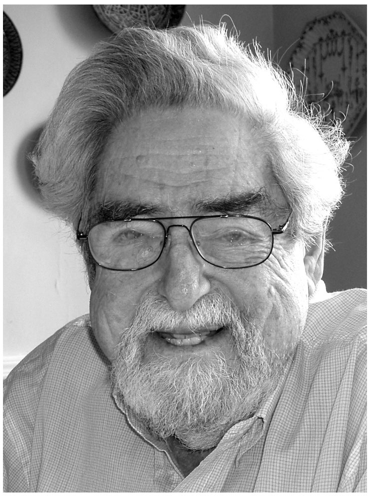
Remarks on Singular Solutions of Fully Nonlinear Elliptic Equations



Louis Nirenberg
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Various maximum principles are extended to some singular solutions of fully nonlinear elliptic equations. Applications are made to symmetry and monotonicity of such solutions via the method of moving planes. Results are extended to viscosity solutions, and to nonlinear parabolic equations. The talk will be expository.

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Multi-Dimensional Traveling Fronts in Bistable Reaction-Diffusion Equations

Masaharu Taniguchi
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Multi-dimensional traveling fronts have been studied in the Allen-Cahn equation (Nagumo equation) and also in multistable reaction-diffusion equations recently. Two-dimensional V-form fronts are studied by Ninomiya and myself (2005) and also by Hamel, Monneau and Roquejoffre (2005). Rotationally symmetric traveling fronts are studied by several authors.

In this talk I will give a brief survey on multi-dimensional traveling fronts, and explain what is different and what gives the difficulties compared with one-dimensional cases. Finally I explain recent works on three-dimensional traveling fronts of pyramidal shapes and those of convex polyhedral shapes.

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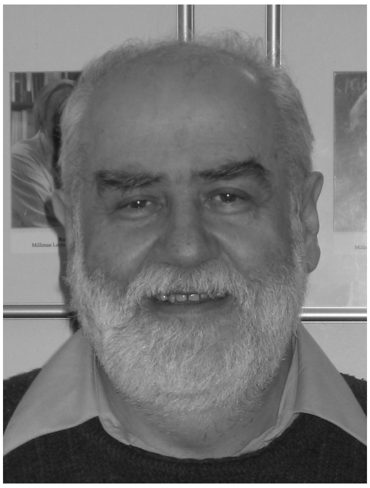

Cloaking, Invisibility and Inverse Problems

Gunther Uhlmann
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We describe recent theoretical and experimental progress on making objects invisible to detection by electromagnetic waves, acoustic waves and quantum waves. For the case of electromagnetic waves, Maxwell's equations have transformation laws that allow for design of electromagnetic materials that steer light around a hidden region, returning it to its original path on the far side. Not only would observers be unaware of the contents of the hidden region, they would not even be aware that something was being hidden. The object, which would have no shadow, is said to be cloaked. We recount the recent history of the subject and discuss some of the mathematical issues involved.

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Nonequilibrium Statistical Mechanics of Certain Hamiltonian Models

Lai-Sang Young
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We consider mechanical models consisting of large arrays of rotating disks and many moving particles. Energy exchange occurs at particle-disk collisions. We assume the system is coupled to unequal heat reservoirs, and is in a nonequlibrium steady state. Easy-to-compute algorithms for macroscopic quantities such as energy and particle density profiles are proposed, and relations between memory, finite-size effects and geometry are discussed. This model, which has chaotic local dynamics, is found to have reasonable thermodynamic properties. For comparison, we present a second model with integrable dynamics and anomolous behavior. These results are from joint works with J-P Eckmann, K Lin and P Balint, and are based on part numerical/part analytical studies.

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Special Sessions

Special Session 1: Nonlinear Schrödinger Equations and Their Applications

Panayotis Kevrekidis, University of Massachusetts, USA
Ricardo Carretero, San Diego State University, USA
Dimitri Frantzeskakis, University of Athens, Greece

Introduction: The nonlinear Schroedinger (NLS) equation is used in a very large variety of physical systems since it describes at the lowest order the nonlinear propagation of modulated waves. Some of the most important applications of the NLS equation emanate from the realm on nonlinear optics and Bose-Einstein condensates. The recent experimental realization of BECs and the ever growing control and experimental advances in nonlinear optical systems has ignited new and exciting developments. From the mathematical point of view, one of the most exciting aspects of these contexts is the broad range of possible configurations including: one to three spatial dimensions, one or many coupled fields, tunable external potentials, and temporally or even spatially variable nonlinearities, among many others.

The aim of this mini-symposium is to bring together experts, as well as young researchers, working on the theory, the numerical simulation and the experimental study of nonlinear Schrödinger equations and their applications. This should be a session appealing to theoretical physicists, experimental physicists and applied mathematicians alike and will be a vehicle for the exchange of ideas that could cross-fertilize different disciplines and promote the initiation of new collaborations that could address some of the pertinent open problems.

Justification of the Nonlinear Schrödinger Equation in Case of Non-Trivial Quadratic Resonances

Christopher Chong
University Stuttgart, Germany
(Guido Schneider)

The nonlinear Schrödinger (NLS) equation can be derived as an amplitude equation describing slow modulations in time and space of an underlying spatially and temporarily oscillating wave packet. For various nonlinear wave equations with nontrivial quadratic resonances, we prove estimates between the formal approximation obtained via the NLS equation and true solutions of the original system. The approximation property holds if the approximation is stable in the system for the three-wave interaction associated to the resonance. The unstable situation is explored with the use of numerical computations for both localized and nonlocalized solutions.

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Multiple Soliton Interaction in DM Optical Transmission Systems under Tod Effects

Francisco J. Diaz-Otero
University of Vigo, Spain
(Pedro Chamorro-Posada)

We analyse the intra-channel interaction properties in optical soliton trains propagating through strongly dispersion managed (DM) time-division multiplexed (TDM) single-wavelength communication systems. We employ an ordinary differential equations model for the parameters describing the propagating pulses which has been obtained using a variational approach. The validity of the model is assessed, for particular cases, by integrating of the underlying full nonlinear partial differential equations.

The variational equations are first solved for an isolated pulse in order to identify the launching parameters of the pulses in the first DM cell of the system. A train of multiple pulses is then introduced in the system in these initial points. We systematically study, for different numbers of solitons in the input train, their evolution and analyse the transmission system in terms of the coalescence distance and mutual interactions among neighbouring pulses as the map strength is varied and the impact of third-order dispersion (TOD) effects and the position of the amplifier within the map period.

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Gap Solitons at Purely Nonlinear Interfaces

Tomas Dohnal
Karlsruhe Institute of Technology, Germany
(Elizabeth Blank)

We study surface gap solitons (SGS) of the 1D nonlinear Schrödinger/Gross-Pitaevskii equation with a

linear periodic potential and a discontinuous nonlinearity coefficient Γ. Families of SGSs are computed using the arclength continuation method for a range of values of the jump in Γ. Using asymptotics, we show that when the frequency parameter converges to the bifurcation gap edge, the size of the allowed jump in Γ converges to 0 for SGSs bifurcating from GSs centered at any $x_c \in \mathbb{R}$. As the results show, bright SGSs can exist even at focusing-defocusing interfaces.

Linear stability of SGSs is next determined via the numerical Evans function method. Zeros of the Evans function detect linear dependence of the stable and unstable manifolds of the linearized problem and thus coincide with eigenvalues of the linearized operator. Far from the SGS location the manifolds are spanned by exponentially decaying/increasing Bloch functions. As we show, numerically stable evolution of the manifolds requires the use of exterior algebra and Grassmanian preserving ODE integrators. Evans function zeros are then detected via the complex argument principle. Both unstable and stable SGSs are found, where stability is obtained even for some SGSs centered in the medium with the less focusing nonlinearity. Direct simulations of the PDE for selected SGS examples confirm the results of Evans function computations.

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Universal Spreading of Wave Packets in Disordered Nonlinear Systems

Sergej Flach
MPIPKS Dresden, Germany

I will analyze mechanisms and regimes of wave packet spreading in nonlinear disordered media. Wave packets can spread subdiffusively in two regimes of strong and weak chaos [1, 2, 3]. I will discuss resonance probabilities, nonlinear diffusion equations, and a dynamical crossover from strong to weak chaos. The crossover is controlled by the ratio of nonlinear frequency shifts and the average eigenvalue spacing of eigenstates of the linear equations within one localization volume. I consider generalized models in higher lattice dimensions [4] and obtain critical values for the nonlinearity power, the dimension, and norm density, which influence possible dynamical outcomes in a qualitative way.

[1] A. S. Pikovsky and D. L. Shepelyansky, Phys. Rev. Lett. 100, 094101 (2008); G. Kopidakis et al, Phys. Rev. Lett. 100, 084103 (2008).
[2] S. Flach, D. O. Krimer and Ch. Skokos, Phys. Rev. Lett. 102, 024101 (2009); Ch. Skokos et al, Phys. Rev. E 79, 056211 (2009).
[3] S. Flach, arXiv:1001.2673 (2010).
[4] Ch. Skokos and S. Flach, arXiv:1001.5171 (2010).

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Dipolar Condensates Confined in a Toroidal Trap

Montserrat Guilleumas
Universitat de Barcelona, Spain
(M. Abad, R. Mayol, M. Pi and D. Jezek)

A Bose-Einstein condensate of ^{52}Cr atoms is a very suitable system for studying the effects of dipolar interactions due to their large magnetic dipole moment. In contrast to s-wave interactions, dipole-dipole interactions are long range and anisotropic. These main features introduce interesting new physics in the field of Bose-Einstein condensation. In particular, their anistropic character can be enhanced in specific configurations, leading to a rich variety of phenomena.

We investigate the physics arising from dipolar interactions in a multiply connected geometry. We consider a system of ^{52}Cr atoms confined in a toroidal potential, with the dipoles aligned perpendicularly to the trap symmetry axis. For small scattering lengths, the interaction in combination with the central repulsive Gaussian barrier creating the toroidal potential produces an azimuthal dependence of the particle density for a fixed radial distance. As the scattering lenght is reduced, this dipolar effect becomes stronger and the density presents two symmetric density peaks in the perpendicular axis, while in the parallel axis the density becomes smaller [1]. This behaviour can be understood in terms of a self-induced energy barrier that the mean-field dipolar interaction creates in the condensate. This barrier-like structure yields an interesting dynamics in the system, governed by the tunneling of atoms.

For very low values of the scattering length, just above collapse, the system undergoes a dipolar-induced symmetry breaking phenomenon [1]. The whole density becomes concentrated in one of the peaks, resembling an origin displaced cigar-shaped condensate.

[1] M. Abad, M. Guilleumas, R. Mayol, M. Pi and D. Jezek, arXiv:1001.3594 (2010).

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Existence Results and Finite Difference Discretization for the Hirota Equation

Nikos Karachalios
University of the Aegean, Greece
(Georgios Zouraris)

We discuss some issues regarding the well posedness for the initial- and periodic boundary- value

problem for Hirota equation and its numerical solution by a finite difference method. For a generalized equation

$$\begin{aligned}\varphi_t + \beta(|\varphi|^2)\,\varphi_x - \mathrm{i}\,\rho\,\varphi_{xx} + \sigma\,\varphi_{xxx} \\ - \mathrm{i}\,\gamma(|\varphi|^2)\,\varphi = 0 \quad \text{on} \quad \mathbb{R}\times(0,T],\end{aligned} \tag{1}$$

where ρ, $\sigma \in \mathbb{R}$ are given constants and

$$\beta, \gamma \in C^3(\mathbb{R};\mathbb{R}),$$

we prove local existence of strong solutions. Global existence of weak solutions is shown for the equation (1) involving the classical cubic nonlinearities, arising in the study of the motion of vortex filaments in 3D. The proof is using rigorously derived energy identities in the appropriate phase space. For the classical version of the equation, a new finite difference method is formulated for which a convergence analysis is carried out.

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Inhibition of Light Tunneling in Waveguide Arrays

Yaroslav Kartashov
ICFO-Institut de Ciencies Fotoniques, Spain
(Y. V. Kartashov, A. Szameit, M. Heinrich, F. Dreisow, R. Keil, S. Nolte, A. Tunnermann, V. A. Vysloukh, F. Lederer, L. Torner)

We describe theoretically and report on experimental observation of almost perfect light tunneling inhibition at the edge and inside laser-written waveguide arrays. When the refractive index of the guiding channels of array is harmonically modulated along the propagation direction and out-of-phase in adjacent guides, light is trapped in the excited waveguide over a long distance even in linear case due to resonances at specific modulation frequencies, which depend, in particular, on the depth of longitudinal refractive index modulation. The phenomenon can be used for tuning the localization threshold power in such z-modulated structures. We address light tunneling inhibition in both one- and two-dimensional settings. We also study an impact of nonlinearity on the width of resonances where tunneling inhibition is achieved. The nonlinearity broadens such resonances so that localization can take place even in detuned systems at power levels well below those required for nonlinearity-mediated localization in unmodulated systems.

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Long-Range Interactions and Phase-Shift Discrete Breathers in Klein-Gordon Chains. A Comparison with the DNLS Results

Vassilis Koukouloyannis
Technol. Educational Institution of Serres, Greece
(P. G. Kevrekidis, J. Cuevas-Maraver, V. Rothos)

A classical 1D Klein-Gordon chain with nearest-neighbor interaction cannot support phase-shift discrete breathers i.e. breathers with phase difference between successive central oscillators different from 0 or π. The situation changes when long-range interactions are introduced. Phase-shift breathers also appear in 2D Klein-Gordon lattices in the form of vortex breathers. In the 2D case the existence of long-range interactions can critically effect the stability of some of these structures. A comparison with the corresponding results in the DNLS is performed.

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Nonlocal Solitons

Fabian Maucher
MPI PKS Dresden, Germany
(Stefan Skupin, Anton Desyatnikov, Yuri Kivshar, Wieslaw Krolikowski)

We discuss the nonlinear Schrödinger equation with spatially nonlocal nonlinearity. We show that this nonlinear model supports formation of 2D and 3D localized structures in a form of solitons and azimuthons. We investigate stability of these solitons and discuss their experimental realization in thermal media.

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Excited States in the Thomas-Fermi Limit

Dmitry Pelinovsky
McMaster University, Canada

Excited states of Bose-Einstein condensates are considered in the semi-classical (Thomas-Fermi) limit of the Gross-Pitaevskii equation with repulsive inter-atomic interactions and a harmonic potential. The relative dynamics of dark solitons (density dips on the localized condensate) with respect to the harmonic potential and to each other is approximated using the averaged Lagrangian method. This permits a complete characterization of the equilibrium positions of the dark solitons as a function of the chemical potential parameter. It also yields an analytical handle on the oscillation frequencies of dark solitons around such equilibria. The asymptotic predictions are generalized for an arbitrary number of dark solitons and are corroborated by numerical computations for 2- and 3-soliton configurations.

The existence results are rigorously proved with the method of Lyapunov-Schmidt reductions.

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Homoclinic Solitons in Spinor Bose-Einstein Condensates

Vassilis Rothos
Aristotle University of Thessaloniki, Greece
(E. Doktorov, Y. Kivshar)

We study the full-time dynamics of modulational instability in $F = 1$ spinor Bose-Einstein condensates for the case of the integrable three-component model associated with the matrix nonlinear Schrödinger equation. We employ methods of integrable systems to construct the homoclinic soliton equations and their stability under small perturbations with physical applications.

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Global Well-Posedness and Small Data Scattering for the Ostrovsky Equation

Atanas Stefanov
The University of Kansas, USA
(Panos Kevrekidis, Yannan Shen)

We consider the generalized Ostrovsky equation $u_{tx} = u + (u^p)_{xx}$. For integers $p \geq 4$, we show that the equation is globally well-posed for small data in and moreover, small solutions scatter. The latter results are corroborated by numerical computations which confirm the heuristically expected decay of $\|u(t)\|_{L^r} \sim t^{-(r-2)/2r}$.

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Josephson Devices with Ultracold Atoms

Andrea Trombettoni
SISSA (Trieste), Italy

Ultracold atoms in double well potentials and optical lattices provide an experimental realization of Josephson junctions and networks. After reviewing analogies and differences with superconducting Josephson devices, I will discuss the dynamics of ultracold atoms in such Josephson devices, focusing in particular on the properties of Josephson oscillations in the 3D nonlinear Schrödinger equation in presence of a double well potential.

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Special Session 2: Dynamics in Neuronal Networks

Qishao Lu, Beihang University, China
Jianzhong Su, The University of Texas at Arlington, USA
Kiyoyuki Tchizawa, Institute of Administration Engineering, Japan

Introduction: The session will include all areas of modeling, simulation and analytical methods for neuron or neuronal network models with a particular emphasis on nonlinear phenomena such as Firing Oscillations, Synchronization and Their Propagation Waves. To further understand experimental or computer-simulated results, one often use dynamical system tools such as bifurcation analysis, singular perturbations, and fast-slow systems to provide mathematical insight. In recent years, there are intense research interest on the effects of noise to these neuronal patterns as well as their related mathematical analysis based on bifurcation structure. This special session will provide a forum for researchers in this area to present current results.

Analysis of a Model for an Excitable Fiber with Myotonia or Periodic Paralysis

Jonathan Bell
University of Maryland Baltimore County, USA
(Kamonwan Kocharoen, Yongwimon Lenbury)

Due to mutations in the coding region of ion channel genes, various diseases of muscle cells can arise. One such class of channelopathies, myotonia, has been studied from a clinical and electrophysiological standpoint, and a mathematical model has been developed to further understand the dynamics of the disease. In our presentation we will discuss reduction of this model to an analytically tractable two-compartment 3D system, and through geometric perturbation theory and simulation, solution behavior corresponding to some rat muscle observations will be explained. In particular, we are able to detect slow-fast limit cycles which generate bursts of action potentials characteristic of the clinical case where active and non-active phases are observed to alternate in a pulsatile fashion, mimicking observations seen in patients with Hyperkalemic periodic paralysis.

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Noise-Induced Firing and Quiescence in a Hodgkin-Huxley Model

Katarina Bodova
Comenius University, Slovak Republic

The electrical behavior of neurons can show a significant amount of variability. Three sources of randomness contribute to this variability: fluctuating pre-synaptic inputs, variability in synaptic transmission and stochastic channel dynamics. Here we focus on how stochastic channel dynamics can influence the behavior of a single neuron. To simulate the classical Hodgkin-Huxley model with stochastic channel dynamics, we track the opening and closing of a fixed number of sodium and potassium channels. Since this is computationally expensive, we also consider directly adding multiplicative noise to the original equations that model channel dynamics. We explore the behavior of neurons for a wide range of noise parameters and external forcing given by a constant applied current. Our experiments show that noise can play significant and counterintuitive roles in determining the firing behavior of a neuron. We construct a simple probabilistic model that captures the dynamical features of the HH model and produces similar dstribution shapes of inter-spike intervals.

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Limit Theorems for Stochastic Neuronal Network Dynamics

Lee Deville
University of Illinois, USA

We consider a network of pulse-coupled oscillators containing randomness both in input and in network architecture. We analyze the scalings which arise in certain limits, demonstrate limit theorems in the correct scaling, and interpret various "finite-size" effects as perturbations of these limits. We also observe that for certain parameters, this network supports both synchronous and asynchronous modes of behavior and will switch stochastically between these modes due to rare events. We also relate the analysis of this network to results in random graph theory, and in particular, those involving the size of the "giant component" in the Erdos-Renyi random graph and use this to understand the phase transitions in the system.

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Bifurcation and Bursting in Morris-Lecar Model for Class I and Class II Excitability

Lixia Duan
North China University of Technology, PR China

Morris-Lecar (ML) neuronal model can demonstrate two different types of neuronal excitability (i.e. class I and class II excitability) when parameters are set appropriately. In this report, the ML model with current-feedback control is considered as a typical fast-slow dynamical system to study the relationship between bifurcation structures and bursting oscillations. Two-parameter bifurcation analysis of the fast subsystem is performed under the parameter sets of class I and class II excitability at first. Then we further show that the bifurcation structures of the fast-subsystem are different under the parameter sets of class I and class II excitability, which lead to different types of bursting oscillations. It is shown that the model exhibits five types of bursting oscillations under class I parameter set while exhibits three types of bursting oscillations under class II parameter set. Different bursting regions are obtained by using two-parameter bifurcation and the fast-slow dynamic analysis based on the Izhikevich's classification scheme of bursting. The results in this paper shown that under the parameter sets of class I and class II, the bifurcation structures of the fast-subsystem can provide crucial information about the possible types of bursting in the neuronal model.

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Fold-Hopf Bifurcations of the Rose-Hindmarsh-Type Model

Zhaosheng Feng
University of Texas-Pan American, USA
(Suqi Ma)

In this talk we are concerned with the finite propagation speed of signals in synapses, when a time delay signal self-feedback mechanism is introduced into the Rose-Hindmarsh Model. The Fold-Hopf bifurcation and the normal form near the Fold-Hopf points are investigated. The complex bursting-spiking firing modes associated with bifurcations are presented.

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Improvement of Image Processing by Using Homogeneous Neural Networks with Fractional Derivatives Theorem

Zbigniew Gomolka
University of Rzeszow, Poland
(Twarog Boguslaw, Bartman Jacek, Kwiatkowski Bogdan)

Paper deals with unique circumvention of designing feed forward neural networks in the task of the interferometry image recognition. In order to bring to the fore the interferometry techniques, we recall

briefly that this is one of the modern techniques of restitution of three dimensional shapes of the observed object on the basis of two dimensional flat like images registered by CCD camera. The preliminary stage of this process is conducted with ridges detection, and to solve this computational task, the discussed neural network was applied. By looking for the similarities in the biological neural systems authors show the designing process of the homogeneous neural network in the task of maxims detection. The fractional derivative theorem has been involved to assume the weight dstribution function as well as transfer functions. To ensure reader that the theoretical considerations are correct, the comprehensive review of experiment results with obtained two dimensional signals have been presented too.

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Application of Jacobi Elliptic Function Expansion Method to Perturbed and Coupled KdV-Type Equations with Variable Coefficients

Cuncai Hua
Yunnan Normal University, Peoples Rep. of China

(Xiujuan Jia and Liping Ren)

In this talk, on the basis of the idea of homogeneous balance method and its transforming forms, the Jacobi elliptic function expansion method is improved and applied to solve perturbed KdV and combined KdV-mKdV equation with variable coefficients, and two types of perturbed and coupled nonlinear KdV equations. A number of significant new analytic solutions are obtained. The solutions include the type of solitary wave solution, periodic solutions of hyperbolic triangle, as well as the type periodic solutions of trigonometric functions.

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Modeling Tristable Perception for Visual Plaids

Gemma Huguet
CRM-NYU, Spain

(Ernest Montbrió (UPF; CNS, NYU) and John Rinzel (CNS & CIMS, NYU))

When observers view an ambiguous visual scene with two or more different interpretations they report alternation in the different perceptions. A well-known example of this phenomenon is binocular rivalry, for which different models have been proposed (e.g., Laing & Chow, 2002; Wilson, 2003). We focus on experiments with two superimposed moving gratings (plaids), identical except for their movement directions (normal to their respective orientations). There are two sources of ambiguity: their relative motion and the depth order (e.g., Moreno-Bote, 2008). If the angle between the normal vectors for motion is small, coherence is favored (the gratings move together as a single pattern). If the angle is large, transparency is favored (the gratings slide across one another). For some range of angles, tristable perception is experienced: a coherent percept and two transparent percepts with alternating depth ordering. In this talk, we present a neural network, firing rate model for the dynamics of alternations during perceptual tristability for plaids. The network is distributed in “space”; the space-like variable corresponds to the angle of motion. The two stimuli are peaked around the two angles of movement direction. Slow adaptation is responsible for the switching.

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Electric Alternans in a Mathematical Model of Ventricular Muscle

Hiroyuki Kitajima
Kagawa University, Japan

(Toru Yazawa)

Electrical alternans is alternating amplitude from beat to beat on the electrocardiogram. It has been associated with ventricular arrhythmias in many clinical settings, however, its dynamical mechanisms remain unknown. In this study, we construct a network model of the heart based on Yazawa’s experiments. Using this model, we find alternating oscillations of the action potential for the ventricular muscle by changing a value of the parameter $[K^+]_o$ (free concentration of potassium ion in extracellular compartment). We are now studying the dependency of generating alternans on several ionic currents (fast sodium current, L-type calcium current and so on).

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Bifurcations of Noisy Phantom Bursting

Morten Pedersen
Lund University, Sweden, Denmark

Phantom bursting is a mechanism by which two slow processes, one much slower than the other, interact to provide bursting on an intermediate “phantom” time-scale. Phantom bursting is believed to underlie regular bursting activity in insulin secreting beta-cells. Stochastic membrane channel activity will introduce noise in the system, and it is therefore relevant to study the dynamics of noisy phantom burster models. Simulations have shown that phantom bursting is much more sensitive to noise than regular bursting with only one slow variable. Here I present results on the bifurcation structure of phantom bursting, and show how we can get analytical

insight in the mechanisms by which noise perturb the system.

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Modeling Indepedent Signaling of Spontaneous and Evoked Neurotransmissions

Jianzhong Su
University of Texas at Arlington, USA
(Justin Blackwell)

In this talk we will discuss a mathematical model to simulate spontaneous and evoked neurotransmission to determine how receptor location on presynaptic density can activate distinct sets of NMDA receptors and signal independently to the postsynaptic side. We will also examine how the release rate of the neurotransmitter Glutamate will affect post-synaptic currents by varying the fusion pore size at the presynaptic side. The numerical study is to explore plausible conditions for a single synapse to accommodate two independent signaling of neurotransmitters.

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On 4-Dimensional Duck Solution with Relative Stability

Kiyoyuki Tchizawa
Institute of Administration Engineering, Japan

This paper gives the existence of a relatively stable duck solution in a slow-fast system in R^{2+2} with an invariant manifold. It has a 4-dimensional duck solution having a relatively stable region when there exist the invariant manifold near the pseudo singular saddle or node point.

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Hierarchical Modeling of Dynamic Functions with Labeling or Time-To-Event Outcomes

Xiaohui Wang
University of Texas-Pan American, USA

Non-linear dynamic functional data with various outcomes, which are binary or multi-level labeling responses, or time-to-event outcomes, are studied by using hierarchical modeling. Analysis of functional predictors with those outcomes can be very challenging because of the features of different data. In our neural models, we use wavelet- and spline-based nonparametric approaches wherein the usage of those bases functions simplifies the parameterizations and the unified modeling framework allows synergistic benefit between the regression of curve predictors and modeling of response data. Examples of applications on real world data sets are illustrated.

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Multiple Time-Scales, Bursts and Canards in Pituitary Lactotroph Cells

Martin Wechselberger
University of Sydney, Australia
(Theodore Vo, Richard Bertram, Joel Tabak)

Bursting oscillations in nerve cells have been the focus of a great deal of attention by mathematicians. These are typically studied by taking advantage of multiple time-scales in the system under study to perform a singular perturbation analysis. Bursting also occurs in hormone-secreting pituitary cells, but is characterized by fast bursts with small electrical impulses. Although the separation of time-scales is not as clear, singular perturbation analysis is still the key to understand the bursting mechanism. In particular, I will show that canards are responsible for the observed oscillatory behavior.

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Sudden Death and Natural Death: DFA and Poincare Map of Heartbeat

Toru Yazawa
Tokyo Metropolitan University, Japan
(Hiroyuki Kitajima)

This is empirical paper including experimental observations and analysis of real-world data. We show cardiac physiology of model animals that could contribute to Kitajima's simulations that will be presented together with this paper. To study differences between the heart at natural-death and the heart at sudden-death, we analyzed EKGs of model animals, by DFA (detrended fluctuation analysis) and Poincare Map (delay time embeddment). Crabs (Birgus latro and Scylla serrata), lobster (Panulirus japonicus) and other crustaceans were used. EKGs were recorded electro-physiologically. At healthy condition, the scaling exponents were maintained at about 1.0. At natural death, scaling exponents gradually went down, approaching to 0.5. At unpredictable death, that is, sudden death, scaling exponents were maintained at high values, 1.3 ∼ 1.5. We found the heart of dying animals exhibited Alternans-rhythm, which is so called as the harbinger of death since the hearts were at risk of catastrophic circulation stoppage. This unique two beat rhythm, which was first described by Traube (1872), has not gotten much attention until physicians recently noticed that ischemic heart diseases are accompanied by the Alternans. Although the

mechanism of generation of Alternans is not known, Kitajima's mathematical model and simulations of this deadly rhythm is of interest.

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Neuronal Waves

Linghai Zhang
Lehigh University, USA

In this talk, by using a nonlinear singularly perturbed system of integral differential equations as a reasonable realistic model, we show how a nonlinear nonlocal neuronal network can generate several stable traveling waves, including homoclinic orbits and heteroclinic orbits. All of these orbits serve as nontrivial local attractors of the dynamical systems. We will build explicit relationships between small waves and large waves and relationships between speeds of small waves and speeds of large waves. Furthermore, we will establish relationships between speed index functions and stability index functions whose zeros are necessary and sufficient to determine the stability of the brain waves.

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Synchronization in Globally Weakly Coupled Neurons

Yong Zhao
Beijing Uni. of Aeron. and Astron., PR of China
(Qishao Lu)

Neural activity is a cooperative process of neurons [1, 2]. It is very important to understand how collective behaviour activity from the interactions between their neurons in neural networks. One of the most important collective behaviours in the brain are synchronization. Many examples of synchronization can be found in the nervous system [3, 4]. In this paper, we discuss the existence and stability of the synchronizing states of the globally weakly coupled neurons. In order to investigate the phenomena, we derive general phase model of globally weakly coupled neurons using phase response curves and averaging theory [5]. Based on the phase approximation, we precict the existence and stability of the synchronizing state. The phase response curve and an expression of the Lyapunov exponent are also analytically to study the stability of synchronizing state. As an application, we consider networks of multi-coupling Morris-Lecar nuerons and examine the effects of inhibitory and exicitory coupling, respectively, or both coupling Morris-Lecar nuerons on the synchronization. Futheremore, we discuss transition among multistability states and the interaction between any two of them effect on synchronization. The results are consistent with our theoretical analysis. It is useful to understand and interpret the collective activity of nervous system.

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Generation Mechanisms of Electrical Bursting Patterns for Pancreatic β-Cells with Subspace Ca^{2+} and Ca^{2+}-Based Nucleotide Ratio

Yang Zhuoqin
Beihang University, Beijing, Peoples Rep. of China
(Qishao Lu)

Pancreatic β-cells located within pancreatic islets of Langerhans can control glucose concentration by secretion of insulin, in response to an elevation in the blood glucose level. Electrical activities are mainly responsible for the $[Ca^{2+}]_i$ oscillations of β-cells and islets, which are in phase with the regular oscillatory patterns of insulin secretion from pancreatic islets.

So several types of models illustrate how these mechanisms can interact to produce pancreatic β-cell electrical activity and its relationship to $[Ca^{2+}]_i$. In this work, a complete pancreatic β-cells model is developed by focusing on two important slow negative feedback mechanisms, that is, a slowly activating subspace Ca^{2+}-dependent K^+ current [1] and ATP-sensitive K^+ current controlled by a Ca^{2+}-based nucleotide oscillation [2]. In this model, three slow processes Css, Cer and a with different time scales can interact to drive fast, medium and slow bursting typically observed in pancreatic β-cells.

Naturally, the study will give emphasis on electrical bursting, as a main pattern of firing activities in pancreatic β-cells. We found diverse patterns of electrical bursting with shapes varying from only waves to the coexistence of waves and spikes to only spikes are robust to fast, medium and slow oscillation periods. Under robustness of the electrical bursting patterns to fast, medium and slow oscillation periods, three different electrical bursting patterns could be explored through an extension of fast/slow analysis [3, 4] for fast bursting. Since the variable a is nearly constant during fast bursting and the slower slow variable Cer has no effect on the fast subsystem, we can consider bifurcations of equilibria and limit cycles of fast subsystem with respect to the faster slow variable Css as only a slow varying parameter, and then acquire dynamic generation mechanisms and topological types of different bursting patterns. In the light of all bifurcations associated with bursting, three topological types of bursting are attained, that is, "fold/fold" point-point hysteresis loop bursting, "fold/Hopf" bursting via the "fold/fold" hysteresis loop and "fold/homoclinic" bursting via the "fold/homoclinic" hysteresis loop.

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Special Session 3: Dynamics of Chaotic and Complex Systems

Jose M. Amigo, Universidad Miguel Hernandez, Spain
Miguel A. F. Sanjuan, Universidad Rey Juan Carlos, Madrid, Spain

Introduction: This special session will focus on current research related to dynamics of chaotic and complex systems in a broadest way, paying a special attention to applications in physics, engineering, biology, neurosciences and others.

The main goal of this special session is to join together physicists, mathematicians and other scientists interested in new developments of chaotic dynamics in its broader sense, the different available methods of controlling chaos and their applications in complex systems arising in many fields of engineering and sciences, and in particular to life sciences such as the analysis of the dynamics of neurons and genetic regulation networks. The topics included, but not limited to, in this session are:

- New developments of chaotic dynamics
- Novel methods of controlling chaotic and complex dynamics
- Hamiltonian and dissipative chaotic systems
- Fractal structures in phase space
- Chaotic dynamics of neuronal and genetic models
- Control and synchronization in neuronal and genetic networks

As the organizers of this Special Session, we would like to invite you to participate and kindly ask you to register and send an abstract at

Quantifying the Effects of Anesthetics on Electrical Brain Activity by Synchronization Measures

Thomas Aschenbrenner
MPI Institute for Extraterrestrial Phys., Germany
(Wolfram Bunk, Roberto Monetti)

Essential functional properties of the brain are the result of synchronization processes on the neural level (e.g. information processing, memory coding, etc.). In this study consciousness is described by measures assessing synchronization of electrical brain activity. Several synchronization measures in time domain as well as in frequency domain are applied to distinguish between conscious and deeply unconscious states. Digitally filtered data is used to determine the influence of several established EEG frequency bands on the classification results. We find that unconsciousness comes along with significantly enhanced synchronization levels. These results offer some valuable clues to the physiological processes of anesthetic-induced unconsciousness.

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SRB Measures for Certain Markov Processes

Wael Bahsoun
Loughborough University, England
(Pawel Gora)

In this talk we study physical SRB (Sinai-Rulle-Bowen) measures for iterated function systems (IFS) whose constituent maps are strictly increasing transformations of the interval. We obtain an upper bound on the number of SRB measures for the IFS. Moreover, when all the constituent maps have common fixed points at 0 and 1, we provide sufficient conditions for δ_0 and/or δ_1 to be, or not to be, SRB measures. To complement our theoretical results, we show at the end of the talk that examples of IFS of this type can describe evolutionary models of financial markets.

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Chaotic Behavior Via Lyapunov Exponents

Francisco Balibrea
Universidad de Murcia, Spain

In experimental dynamics, it is frequently used to associate a positive Lyapunov exponent of the orbit of a dynamical system (X, f) with the sensitive dependence on initial conditions of such orbit and negative Lyapunov exponent with non-sensitive dependence and take them as criteria for a chaotic behavior. Recent examples introduced in [1] prove that such statements are true under additional properties on f.

When we consider non-autonomous systems $(X, f_{0,\infty})$ where $f_{0,\infty} = (f_n)_{n=0}^{\infty}$ and all maps are continuous of X into itself, the notions of Lyapunov exponents and sensitive dependence on initial conditions can be extended to such setting.

In the lecture we give a partial answer to the above statements when we consider non-autonomous systems and the sequence $f_{0,\infty}$ is periodic of prime period p, that is $f_{n+p} = f_n$.

[1] B. Demir and S. Kocak. A note on positive Lyapunov exponent and sensitive dependence on initial conditions. Chaos, Solitons and Fractals 12 (2001), 2119-2121.

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Qualitative Analysis of Dissipative Systems: Rössler Equations and Numerical Tools

Roberto Barrio
University of Zaragoza, Spain
(R. Barrio, F. Blesa, S. Serrano)

In this talk we study different aspects of the paradigmatic Rössler model. We present a detailed study of the local and global bifurcations of codimension one and two of limit cycles. This provides us a global idea of the three-parametric evolution of the system. We also study the regions of parameters where we may expect a chaotic behaviour by the use of different Chaos Indicators. The combination of the different techniques gives an idea of the different routes to chaos and the different kind of chaotic attractors we may found in this system.

In the analysis we use different numerical techniques and we make an extensive use of the new freeware package TIDEs (`http://gme.unizar.es/software/tides`), that is an state-of-te-art numerical ODE integrator based on the Taylor series method that permits to solve an ODE system up to any arbitrary precision level and a direct computation of the solution of the variational equations up to any order, what is especially useful when one uses chaos indicators.

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Energy Localization on q-Tori, Long Term Stability and the Interpretation of the FPU Paradox

Tassos Bountis
University of Patras, Greece
(T. Bountis (*) and H. Christodoulidi, Department of Mathematics, University of Patras, Patras, Greece (*) On leave at the Max Planck Institute for Physics of Complex Systems, April – June 2010, bountis@pks.mpg.de, tassos50@otenet.gr)

Two approaches have been proposed in recent years for the explanation of the so-called FPU paradox, i.e. the persistence of energy localization in the 'low-q' Fourier modes of Fermi Pasta Ulam nonlinear lattices, preventing equipartition among all modes at low energies. In the first approach, a low-frequency fraction of the spectrum is initially excited leading to the formation of 'natural packets' exhibiting exponential stability, while in the second, emphasis is placed on the existence of 'q-breathers', i.e. periodic continuations of the linear modes of the lattice, which are exponentially localized in Fourier space. Following ideas of the latter, we introduce in this presentation the concept of 'q-tori' representing exponentially localized solutions on low–dimensional tori and use their stability properties to reconcile these two approaches and provide a more complete explanation of the FPU paradox.

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Synchronization Patterns Revealed by Time-Scale Resolved Symbol Maps

Wolfram Bunk
MPI Institute for Extraterrestrial Phys., Germany
(R. Monetti, Th. Aschenbrenner)

Synchronization is governing the interplay and the individual development of components of coupled systems. Monetti et al. (2009) presented a method, which uses transcription symbols between the observables of the system components and associated order classes to classify different types and the strength of synchronization. According to the construction, the delay time τ and the vector dimension d, define the time horizon and the set of transcription symbols S, respectively. By scanning τ over a reasonable range, each pair of time series can be mapped in a "τ versus S-plane". The resulting density dstribution of the generated transcription symbols together with the definition of order classes allow for a detailed characterization of synchronization patterns on different time scales, e.g. by means of entropy measures or structural analysis of the map. To discuss the methodology, we investigate a coupled Rössler system in different synchronization regimes and the spatio-temporal structuring of brain activity based on multi-channel surface EEGs.

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On the Distribution of the Critical Values of the Iterated Differentiable Functions

Maria Correia
University of Evora, Portugal
(Carlos C. Ramos and Sandra M. Vinagre)

We consider the dynamical system $(\mathcal{A}, T_f)$, where $\mathcal{A}$ is a class of differentiable real functions defined on some interval and $T_f : \mathcal{A} \to \mathcal{A}$ is an operator $T_f \varphi := f \circ \varphi$, where f is a function on the real line. We analyze in detail the case in wich T arises from a differentiable m-modal map f. We obtain a combinatorial description of the orbits of $(\mathcal{A}, T_f)$ and we study the evolution of the dstribution of the critical values of the iterated functions..

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Shil'nikov Chaos and Mixed-Mode Oscillation in Coupled Chua Oscillator: Theory and Experiment

Syamal Dana
Indian Inst. of Chem. Biology, Kolkata, India
(Chittaranjan Hens, Sourav K. Bhowmick)

A method of observing homoclinic bifurcation and related Shil'nikov scenario in asymmetric Chua oscillator is reported. The asymmetry is created in one Chua oscillator by coupling either unidirectionally or bidirectionally to a second Chua oscillator in resting state. The coupling parameter act as a tuning parameter to reveal Shil'nikov scenario near a saddle focus and a sequence of mixed-mode oscillations. The model clearly shows the related homoclinic bifurcations and cycle-cycle bursting near the bifurcation point. The role of the asymmetry in observing Shil'nikov scenario is well established. Different regimes of homoclinic chaos are thus observed in parameter space. Results are all supported by strong experimental evidences in electronic circuit.

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A General Characteristic Relation for Type-II and -III Intermittencies

Ezequiel Del Rio
ETSI Aeronauticos UPM, Spain
(Sergio A. Elaskar and Jose Manuel Donoso)

The reinjection probability density (RPD) for type-II and type-III intermittency phenomena is analytically and numerically studied. In this analysis our previously introduced function M [1] is stated as a powerful tool to determine the RPD from numerical or experimental data. A new one-parameter class of RPD has been found from which the usual uniform one is a particular case. The new RPD has been detected to be valid in a wide class of maps and dynamical systems. Therefore, form this RPD in any particular case, it is also possible to derive new expressions for the duration probability density of the laminar phase and to provide new characteristic relations which depend on the lower bound of the reinjection (LBR). The LBR is included as a special value of a sifting parameter which can be also obtained form M. A necessary condition has been set for the existence of the characteristic relation corresponding to the widely used case of uniform RPD. We guess that a non-linear mechanism is responsible for this general RPD.

[1] Ezequiel del Rio and Sergio Elaskar. IJBC (to appear on April, 2010).

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Computing the Topological Entropy of Unimodal Interval Maps

Rui Dilão
IST, NonLinear Dynamics Group, Lisbon, Portugal
(José María Amigó)

We derive an algorithm to determine recursively the lap number (minimal number of monotonous pieces) of the iterates of unimodal maps of an interval, not necessarily symmetric around the critical point. This enables to calculate numerically the topological entropy of unimodal maps. As bifurcations of parameterized families of maps are associated with the break of topological conjugacy, the lap number and the topological entropy can be used to predict the reversion of bifurcations in families of maps of an interval.

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Phase Synchronization in Coupled Sprott Chaotic Systems Presented by Fractional Differential Equations

G. Hussian Erjaee
Qatar University
(Modi Alnasr)

Phase synchronization occurs whenever a linearized system describing the evolution of the difference between coupled chaotic systems has at least one eigenvalue with zero real part. We illustrate numerical phase synchronization results and stability analysis for some coupled Sprott chaotic systems presented by fractional differential equations.

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Dynamics of Generalized Hydrodynamical Models

Carlos Escudero
Consejo Sup. de Investigaciones Científicas, Spain

The equations of continuum hydrodynamics, the Euler and Navier-Stokes equations, can be derived from the Boltzmann equation, which describes rarefied gas dynamics at the kinetic level, by means of the Chapman-Enskog expansion. This expansion assumes a small Knudsen number, which is the ratio among the mean free path of the gas molecules and the macroscopic characteristic length. As a consequence, the Navier-Stokes equations are able to successfully describe sound propagation when the frequency of a sound wave is much higher than the collision frequency of the particles. When both frequencies become comparable, these equations give a poor account of the experimental measurements. Continuing the Chapman-Enskog expansion to higher orders does not improve consid-

erably the Navier-Stokes results. A series of generalized hydrodynamic equations has been introduced in the literature along the years in order to improve the continuous description of small scale properties of fluid flow, as ultrasound propagation. We will describe some of the proposed approaches, and we will analyze the resulting equations of fluid motion. In some cases the dynamics becomes more complex than in the original fluid mechanical equations, creating unexpected effects and giving rise to new mathematical structures in the equations.

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Spatial Chaos of Traveling Waves Has a Unique Velocity

Bastien Fernandez
CNRS, France
(B. Luna, E. Ugalde)

This talk is about the complexity of stable waves in unidirectional bistable coupled map lattices. Numerical calculations reveal that, grouping traveling patterns into sets according to their velocity, at most one set has positive topological entropy for fixed parameters. The chaotic set's velocity has a mode-locking structure in parameter space and the entropy shows non-monotonous features. By using symbolic dynamics, we analytically determine velocity-dependent parameter domains of existence of pattern families with positive entropy. These estimates show excellent agreement with numerical results.

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Weak Stability Boundary and Invariant Manifolds

Marian Gidea
Northeastern Illinois University, USA
(Edward Belbruno; Francesco Topputo)

The concept of the weak stability boundary has been successfully used in the design of several fuel efficient space missions. We give a rigorous definition of the weak stability boundary in the context of the planar circular restricted three-body problem, and we provide a geometrical argument for the fact that, under some conditions, the points on the stable manifold of the Lyapunov orbits about the libration points L_1 and L_2 are points in the weak stability boundary. The geometrical method is based on the property of the invariant manifolds of Lyapunov orbits being separatrices of the energy manifold. We support our geometrical argument with numerical experiments.

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Fractal Traps and Fractional Dynamics

Pierre Inizan
IMCCE - Observatoire de Paris, France

Anomalous diffusion may arise in typical chaotic Hamiltonian systems. According to G. M. Zaslavsky's analysis, a description can be done by means of fractional kinetics equations. However, the dynamical origin of those fractional derivatives is still unclear. In this talk we study a general Hamiltonian dynamics restricted to a subset of the phase space. Starting from R. Hilfer's work, an expression for the average infinitesimal evolution of trajectories sets is given by using Poincaré recurrence times. The fractal traps within the phase space which are described by G. M. Zaslavsky are then taken into account, and it is shown that in this case, the derivative associated to this evolution may become fractional, with order equal to the transport exponent of the diffusion process.

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More Topology of Tank Stirring

Judy Kennedy
Lamar University, USA
(Barry Peratt)

We continue our study of stirring in a cylindrical tank, by considering a tank with a centrally located impeller, and eccentrically located recycle loop. We discuss work in progress, and investigate our idealized model both numerically and rigorously.

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Identification of Chaos in Systems with Non-Continuities and Memory by the 0-1 Test

Grzegorz Litak
Lublin University of Technology, Poland
(Ramupillai Gopal, A. Venkatesan)

We examine the nonlinear systems with non-continuities and memory. Instead of the standard Lyapunov exponent treatment a statistical '0-1' test based on the asymptotic properties of a non-harmonic Brownian motion chain has been successively applied to reveal the chaotic nature. In this test we calculated the control parameter K which is approaching asymptotically to 0 or 1 for regular and chaotic motions, respectively. The presented approach is independent on the integration procedure as we defined a characteristic distance between the points forming the time series used in the test separately.

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Suppression of Spiral-Wave Turbulent Dynamics in Excitable Media

Alexander Loskutov
Moscow State University, Russia
(Semen A. Vysotsky)

Suppression and control of the turbulent dynamics of excitable media, which appears through a set of coexisting spiral waves is a very important area of investigations in view of its application in cardiology. The dominating hypothesis in the current theory of excitable systems is that the fatal arrhythmias (fibrillations) occur due to the creation of numerous spiral waves or vortex structures (i.e., spatiotemporal chaos), in cardiac tissue.

The methods for suppression of such regimes by means of electrical shock are very inflexible and are not necessarily successful. Fortunately, recent investigations open new possibilities. A large-amplitude action is not necessary and can be weakened. Moreover, the turbulent regime in many excitable media can be stabilized by a weak periodic parametric or force actions applied to certain medium regions.

We consider 2D excitable media and show that the turbulent dynamics appearing owing to the decay of spiral waves can be basically suppressed using low-amplitude point stimulations. To overcame certain difficulties concerned with essential limitations of this approach, we propose to use several pacemakers and moving pacemakers. The method of retrieval of shapes, frequencies, amplitudes and duration of external stimuli is discussed. Applications of the obtained results to cardiology are considered.

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Estimation of the Unknown Variables and Parameters of Chaotic Systems by Means of Synchronization

Ines Mariño
Universidad Rey Juan Carlos, Madrid, Spain
(J. Miguez, R. Meucci)

We propose a Monte Carlo methodology for the joint estimation of unobserved dynamic variables and unknown static parameters in chaotic systems. The technique is sequential, i.e., it updates the variable and parameter estimates recursively as new observations become available, and, hence, suitable for online implementation. We demonstrate the validity of the method by way of two examples. In the first one, we tackle the estimation of all the dynamic variables and one unknown parameter of a five-dimensional nonlinear model using a time series of scalar observations experimentally collected from a chaotic CO_2 laser. In the second example, we address the estimation of the two dynamic variables and the phase parameter of a numerical model commonly employed to represent the dynamics of optoelectronic feedback loops designed for chaotic communications over fiber-optic links.

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Exploring a Spatio-Temporal Coupled System Using Symbolic Dynamics

Steffen Mihatsch
MPI Institute for Extraterrestrial Phys., Germany
(W. Bunk, R. Monetti)

We describe dynamical features of coupled systems using mappings of time series into the finite group of permutations. Group and information theoretic arguments allow us to define different categories of similarities between the resulting symbol series. Here we describe interdependencies between a pair of symbol sequences by starting with a product of groups, modding out a group operation and working on the quotient. This approach can be generalized to more than two simultaneous time series. The method is applied to a prototype system of chaotic oscillators, where local coupling induces a rich global synchronization behaviour. The elementary approach is efficient and applicable to a wide variety of systems.

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Recurrence Analysis of Transcription Symbols to Quantify Coupling between Time Series

Roberto Monetti
MPI Institute for Extraterrestrial Phys., Germany
(Wolfram Bunk, Thomas Aschenbrenner)

The characterization of recurrences in phase space provides a great deal of information about the dynamical properties of time series. Within this context, recurrence quantification analysis (RQA) is a well-established methodology. In the case of order patterns (symbols), a similar approach can be applied to a symbolic representation of a time series. RQA has been extended to more than one time series, where joint dynamical properties are characterized using co-recurrences. Here, using the concepts of transcription symbols and associated order classes introduced by the authors (Phys Rev. E 79, 046207 2009), we develop a novel approach to quantify co-recurrence patterns for studying synchronization. The method is applied to investigate synchronization features of a typical coupled chaotic model system and also to real world data.

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Topological Entropy for Local Processes

Piotr Oprocha
Universidad de Murcia, Spain
(Pawel Wilczynski)

The main aim of this talk is to present a definition of topological entropy for dynamics of processes described by nonautonomus differential equations. We also provide tools for estimation of the value of entropy of a process (its upper or lower bounds) in terms of Poincaré sections.

⟶ ∞ ⋄ ∞ ⟵

Boundary Crisis: Mind the Gaps!

Hinke Osinga
University of Bristol, England

Boundary crisis is a mechanism for destroying a chaotic attractor when one parameter is varied. In a two-parameter setting the locus of boundary crisis is associated with curves of homoclinic or heteroclinic bifurcations of periodic saddle points. It is known that this locus has non-differentiable points. We show here that the locus of boundary crisis is far more complicated than previously reported. It actually contains infinitely many gaps, corresponding to regions (of positive measure) where the attractor persists.

⟶ ∞ ⋄ ∞ ⟵

Lyapunov Vectors and Modes Splitting of Extended Systems

Ulrich Parlitz
University of Göttingen, Germany
(Pavel V. Kuptsov)

The dynamics in tangent space of some spatially extended dissipative systems may be decomposed in physical modes and isolated modes. The so-called physical modes are (most) relevant for the system dynamics and are decoupled from the remaining set of hyperbolically isolated modes representing degree of freedom corresponding to strongly decaying perturbations. This modes splitting is studied for the Ginzburg–Landau equation at different strength of the spatial coupling. We show that isolated modes coincide with eigenmodes of the homogeneous steady state of the system and that there is a local basis where the number of non-zero components of the state vector coincides with the number of the physical modes. In a system with finite number of degrees of freedom the strict modes splitting disappears at finite value of coupling and above this value a fussy modes splitting is observed.

⟶ ∞ ⋄ ∞ ⟵

The Interaction between Grazing Bifurcations and Boundary Crises in an Impacting System

Petri Piiroinen
National University of Ireland, Galway, Ireland
(Joanna F Mason)

It is well known that impacting systems can exhibit very rich dynamics, and attractors can be born or disappear in grazing bifurcations. This has lead to an increased research effort to understand the dynamics in the vicinity of such bifurcations. However, in this talk we will show how grazing bifurcations interact with boundary crises in an impacting model of gear rattle. In particular the focus will be on the differences and similarities with behaviour found in smooth systems. In addition, we will also show why codimension-2 bifurcations cause unwanted behaviour in our gear system. To analyse this we will use a classical approach in which limit cycles are found and their corresponding stable and unstable manifolds are explored. Together with basin of attraction calculations we can thus describe why some periodic and chaotic attractors suddenly disappear, which is not obvious from the analysis of grazing bifurcations on their own.

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Calming a Chaotic Network of Dendritic Neurons Using Weak Phase Control Strategy Coupled with a Single Control Pulse

Edita Sakyte
Kaunas University of Technology, Lithuania
(Minvydas Ragulskis, Miguel A.F. Sanjuan)

A chaotic network of dendritic neurons is analyzed in this paper. The complete network exhibits bistability under continuous weak stimulation: the oscillatory synchronized regime and the quite regime coexist. Simple control strategies based on a single control pulse can be effectively used to calm such dendritic networks. Complex nonlinear network dynamics is observed when it undergoes not only phase-dependent continuous weak stimulation but also when it is additionally driven by an external phase-independent stimulation. In that case basin boundaries between the synchronized and the quite regime become highly complex and fractal. Simple strategies based on a single control pulse do not work then because it is impossible to predict the dynamics of the network after the pulse. We propose a new network control method. Initially a weak phase control strategy is applied and fractal basin boundaries evolve into a deterministic manifold. Consequently a single control pulse is immediately applied and the network calms down.

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Basin Boundary Metamorphosis Produced by Parametric Harmonic Perturbations

Jesus Seoane
Universidad Rey Juan Carlos, Madrid, Spain

Basin boundary metamorphosis are characteristic in some kinds of chaotic dynamical systems. They take place when one parameter of the system is varied and it passes thorough certain critical value. Previous works shown that this phenomenon involves certain particular unstable orbits in the basin boundary which are accessible from inside one of the basins. In this talk, we show that parametric harmonic perturbations can produce basin boundary metamorphosis in chaotic dynamical systems. The main findings of our research are oriented in both, the study of the fractal dimension of the basin boundaries and in the study of the variation of the area of the basin once we change the value of one suitable parameter. The physical context of this work is related with the phenomenon of particles escaping from a potential well, which is illustrated by using as prototype model the Helmholtz oscillator. Finally, Melnikov analysis of the reported phenomenon has also been carried out. This is joint work with S. Zambrano, Inés P. Mariño, and Miguel A. F. Sanjuán (Spain).

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On the Numerical Integration of Variational Equations

Charalampos Skokos
MPI for the Physics of Complex Systems, Germany
(Ch. Skokos and E. Gerlach)

We investigate the efficiency of different numerical schemes for the integration of the variational equations of Hamiltonian systems whose kinetic energy is quadratic in the generalized momenta and whose potential is a function of the generalized positions. We consider numerical techniques based both on symplectic and non-symplectic integrations schemes, like the SBAB integrators and the DOP853 method respectively, and apply them to Hamiltonian systems with different numbers of degrees of freedom. The numerical verification of well-known properties of chaos indicators like the Lyapunov Characteristic Exponents (LCEs) and the Generalized Alignment Indices (GALIs) is used for characterizing the efficiency of the various integration schemes. In particular, we check whether the set of computed LCEs consists of pairs of values having opposite signs, with two of them being equal to zero, and if the time evolution of GALIs follows specific theoretically predicted laws. Since symplectic integrators approximate the solution of the Hamilton equations of motion by the repeated action of a symplectic map S, the tangent map of S can be used for the integration of the variational equations (tangent map method). We show that this method is quite fast and accurate, exhibiting a better numerical performance than other integration schemes.

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Bounds for Compact Invariant Sets of Nonlinear Systems Possessing First Integrals with Applications to Hamiltonian Systems

Konstantin Starkov
Instituto Politecnico Nacional, Mexico

We consider the problem of finding a domain which contains all compact invariant sets for a nonlinear system possessing a first integral. Finding such domain signifies its description by means of equations and inequalities depending on parameters of a system and is important for analysis of its long-time dynamics. As examples, we examine two systems: the generalized Yang-Mills system defined by Hamiltonian $H_1 = \frac{1}{2}(\alpha_1 x_1^2 + \alpha_2 x_2^2 + y_1^2 + y_2^2) + \frac{1}{4}ax_1^4 + \frac{1}{4}bx_2^4 + \frac{1}{2}cx_1^2x_2^2 + dx_1x_2^3 + ex_1^3x_2$, and the system defined by Hamiltonian

$$\begin{aligned} H_2 =& \frac{1}{2}(y_1^2 + y_2^2) + \frac{e}{2}(x_1^2 + x_2^2) + \frac{m^2}{2}(\frac{1}{x_1^2} + \frac{1}{x_2^2}) + \\ & A(x_1^6 + x_2^6) + B(x_1^4x_2^2 + x_1^2x_2^4) + C(x_2^4 - x_1^4), \end{aligned}$$

appeared in studies of dynamics of hydrogenic atoms; $e0; B \neq 0$. This work is a continuation of the study fulfilled in the paper by Starkov K. E. [2008], Physics Letters A 372, 6269-6272.

⟶ ∞ ⋄ ∞ ⟵

Piecewise Isometries and Mixing in Granular Tumblers

Rob Sturman
University of Leeds, England
(G. Juarez, J. Ottino, R. Lueptow, S. Wiggins)

The understanding of fluid mixing has been transformed as a result of dynamical systems theory, its fingerprint being “stretching and folding”, the hallmark of deterministic chaos. Here we introduce a different mixing mechanism – cutting and shuffling – that has a theoretical foundation in a relatively new area of mathematics called “piecewise isometries” (PWIs). The mixing properties of PWIs that arise from the study of the kinematics of flows are fundamentally different from the stretching and folding mechanism of the familiar chaotic advection; PWIs have zero Lyapunov exponents, zero topological entropy, and exhibit no hyperbolic behavior. As a physical example we show experimentally that

PWIs capture essential aspects of mixing of granular materials in a three-dimensional tumbler. Simulations based on a continuum model connect the PWI theory with experiments and demonstrate how the combination of cutting and shuffling (the PWI framework) with stretching due to advection contribute to increasing interfacial boundaries leading to effective mixing.

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Broken Egg Phenomenon Was Identified to the First Experimetal Data of Chaos

Yoshisuke Ueda
Waseda University, Japan

The appearance of the data I accidentally came upon during our hand-made analog computer (using vacuum-tubes) experiments on the 27th of November, 1961 was like a broken egg with jagged edges. The original sheet of data was now kept at Brookheaven National Laboratory (BNL Photography Division Negative No. 1-380-90). The motive of the experiment was to clarify what happend between frequency entrainment (pull-in) and quenching; these were well known two mechanisms of synchronization. All I did for a long period of time was to keep on pursuing my stubborn desire to understand this unsettling phenomenon. "What are the possible steady states of a nonlinear system?" – this has always been my question. During this period I had been scuffling with the precious heritages of G. D. Birchoff, N. Levinson and H. Poincaré. This talk will disclose the origin of experimental chaos research in the world.

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How to Find Rare Periodic and Rare Chaotic Attractors?

Mikhail Zakrzhevsky
Riga Technical University, Riga, Latvia

The problems of the global dynamics of nonlinear systems, described by nonlinear ODE or discrete equations, are under consideration. The paper is a continuation of our publications on rare attractors (RA) and a method of complete bifurcation groups (MCBG) recently proposed by the author [1-2]. In this paper some new periodic and chaotic RA have been obtained for different archetypical driven and autonomous nonlinear systems. Among them there are polynomial, piecewise-linear, impact, pendulum, escape, rotor and discrete systems. The main method for finding of rare attractors is the MCBG. This method allows to do complete bifurcation analysis for each separate group: stable and unstable nT-branches continuation without their break in bifurcation points. These branches are connected with protuberances born from bifurcation points. Unstable periodic solutions, during branch continuation, are corner-stone meaning in the MCBG. New bifurcation sub-groups, obtained by the MCBG, such as complex protuberances and unstable periodic infinitiums (UPI) allow predicting and finding new (unknown) regular and chaotic rare attractors (RA) in the system. The concept of rare attractors may be an essential part of the theory of rare dynamical phenomena and the modern theory of catastrophes. We suppose that new important applications of the method of complete bifurcation groups and rare attractors may be found in celestial mechanics, dynamics of natural and artificial satellites, in the three- and n-body problems, medicine and biology, dynamical nano-materials, nonlinear economics and in engineering.

[1] Zakrzhevsky, M. V., 2008, New Concepts of Nonlinear Dynamics: Complete Bifurcation Groups, Protuberances, Unstable Periodic Infinitiums and Rare Attractors, J. of Vibroengineering, Kaunas, Lithuania, Volume 10, Issue 4, pp.421-441.
[2] Zakrzhevsky M. V., 2009, Global Nonlinear Dynamics Based on the Method of Complete Bifurcation Groups and Rare Attractors, Proceedings of the ASME 2009 (IDETC/CIE 2009), San Diego, USA, 8 pages.

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Recent Developments in Partial Control of Chaotic Systems

Samuel Zambrano
Universidad Rey Juan Carlos, Madrid, Spain

We call partial control of chaos to a technique that allows one to keep the trajectories of a dynamical system close to a chaotic saddle, from which they typically diverge, in presence of environmental noise. The main advantage of this technique is that it makes this possible even in presence of a noise stronger than the applied control. This is due to a consequence of the existence of a horseshoe-like mapping in phase space: the presence of certain *safe sets* close to the saddle. Here we present new results concerning this control strategy. We show that it can also be applied if we have just access to one of the system's parameters, and we show that this approach is advantageous when compared with other approaches to achieve the same goal. We also provide a generalization of the concept of safe sets and we provide examples of application of our technique to chaotic scattering problems of interest in different fields of physics. Finally, we show a circuital implementation of our control technique.

This is an ongoing work with Juan Sabuco, Mattia T. Coccolo, Jesús Seoane, Alexandre Wagemakers and Miguel A. F. Sanjuán.

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Special Session 4: Mathematical Analysis of PDE Models in Materials Science

Toyohiko Aiki, Gifu University, Japan
Nobuyuki Kenmochi, Chiba University, Japan
Adrian Muntean, TU Eindhoven, The Netherlands

Introduction: This special session will address analytic aspects of nonlinear systems of PDEs arising particularly in modeling the behavior of materials such as shape-memory alloys, porous materials (e.g. concrete), biological membranes. The discussions will be around topics like well-posedness, specific limits (e.g. fast-reaction asymptotics and its connection with the large-time behavior), nonlinear transmission conditions, coupling mechanics equations with thermal and chemistry models.

On Global Existence of a Weak Solution to a Mathematical Model for a Valve Made of a Spring of a Shape Memory Alloy

Toyohiko Aiki
Gifu University, Japan

In our previous works we proposed a mathematical model for some device. The devise has a function to open a valve when the temperature of neighborhood of the device is a critical temperature, and consists of a shape memory alloy and a bias springs. It is very important to know the position of these two springs in order to control the valve. We have already established the well-posedness for our problem in case the temperature is given. However, in case the temperature is unknown, we had proved only local existence of a weak solution in time. In this talk we can provide the well-posedness of our model even if the temperature is unknown. The proof of the result is based on an approximation theory by using the standard molifier.

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A Transformation Approach for the Derivation of Boundary Conditions at Curved Porous-Liquid Interfaces

Sören Dobberschütz
University of Bremen, Germany
(Michael Böhm)

The interface conditions coupling a free fluid and a flow inside a porous medium are of great interest in various fields. A classical condition is the law of Beavers and Joseph, which has been proven rigorously using the theory of homogenization. However, to this day all works dealing with this type of law only considered the case of a planar interface.

We present an approach to deal with the case of a curved interface, by transforming a reference geometry with a straight interface to a domain with a curved one. It is assumed that the porous part in the reference geometry consists of a periodic array of a scaled reference cell and that the flow in the transformed geometry is governed by the stationary Stokes equation. Therefore one obtains a set of transformed differential equations in the reference configuration, for which boundary layer functions (in the sense of Jäger and Mikelić) are constructed. When the size of the cells tends to 0, interface conditions for the limit velocity between the porous and free fluid part can be derived: Whereas the velocity is continuous in normal direction, a jump appears in tangential direction, whose magnitude seems to be related to the slope of the interface.

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Obstacle Problem of Navier-Stokes Equations in Thermohydraulics

Takeshi Fukao
Kyoto University of Education, Japan

In this talk, the existence of the solution for the obstacle problem of Navier-Stokes equations in thermohydraulics is considered. It is the system between the heat equation and the Navier-Stokes equations with the temperature dependent constraint. The oneside pointwise constraint by the obstacle function depends on the unknown. They are formulated to the evolution equation governed by the subdifferential of the indicator function on the unknown convex constraint. By using the truncation to the nonlinear term, the existence for the obstacle problem of the Navier-Stokes equations is proved.

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Motion of Polygonal Curved Fronts by Crystalline Motion: V-Shaped Solutions and Eventual Monotonicity

Tetsuya Ishiwata
Shibaura Institute of Technology, Japan

In this talk, we shall discuss the behavior of polygonal curves in the plane which is governed by crystalline curvature flow with the bulk effect. In particular, we are interested in the dynamics of solution curves with two asymptotic lines. We investigate the deformation patterns of solution curves and also show eventual monotonicity of the shape, that is, solution curves become V-shaped in finite time.

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Fractal Dimension of a Global Attractor for a Phase Field Model with Constraints

Akio Ito
Kinki University, Japan

We consider a phase field model of Fix-Caginalp type with constraints. In this talk, we show the boundedness of a fractal dimension of the global attractor associated with the dynamical system of a phase field model with constraints. Actually, the difficulty to show the boundedness of fractal dimension comes from the non-smoothness of the dynamical system. To overcome it, we use the notion of l-trajectory.

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The Existence of Time Global Solutions for Tumor Invasion Models with Constraints

Risei Kano
Kinki University, Japan

I talk about the existence of time global solutions in tumor invasion models with constraint. In this talk, it is important that my non-linear evolution problem I have seen as quasi-variational inequalities, to prove the existence of solutions by abstract theory. The quasi-variational inequality is to find a function which satisfies a variational inequality in which the constraint depend upon the unknown function.

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Temperature Dependent Preisach Hysteresis Model

Jana Kopfová
Silesian University in Opava, Czech Republic
(Pavel Krejčí, Michela Eleuteri)

We use temperature-dependent hysteresis operators to describe the rate-independent solid-solid phase transitions in shape memory alloys, with a special attention to its thermodynamic consistency. So far, only few attempts to incorporate the hysteresis dissipation into the framework of general thermodynamics have been done. The main difficulty consists in defining correctly the state variables to account for the instantaneous memory state.

We suggest a thermodynamically consistent temperature dependent Preisach operator, based on a variational inequality of the following form:

$$\begin{aligned} &|\varepsilon(t) - \eta_r(t)| \le r \quad \forall t \in [t_1, t_2]; \\ &(\mu_1(\vartheta)\dot{\eta}_r(t) + \mu_2(\vartheta)(\eta_r(t) - \varepsilon(t))) \\ &\qquad \cdot (\varepsilon(t) - \eta_r(t) - y) \ge 0 \quad \text{a.e. } \forall |y| \le r\,, \end{aligned}$$

with given functions μ_1, μ_2 such that μ_1 vanishes for high temperatures (i.e. no memory) and μ_2 vanishes for low temperatures (i.e. the standard play operator). We propose the Preisach-like model

$$\sigma = E\varepsilon - \int_0^\infty g(r, \eta_r, \vartheta)\mathrm{d}r,$$

with a given constitutive function g, and show that the model is thermodynamically consistent and that the full system of dynamical balance equations is well posed.

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Reactive Transport Problems in Porous Media: Existence of Global Solutions and Practical Computations

Serge Kräutle
University Erlangen-Nürnberg, Germany

Reactive transport problems in porous media (such as soils) are described by a system consisting of PDEs (for the concentrations of the species dissolved in the groundwater) and/or of ODEs (for the concentrations of immobile species attached to the soil matrix). If chemical reactions are described by kinetic rate laws, then the PDEs and ODEs are coupled through these (nonlinear) rate terms. If local equilibrium is assumed for the reactions or for some of the reactions, then a coupling of the PDEs and ODEs is given by a set of (nonlinear) algebraic equations, leading to a differential-algebraic system of equations (DAE). When we assume mass action law for the kinetic and for the equilibrium reactions (which is a very common assumption in computational geosciences), then it is possible to prove global existence of solutions of the system, even if the order of the nonlinearities is high, by exploiting the structure of the reaction terms. Besides existence of solutions also questions of efficient numerical solution of the problem will be addressed. A reformulation of the system attained by a variable transformation and some substitutions, leading to a decoupling of some (linear) PDEs and a much smaller remaining nonlinear problem, is given. Such a reduction of the problem size often leads to a drastic reduction of the cpu time required.

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Periodic Solutions for Non-Isothermal Phase Transition Models

Kota Kumazaki
Nagoya Institute of Technology, Japan

In this talk, we consider periodic solutions for a phase transtion model of the Penrose-Fife type. This model is the system of two parabolic equations for the absolute temperature and the order parameter which represents the material state. For this model, we impose a non-homogeneous Dirichlet

boundary condition on the nonlinear heat flux and a time dependent constraint on the order parameter, and prove the existence and uniqueness of periodic solutions for our model.

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Critical Point Theory and Applications in Quantum Chemistry

Michael Melgaard
Dublin Institute of Technology, Ireland

(Mattias Enstedt)

We establish existence of infinitely many distinct solutions to the Hartree-Fock equations for Coulomb systems with quasi-relativistic kinetic energy $\sqrt{-\alpha^{-2}\Delta_{x_n}+\alpha^{-4}}-\alpha^{-2}$ for the n^{th} electron. Moreover, we prove existence of a ground state. The results are valid under the hypotheses that the total charge Z_{tot} of K nuclei is greater than $N-1$ and that Z_{tot} is smaller than some critical charge Z_{c}. The proofs are based on a new application of the Fang-Ghoussoub critical point approach to multiple solutions on a noncompact Riemannian manifold, in combination with density operator techniques.

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Homogenization of a Locally-Periodic Medium with Areas of Low and High Diffusivity

Adrian Muntean
TU Eindhoven, The Netherlands

We aim at understanding reaction and transport in those porous materials that present regions with both high and low diffusivities. For such scenarios, the transport becomes structured (here: *micro-macro*), while the reaction will be mainly hosted by the micro-structures of the low-diffusivity region. The geometry we have in mind include perforations (pores) arranged in a locally-periodic fashion. We choose a prototypical reaction-diffusion system (of minimal size), discuss its formal homogenization – the heterogenous medium being now assumed to be made of zones with circular areas of low diffusivity of x-varying sizes. We report on two type of results. On one hand, we rely on formal asymptotic homogenization, suitable use of the level sets of the oscillating perforations combined with a boundary unfolding technique to derive the upscaled model equations. On the other hand, we prove the weak solvability of the limit two-scale reaction-diffusion model. A special feature of our analysis is that most of the basic estimates (positivity, L^∞-bounds, uniqueness, energy inequality) are obtained in x-dependent Bochner spaces.

This is joint work with Tycho van Noorden (TU Eindhoven, NL).

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Optimal Control Problems for Quasi-Variational Inequalities and Its Numerical Approximation

Yusuke Murase
Hiroshima Shudo University, Japan

(Atsushi Kadoya, Nobuyuki Kenmochi)

We study about optical control problem of the following reaction diffusion equation which is quasi-variational inequality type.

$$\mathrm{RD}(f,0)\quad\begin{cases} u_t-\nu\triangle u+g(u)+\partial I_{K_0(L\vartheta)}(u)\ni 0\\ \qquad \text{in } Q:=(0,T)\times(0,1)\\ u_x=0 \qquad \text{on } \Sigma:=(0,T)\times\{0,1\}\\ \vartheta_t-\kappa\triangle\vartheta+h(x,\vartheta,u)=f \qquad \text{in } Q\\ \vartheta=0 \qquad \text{on } \Sigma\\ u(0,\cdot)=u_0,\vartheta(0,\cdot)=\vartheta_0 \qquad \text{in } (0,1)\end{cases}$$

In this talk, we discuss about existence of optimal control of $\mathrm{RD}(f,0)$, its approximated parabolic problem, time-discrete elliptic problem, and relationship between these problems. These problems are very important not only for analytical results, but also from numerical point of view.

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Modelling and Analysis of the Influence of Chemical Factors on the Mechnical Properties of Cellular Tissue

Maria Neuss-Radu
University of Heidelberg, Germany

(Willi Jäger, Andro Mikelić)

Experimental research is providing increasing information on biophysical and biochemical processes in cells and tissue. This information on cellular level has to be included in mathematical modelling of the dynamics of biological tissue. Describing flow, transport and reactions of substances in and their interactions with mechanics of the solid structures on a cellular level leads to a coupled system of nonlinear partial differential equations in complex geometric structures. Using experimental information, the relevant parameters of this system have to be determined in order to pass to a macroscopic scale limit. Starting from the microscopic model at cell level, we derive a macroscopic modell by performing the scale limit. It is important that the resulting system of equations including micro- and macro-variables are linking the different levels in a computable form. Thus, coupling information on the processes in the micro-scale with macroscopic properties we provide theoretical, but quantitative answers to questions posed by physiologists studying

experimentally the transport through tissues and cell layers.

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Sixth Order Cahn-Hilliard Type Equation

Irena Pawłow
PAN and Military Univ. of Warsaw, Poland
(Wojciech M. Zajączkowski)

A three-dimensional sixth order Cahn-Hilliard type equation for a conserved order parameter is considered. The equation arises as a phenomenological model for the kinetics of oil-water-surfactant mixtures. The model is based on a second-order Landau-Ginzburg free energy for a single scalar order parameter describing the local concentration difference between oil and water. The model has been proposed and examined in a line of papers by G. Gompper and associates, see e.g. [1]. In the present paper the well-posedness of an initial-boundary-value problem for such Cahn-Hilliard type equation is studied. The existence and uniqueness of a large time regular solution is proved by applying recent solvability results for parabolic problems in Sobolev spaces with a mixed norm.

[1] G. Gompper. M. Kraus, Ginzburg-Landau theory of ternary amphiphilic systems. I. Gaussian interface fluctuations, Physical Review E 47, No 6 (1993), 4289-4300.

⟶ ∞ ◇ ∞ ⟵

A Reaction/Diffusion System with a Very Fast Reaction

Thomas Seidman
University of Maryland, Baltimore County, USA

A variety of settings involve reaction/diffusion systems in a thin film or membrane, e.g., in the 'film' surrounding an individual bubble in a bubble reactor. With suitable scaling, most reactions are negligible but it remains necessary to consider some few fast reactions.

Here, we consider a model problem involving a compound reaction [2A+B→*], consisting of an extremely fast reaction [A+B→C] (producing the intermediate compound C) coupled with a somewhat slower reaction [A+C→*]. We are interested in what happens in the limit as this fast reaction rate goes to infinity. For the steady state problem in one space dimension, it has been known that the solution goes to a well-defined limit and we now see how that result can be obtained in the time-dependent context in n spatial dimensions.

[This is joint work with A. Muntean (Eindhoven).]

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Regularity Criterion Via Pressure in Lorentz Spaces to the MHD Equations

Tomoyuki Suzuki
Kanagawa University, Japan

We consider the regularity problem of weak solutions to the three-dimensional MHD equations and give a sufficient condition in terms of the pressure. The condition is that the pressure belongs to a scaling invariant Lorentz space with the corresponding small norm.

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Compressible Phase Change Flows and Their Sharp Interface Limit

Gabriele Witterstein
TU München, Germany

In this talk we present diffusive interface models describing flows which undergo a phase change, for example caused by biochemical reactions in a material flow. In mathematical terms, the equations consist of the compressible Navier-Stokes system coupled with an Allen-Cahn equation, and are based on an energetic variational formulation. We investigate density-dependent viscosity and a density-dependent transition layer of thickness $\delta(\rho)$. For this model we establish the existence of a unique weak global solution in one dimension.

Further we consider various diffusive interface models. The settings adopt several applications by choosing different definitions of the double well potential and the relaxation parameters. We use the method of matched asymptotic expansion to determine the sharp interface limit. As it turns out, there are completely different interface models. In one case, the characteristics don't cross the interface, and in another case, they can cross, and phase transition takes place. Further concerning the velocity at the interface we consider cases in which jumps arise.

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Enzyme Kinetics in the Presence of Diffusion

Antonios Zagaris
University of Twente, Netherlands
(L. V. Kalachev, H. G. Kaper, T. J. Kaper, Nikola Popović)

The Michaelis-Menten-Henri (MMH) mechanism is a paradigm reaction mechanism in biochemistry. In its simplest form, it involves a substrate that reacts reversibly with an enzyme, forming a complex which is transformed irreversibly into a product and the enzyme. Given these basic kinetics, a dimen-

sion reduction has traditionally been achieved in two steps: by using conservation relations to reduce the number of species and by exploiting the inherent fast–slow structure of the resulting equations. In the present article, we investigate how the dynamics change if the species are additionally allowed to diffuse. We study the two extreme regimes of large diffusivities and of small diffusivities, as well as an intermediate regime in which the time scale of diffusion is comparable to that of the fast reaction kinetics. We show that reduction is possible in each of these regimes, with the nature of the reduction being regime dependent. Our analysis relies on the classical method of matched asymptotic expansions to derive approximations for the solutions that are uniformly valid in space and time.

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Special Session 5: Qualitative Behavior of Dissipative Dynamical Systems

Alain Miranville, Universite de Poitiers, France
Maurizio Grasselli, Politecnico di Milano, Italy

Introduction: The aim of the special session is to address important qualitative features of dissipative evolution equations arising in mechanics, physics, biology, ... This session will cover topics such as well-posedness and regularity, stability and asymptotic behavior (considered here in a broad sense).

Qualitative Behavior of a Diffuse Interface Model with Singular Potentials

Helmut Abels
University Regensburg, Germany

In this presentation we discuss a Navier-Stokes/Cahn-Hilliard system, which arises as a “diffuse interface model” for the flow of two viscous incompressible Newtonian fluids of the same density in a bounded domain. The fluids are assumed to be macroscopically immiscible, but a partial mixing in a small interfacial region is assumed in the model. For this system we prove existence of weak solutions in two and three space dimensions for a class of physical relevant and singular free energy densities. Moreover, we present some results on regularity and uniqueness of weak solutions and show that any weak solution converges as $t \to \infty$ to a solution of the stationary system. Finally, we consider the so-called “double obstacle limit”, when the binary mixture is much below the critical temperature, where phase separation occurs.

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Long-Term Dynamics of Some Composite Systems of Partial Differential Equations

Francesca Bucci
Università degli Studi di Firenze, Italy
(Daniel Toundykov (Nebraska-Lincoln))

The talk will report on the study of the long-time behaviour of solutions to some composite systems of evolutionary Partial Differential Equations (PDE), comprising hyperbolic dynamics that may feature localised nonlinear dissipation and ‘critical’ nonlinear forces. Issues of concern are, besides existence of a global attractor (for the corresponding dynamical system) having finite fractal dimension, the description of the rates of convergence to equilibria, and the existence of exponential attractors.

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A Variational Formulation for the Caginalp System with Singular Potentials and Dynamic Boundary Conditions

Laurence Cherfils
University of La Rochelle, France
(S. Gatti, Italy)

This talk deals with the Caginalp system, which models melting-solidification phenomena in certain classes of materials. When this system is endowed with a singular potential and dynamic boundary conditions, the existence of solutions in the usual (dstribution) sense is not assured. Therefore, following a recent work of A. Miranville and S. Zelik on the Cahn–Hilliard equation, we define a notion of a variational solution, constructed as the limit of a sequence of regular solutions and satisfying a variational system of inequalities. Then, we give sufficient conditions on the potential or on the boundary conditions in order to ensure that these variational solutions are in fact usual ones. Finally we prove the existence of finite dimensional attractors.

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Some Recent Applications of the Lojasiewicz-Simon Gradient Inequality

Ralph Chill
Universite de Metz, France

(Eva Fasangova, Reiner Schätzle, Alain Haraux, Mohamed Ali Jendoubi)

In the last about 12 years, the Lojasiewicz-Simon gradient inequality has shown its importance in proving stabilization and decay estimates for solutions of gradient or gradient-like systems. We give a personal overview over some variants and recent applications of the Lojasiewicz-Simon gradient inequality, including global existence of solutions of gradient systems and a result about the formation of singularities of the Willmore flow.

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Solutions to Phase Segregation and Diffusion Problems

Pierluigi Colli
University of Pavia, Italy

Models of phase segregation and diffusion of Allen-Cahn and Cahn-Hilliard types will be considered. Systems of two differential equations, interpreted as balances of microforces and microenergy, characterize such models that have been proposed by Paolo Podio-Guidugli. The two unknowns entering the equations are the order parameter and the chemical potential. The analysis of these systems is part of a research collaboration with G. Gilardi, P. Podio-Guidugli and J. Sprekels. Some results will be reported in my presentation.

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On the Memory Relaxation of the Cahn-Hilliard Equation

Monica Conti
Politecnico di Milano, Italy

We discuss some recent results concerning with the memory relaxation of the Cahn-Hilliard equation, covering the well known hyperbolic version of the model.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Generalized Korteweg-de Vries-Burgers Equation

Tomasz Dlotko
Silesian University, Katowice, Poland

Cauchy problem in $\mathbb{R}$ for the *generalized Korteweg-de Vries-Burgers equation* has the form:

$$\begin{aligned} &u_t + \delta u_{xxx} + g(u)_x - \nu u_{xx} = f(u,x), \\ &\qquad t>0,\ x \in \mathbb{R}, \\ &u(0,x) = u_0(x), \end{aligned} \tag{1}$$

$(\delta, \nu > 0)$ where $f : \mathbb{R}\times\mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ are given functions. Local and global solvability of (1) in the phase spaces $H^2(\mathbb{R})$ and $H^1(\mathbb{R})$ will be discussed. The solution of (1) is obtained as a viscosity limit of solutions to the corresponding parabolic regularizations (we add first to (1) an extra term εu_{xxxx}, $\varepsilon > 0$, next let $\varepsilon \to 0^+$). Under a slightly stronger regularity assumption on f we construct a $(H^1(\mathbb{R}), H^3(\mathbb{R}))$ global attractor for the semigroup $\{S(t)\}_{t\geq 0}$ generated by (1) on the phase space $H^1(\mathbb{R})$. We also show that the family of the global attractors $\{\mathcal{A}_\varepsilon\}_{\varepsilon\in[0,1]}$ corresponding to parabolic regularizations is *upper semicontinuous* at $\varepsilon = 0$. Let us mention that, with our assumptions, the set of the stationary solutions to (1) is not trivial; it contains more than one element. Consequently, the global attractor has a complicated structure.

[1] T. Dlotko, *Generalized Korteweg-de Vries-Burgers equation in $H^2(\mathbb{R})$*, submitted,
[2] T. Dlotko, Chunyou Sun, *Asymptotic behavior of the generalized Korteweg-de Vries-Burgers equation*, Journal of Evolution Equations, DOI: 10.1007/s00028-010-0062-2.

$\longrightarrow \infty \diamond \infty \longleftarrow$

On the Asymptotic Behavior of Solutions for the Cahn-Hilliard and the Convective Cahn-Hilliard Equations

Alp Eden
Bogazici University, Turkey

(Varga Kalantarov, Sergey Zelik)

We will report on some of our recent results on the Cahn-Hilliard equation on an unbounded domain and on the convective Cahn-Hilliard equation on a bounded domain in n-dimensions.

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Long-Time Behavior of Driven Fluid Systems

Eduard Feireisl
Academy of Sciences, Prague, Czech Republic

We discuss several issues concerning the long-time behavior of solutions to systems of partial differential equations arising in fluid dynamics. In particular, we concentrate on the existence of time-periodic or almost-periodic solutions to periodically driven energetically insulated systems.

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Continuous Families of Exponential Attractors for Singularly Perturbed Equations with Memory

Stefania Gatti
Università di Modena e Reggio Emilia, Italy
(Alain Miranville, Vittorino Pata, Sergey Zelik)

This talk is devoted to an abstract result obtained in a joint work with Alain Miranville, Vittorino Pata and Sergey Zelik, in press on the Proceedings of the Royal Society of Edinburgh, Section A. We establish, for a family of semigroups $S_\varepsilon(t) : \mathcal{H}_\varepsilon \to \mathcal{H}_\varepsilon$ depending on a perturbation parameter $\varepsilon \in [0,1]$, where the perturbation is allowed to become singular at $\varepsilon = 0$, a general theorem on the existence of exponential attractors $\mathcal{E}_\varepsilon$ satisfying a suitable Hölder continuity property with respect to the symmetric Hausdorff distance at every $\varepsilon \in [0,1]$. The result is applied to the abstract evolution equations with memory

$$\partial_t x(t) + \int_0^\infty k_\varepsilon(s) B_0(x(t-s))\,\mathrm{d}s + B_1(x(t)) = 0, \quad \varepsilon \in (0,1],$$

where $k_\varepsilon(s) = (1/\varepsilon)k(s/\varepsilon)$ is the rescaling of a convex summable kernel k with unit mass. Such a family can be viewed as a memory perturbation of the equation

$$\partial_t x(t) + B_0(x(t)) + B_1(x(t)) = 0,$$

formally obtained in the singular limit $\varepsilon \to 0$.

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An Integro-Differential Equation for the Alignment of Fishes

Danielle Hilhorst
University of Paris-Sud 11, France
(Khashayar Pakdaman, Delphine Salort)

We consider a nonlocal parabolic equation with memory together with periodic boundary conditions. After having proved the existence and uniqueness of the solution, we show the existence of a global attractor.

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Lower Bounds for the Spectrum of the Stokes and Laplace Operators

Alexei Ilyin
Keldysh Inst. of Appl. Math., Moscow, Russia

We prove Berezin–Li–Yau-type lower bounds with additional term for the eigenvalues of the Stokes operator and improve the previously known estimates for the Laplace operator. Generalizations to higher-order operators and applications to the attractors of the Navier–Stokes equations are given.

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Parabolic Variational Inequalities with Environmental Constraints

Nobuyuki Kenmochi
Chiba University, Japan

We consider a nonlinear system of the form

$$w_t - \nu\Delta w + g(x,\vartheta,w) + \partial I_{K(E)}(w) \ni 0 \quad \text{in } Q, \quad (1)$$

$$\vartheta_t - \kappa\Delta\vartheta + h(x,\vartheta,w,w_t) = f \quad \text{in } Q, \quad (2)$$

$$E \in \mathcal{E}(\vartheta,w) \quad \text{in } Q. \quad (3)$$

Here $w := (u,v)$ is a vector function on $Q := \Omega\times(0,T)$ and ϑ is a scalar function on Q, where Ω is a bounded domain in R^3 with smooth boundary and T is a finite positive number; ν, κ are positive constants; g and h are Lipschitz continuous functions with respect to all the arguments; $K(E)$, $E \in R$, is a bounded compact convex set in R^2, $I_{K(E)}$ is the indicator function of $K(E)$ and $\partial I_{K(E)}(\cdot)$ is its subdifferential. One of the characteristics of our system (1)-(3) is found in (3), where $E := E(x,t)$ is called the environmental index and is given by (3) via a non-local operator $\mathcal{E}(\vartheta,w)(\cdot)$. The system (1)-(3) is a sort of quasi-variational problem, since the constraint set $K(E)$ for w depends on the unknowns ϑ and w.

In this talk we discuss the existence question of the initial-boundary problem for (1)-(3) and some delicate points in the construction of solutions.

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Smooth Attractors for Kirchhoff Bussinesque Flows with Supercritical Nonlinearity

Irena Lasiecka
University of Virginia, USA
(Igor Chueshov)

We consider Kirchhoff-Boussinesque models defined on 2-D bounded domains and subjected to supercritical nonlinear forcing. The distinctive features of the system are: (i) the model considered is hyperbolic like, (ii) the nonlinearity is above critical Sobolev's exponent, (iii) the system lacks gradient structure due to the presence of Boussinesque source.

The resuts presented include: (i) existence of nonlinear semigroup corresponding to finite energy solutions, (ii) existence and finite dimensionality of global attractors (ii) C^∞ smoothness of strong

atractors. This latter property is rather unexpected in hyperbolic flows with above critical nonlinear forcing. The proofs are based on methods recently reported in I. Chueshov and I. Lasiecka, Long time behavior of second order evolutions, Memoires AMS, vol 917, 2008.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Approximating Infinite Delay with Finite Delay

Elsa Marchini
Politecnico di Milano, Italy
(M. Conti, V. Pata)

We consider a linear integro-differential equation of hyperbolic type in a Hilbert space, which is an abstract version of the equation

$$u_{tt}(t) - \Delta\Big[u(t) + \int_0^\infty \mu(s)[u(t) - u(t-s)]ds\Big] = 0$$

modelling linear viscoelasticity. The longterm memory appearing in this equation raised some criticism in the scientific community from the very beginning, due to the conceptual difficulty in accepting the idea of a past history defined on an infinite time interval (when even the age of the universe is finite). Aiming to overcome such a philosophical objection, we study equations with finite delay kernels μ_ε suitably approximating μ and we provide some comparison results which guarantee the closeness of the corresponding solutions.

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Exponential Stability in Linear Viscoelasticity with Almost Flat Memory Kernels

Vittorino Pata
Politecnico di Milano, Italy

We deal with the solution semigroup in the history space framework arising from an abstract version of the boundary value problem with memory

$$\partial_{tt}u(t) - \Delta\Big[u(t) + \int_0^\infty \mu(s)[u(t) - u(t-s)]ds\Big] = 0,$$
$$u(t)_{|\partial\Omega} = 0,$$

modelling linear viscoelasticity. The exponential stability of the semigroup is discussed, establishing a necessary and sufficient condition involving the memory kernel μ.

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Renormalization Group Method for the Three Dimensional Primitive Equations

Madalina Petcu
Universite de Poitiers, France
(Roger Temam)

In this article we study the limit, as the Rossby number goes to zero, of the three dimensional primitive equations of the ocean and of the atmosphere. We study the averaging of a penalization problem displaying oscillations generated by an antisymmetric operator and the presence of two time scales.

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Regularity and Long Time Behavior of a Nonlocal Phase Separation System

Hana Petzeltova
Academy of Sciences, Prague, Czech Republic

We will discuss regularity an convergence to equilibria of solutions to a nonlocal two-phase separation system

$$u_t = \nabla(\mu\nabla v), \quad v = f'(u) + \Psi(u),$$

where f is a convex function and the operator Ψ represents the nonlocal interactive term.

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A Numerical Analysis of the Cahn-Hilliard Equation with Inertial Term

Morgan Pierre
University of Poitiers, France
(Maurizio Grasselli)

P. Galenko et al. proposed a Cahn-Hilliard model with inertial term in order to model spinodal decomposition caused by deep supercooling in certain glasses. Here we analyze a finite element space semidiscretization of their model based on a scheme introduced by Elliott, French and Milner for the Cahn-Hilliard equation. We prove the convergence of the semidiscrete solution to the continuous solution; we obtain optimal a priori error estimates in energy norm and related norms; we also show that the semidiscrete solution converges to an equilibrium as time goes to infinity, thanks to the Łojasiewicz gradient inequality. Some finite difference time discretizations of the PDE are also studied in the same spirit.

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Orthogonal Sequences and Regularity of Embeddings into Finite-Dimensional Spaces

Eleonora Pinto de Moura
University of Warwick, England
(James C. Robinson)

I shall begin by introducing the 'Assouad dimension' $\dim_A(X)$ of a set X, a notion which has proved useful in the study of embeddings of finite-dimensional sets into Euclidean spaces. In particular, Robinson (2010) showed that if X is a subset of a Hilbert space with $\dim_A(X-X) < d < N$, then X can be embedded into some $\mathbb{R}^N$ using a linear map L whose inverse is Lipschitz continuous with a logarithmic correction term. More precisely, there exist $\delta > 0$ and $C > 0$ such that

$$\frac{1}{C}\frac{\|x-y\|}{|\log\|x-y\||^{\gamma}} \le |Lx - Ly| \le C\|x-y\|,$$

for $x, y \in X$ with $\|x-y\| \le \delta$, provided

$$\gamma > \frac{N+2}{2(N-d)}.$$

In this talk I will discuss this result and its generalisation to the Banach space case. Finally I will show that the bound on the exponent γ is sharp as $N \to \infty$.

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Long-Time Behavior of Cahn-Hilliard Equation Coupled with a Three-Dimensional Non-Newtonian Fluid

Dalibor Prazak
Charles University, Prague, Czech Republic
(M. Grasselli)

We consider a class of models which describes the behavior of binary mixture of incompressible non-Newtonian viscous fluids by means of the diffuse interface approach. These models lead to the formulation of Navier-Stokes type equations characterized by a nonlinear stress-strain law and suitably coupled with a convective Cahn-Hilliard equation for the order parameter. We analyze the corresponding dynamical systems proving the existence of global and exponential attractors.

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Attractors for Reaction-Diffusion Systems in Unbounded Domains

Giulio Schimperna
University of Pavia, Italy
(Maurizio Grasselli, Dalibor Pražák)

In this talk we consider a nonlinear reaction-diffusion system settled on the whole euclidean space. We prove the well-posedness of the corresponding Cauchy problem in a general functional setting, namely, when the initial datum is uniformly locally bounded in L^2. Then we adapt the short trajectory method to establish the existence of the global attractor and, in the three-dimensional case, we find an upper bound of its Kolmogorov ε-entropy.

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Finite Dimensional Reduction for a Reaction Diffusion Problem with Obstacle Potential

Antonio Segatti
Universitá di Pavia, Italy
(Sergey Zelik)

A reaction-diffusion problem with an obstacle potential is considered in a bounded domain of $\mathbb{R}^N$. Under the assumption that the obstacle K is a closed convex and bounded subset of $\mathbb{R}^n$ with smooth boundary or it is a closed n-dimensional simplex, we prove that the long-time behavior of the solution semigroup associated with this problem can be described in terms of an exponential attractor. In particular, the latter means that the fractal dimension of the associated global attractor is also finite.

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3D-2D Asymptotic Observation for Minimization Problems Associated with Degenerative Energy-Coefficients

Ken Shirakawa
Kobe University, Japan
(Rejeb Hadiji, Univ. Paris 12, France)

This is a jointwork with Prof. Rejeb Hadiji, University of Paris 12, France.

Let $0 < h < 1$, let $S \subset \mathbb{R}^2$ be a bounded 2D-domain with a Lipschitz boundary, and let $\Omega^{(h)} \subset \mathbb{R}^3$ be a cylindrical 3D-domain, given by $\Omega^{(h)} := S \times (0, h)$.

In this talk, a class $\{\mathcal{E}^{(h)} \,|\, 0 < h < 1\}$ of functionals $\mathcal{E}^{(h)}$ on the spaces $L^2(\Omega^{(h)}; \mathbb{R}^3)$ of three-variable functions, is considered. Here, for any $0 < h < 1$, $\mathcal{E}^{(h)}$ is supposed to be a possible free-energy in a magnetic thin film, shaped by $\Omega^{(h)}$, and its principal part consists of the lower semi-continuous envelopment of the following nonconvex functional:

$$\varphi \in C(\Omega^{(h)}; \mathbb{S}^2) \cap C^1(\Omega^{(h)}; \mathbb{R}^3) \mapsto \fint_{\Omega^{(h)}} \alpha(x)^2 |\nabla\varphi(x)|^2 \, dx;$$

with a given Lipschitz function $\alpha : \mathbb{R}^3 \longrightarrow \mathbb{R}$.

The main focus in this talk is on the limiting observation for the minimization problems of $\mathcal{E}^{(h)}$, as $h \searrow 0$, and this limiting problem is discussed under

the situation that the effective domain of $\mathcal{E}^{(h)}$ may vary with respect to $0 < h < 1$. Then, such variable situation is involved in the degenerative situation of the energy-coefficient $\alpha^2 \in C(\mathbb{R}^3)$.

As a consequence, a certain limiting energy, on the space $L^2(S; \mathbb{R}^3)$ of binary functions, will be presented with some continuous dependence of the minimizers.

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Phase Transition Solutions for the Cahn-Hilliard Equation with a Degenerate Diffusion

Peter Takac
Universität Rostock, Germany
(Pavel Drabek, Raul F. Manasevich)

We show the existence of phase transition solutions in space dimension one and the nonexistence of such solutions in a higher space dimension by Pohozhaev's identity. We use the p-Laplace operator to model degenerate or singular diffusion. We prove new sharp regularity results in order to derive Pohozhaev's identity. Our results are much different from the case of regular diffusion ($p = 2$), especially in the case of space dimension one when the set of stationary points forms a k-dimensional manifold.

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On Quasilinear Parabolic Evolution Equations in Weighted L_p-Spaces

Mathias Wilke
University of Halle-Wittenberg, Germany
(M. Köhne, J. Prüss)

In this talk we present a geometric theory for quasilinear parabolic evolution equations in weighted L_p-spaces (w.r.t. time). We prove existence and uniqueness of solutions as well as the continuous dependence on the initial data. Moreover, we make use of a regularization effect to study relative compactness of bounded orbits in the natural phase space. This in turn allows us to prove some results concerning the long-time behaviour of the solutions.

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Strong Trajectory Attractors for the Dissipative Euler Equations

Sergey Zelik
University of Surrey, Guildford UK, England

We consider the dissipative 2D Euler equations with periodic boundary conditions in the standard energy phase space where the gradient of the velocity field is square integrable. We prove the uniqueness and the energy equality on the properly defined trajectory attractor and based on that fact establish that the attraction to that attractor holds in a strong topology.

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Special Session 6: Continuous and Discrete Dynamical Systems in Physics

Atanas Stefanov, University Of Kansas, USA
Dmitry Pelinovsky, McMaster University, Canada

Introduction: The special session will highlight recent advances in mathematical analysis of partial differential equations and differential difference equations with applications to the physics of Bose-Einstein condensates, DNA modeling, and other physical contexts. Analysis of the continuous and discrete nonlinear Schrödinger equations is the main subject of recent studies with a particular emphasis on existence and dynamics of localized modes in these equations. The special session will bring together both specialists in mathematical analysis of nonlinear waves and applied researchers with interests in numerical simulations.

A Variational Theory for Point Defects in Patterns

Nicholas Ercolani
University of Arizona, USA
(S. C. Venkataramani)

We will discuss a rigorous scaling law for minimizers in a natural version of the regularized Cross-Newell model for pattern formation far from threshold. These energy-minimizing solutions support defects having the same character as what is seen in controlled experimental studies of Rayleigh-Benard

convection and numerical simulations of the microscopic equations describing these systems.

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Mean-Field Approxiomation of Transfer Operators in High-Dimensional Conformation Dynamics

Gero Friesecke
TU Munich, Germany
(Oliver Junge, Peter Koltai)

Conformation transitions of molecules reflect the global spatial/temporal behaviour of the system, and occur at much slower timescales compared to the elementary frequencies of the system. They are therefore difficult territory for trajectory based methods. Since the 1990s an alternative approach based on eigenfunctions of transfer operators has been developed, notably by Deuflhard, Schuette and coworkers. In the latter approach, the number of computational degrees of freedom of a direct discretization scales exponentially in the number of atoms. To overcome this problem we have developed a mean field method based on a statistical independence ansatz for the eigenfunctions of the operator related to a partitioning into subsystems. This strategy is reminiscent of the passage from the many-particle Schrödinger equation to Hartree-Fock theory in electronic structure computations. A key theoretical property of the mean field model is that it resolves weak subsystem interactions correctly to leading order in the coupling constant.

Numerical tests for small molecules show excellent qualitative agreement between mean field and exact model, at greatly reduced computational cost.

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Rigorous Justification of Effective Dynamics in Lattices

Johannes Giannoulis
TU München, Germany

We present several results concerning the justification of amplitude equations for macroscopically modulated plane waves in atomic chains and lattices. We discuss the possible generalizations but also the limitations of the presented method with respect to effective dynamics where the macroscopic object is not a modulation or the leading order terms of the interaction potentials are not harmonic.

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Existence and Conditional Energetic Stability of Three-Dimensional Fully Localised Solitary Gravity-Capillary Water Waves

Mark Groves
Universität des Saarlandes, Germany

A solitary wave of the type advertised in the title is a critical point of the Hamiltonian, which is given in dimensionless coordinates by

$$H(\eta,\xi)=\int_{\mathbb{R}^2}\Big\{\frac{1}{2}\xi G(\eta)\xi+\frac{1}{2}\eta^2+\beta\sqrt{1+\eta_x^2+\eta_z^2}-\beta\Big\},$$

subject to the constraint that the impulse

$$I(\eta,\xi)=\int_{\mathbb{R}^2}\eta_x\xi$$

is fixed. Here $\eta(x,z)$ is the free-surface elevation, ξ is the trace of the velocity potential on the free surface, $G(\eta)$ is a Dirichlet-Neumann operator and $\beta>1/3$ is the Bond number.

In this talk I show that there exists a minimiser of H subject to the constraint $I=2\mu$, where μ is a small positive number. The existence of a solitary wave is thus assured, and since H and I are both conserved quantities its stability follows by a standard argument. ‘Stability’ must however be understood in a qualified sense due to the lack of a global well-posedness theory for three-dimensional water waves.

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Travelling Pulses for the Discrete Fitzhugh-Nagumo System

Hermen Jan Hupkes
Brown University, USA
(B. Sandstede)

The existence of fast travelling pulses of the discrete FitzHugh-Nagumo equation is obtained in the weak-recovery regime. This result extends to the spatially discrete setting the well-known theorem that states that the FitzHugh-Nagumo PDE exhibits a branch of fast waves that bifurcates from a singular pulse solution. The key technical result that allows for the extension to the discrete case is the Exchange Lemma that we establish for functional differential equations of mixed type.

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On the Existence of Steady Quasipatterns of the Swift-Hohenberg Equation and of the Bénard-Rayleigh Convection Problem

Gerard Iooss
IUF, Universite de Nice, France

This work is made in collaboration with A. M.

Rucklidge (Leeds). Quasipatterns (two-dimensional patterns which are quasiperiodic in any spatial direction) remain one of the outstanding problems of pattern formation. As with problems involving quasiperiodicity, there is a small divisor problem. For any even order $Q \geq 8$, we consider SQ-fold quasipattern solutions of the steady Swift–Hohenberg equation (notice that for $Q \leq 6$ the pattern is periodic). We prove that a formal solution, given by a divergent series, may be used to build a smooth quasiperiodic function which is an approximate solution of the pattern-forming PDE up to an exponentially small error. Choosing such a function as an initial data, the solution of the evolution problem stays exponentially close to this function for a very long time. Considering now the hydrodynamic Bénard - Rayleigh convection problem, we are able to prove the same type of result for the bifurcating steady quasipattern convective regime.

[1] G. Iooss, A. M. Rucklidge. *On the existence of quasipattern solutions of the Swift-Hohenberg equation.* J. Nonlinear Science (to appear).
[2] G. Iooss. *Quasipatterns in steady Bénard-Rayleigh convection.* Volume in honor of 75th anniversary of the birth of V. Yudovich. Zhukov ed. (to appear).

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Continuation of Breathers from Infinity and Application to Dna Breathing

Guillaume James
Grenoble University and CNRS, France

(S. Cuesta-Lopez, C. Ferreira, A. Levitt, D. Pelinovsky, M. Peyrard)

The base pairs that encode the genetic information in DNA show large amplitude localized excitations called DNA breathing. This phenomenon can be modeled using different types of nonlinear lattices that sustain spatially localized and time-periodic oscillations called discrete breathers. In this talk we discuss how to prove the existence of such solutions for weakly-coupled oscillators, in the large amplitude and low frequency limit. This requires a nontrivial adaptation of the anticontinuum limit technique involving continuation from infinity.

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Nonlinear Waves in Granular Crystals

Panayotis Kevrekidis
University of Massachusetts, Amherst, USA

In this talk, we intend to give an overview of our recent theoretical, numerical and experimental activity in the theme of solitary nonlinear waves that arise in granular crystals made up of elastically interacting spherical beads subject to Hertz's law. We start by examining beads of a single type, reviewing the theory of highly localized (nearly compact) solitary traveling waves that exist therein and we consider the possibility of discrete breathers in the 1d granular chain. We then insert a single or multiple defect beads (e.g. of different mass) and examine how this insertion modifies the underlying linear spectrum and the nonlinear states emanating from it. We find interesting bifurcation phenomena especially when inserting multiple defects, such as next-nearest-neighboring ones. We illustrate how to generalize this paradigm of "defect insertion" to an infinite limit of the so-called dimer lattice. In that limit, we consider both solitary traveling waves and discrete breathers. Time permitting, generalizations to other heterogeneous or even disordered chains, as well as to higher dimensional examples will also be given.

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Strong Turbulence from Multiple Collapses in Nonlinear Schrödinger Equation

Pavel Lushnikov
University of New Mexico, USA

(Y. Chung and N. Vladimirova)

We consider a nonlinear Schrödinger equation (NLS) with dissipation and forcing in critical dimension. Without both linear and nonlinear dissipation NLS results in a finite-time singularity (collapse) for any initial conditions. Dissipation ensures collapse regularization. If dissipation is small then multiple near-singular collapses are randomly distributed in space and time forming collapse turbulence. Collapses are responsible for non-Gaussian tails in the probability dstribution function of amplitude fluctuations which makes turbulence strong. Power law of non-Gaussian tails is obtained for strong NLS turbulence.

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From Breathers to Q-Breathers in the Finite DNLS Lattice

Simone Paleari
Università di Milano, Italy

(Tiziano Penati)

We consider the finite discrete Nonlinear Schrödinger lattice with Dirichlet boundary conditions in both the focousing and defocousing cases. We construct and approximate breathers type solutions from the continuum, as perturbations of the NLS ground state in $H^1_0([-L,L])$, using ideas from the Finite Elements Method. We control the

solutions with respect to the relevant parameters like the number of particles in the chain and the energy of the solution; with these informations it is possible to explain numerical results concerning Fourier localization properties of such (q-)breathers and energy thresholds for their existence.

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Continuous Approximation of Breathers in One and Two Dimensional Lattices

Tiziano Penati
University of Milan, Italy
(D. Bambusi, S. Paleari)

We construct and approximate breathers in 1D and 2D infinite lattices starting from the NLS continuous limit.

We first deal with the dNLS model [1]: such periodic solutions are obtained as perturbations of the ground state of the NLS model in $H^1(\mathbb{R}^n)$, with $n = 1, 2$. This result is based on the interpolation of the lattice using the Finite Element Method (FEM).

We then move to the Klein-Gordon model [2]: in this case the proof is based on a suitable Lyapunov-Schmidt decomposition which exploits the relation between the KG lattice and the dNLS lattice.

Work is in progress on finite dNLS lattices, with fixed and periodic boundary conditions.

[1] Bambusi-Penati: Nonlinearity 23 (2010) 143-157.
[2] Bambusi-Paleari-Penati: Applicable Analisys, 2010 (to appear).

$\longrightarrow \infty \diamond \infty \longleftarrow$

Travelling Waves in Forced Discrete NLS Equation

Vassilis Rothos
Aristotle University of Thessaloniki, Greece
(M. Feckan)

We study the existence and bifurcation of periodic travelling wave solutions in forced spatially discrete nonlinear Schrödinger equations with local interactions. We consider polynomial type and bounded nonlinearities. The mathematical methods are based in using Palais-Smale conditions and variational methods.

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Spatial Pattern Selection in Recurrent Precipitation and Liesegang Rings

Arnd Scheel
University of Minnesota, USA

We discuss propagation into unstable states in a simple, yet surprisingly rich, 2-species reaction diffusion system that arises as a model for recurrent precipitation, and for solidification in undercooled liquids. We show that the invasion of unstable states can create homogeneous bulk states or transient periodic patterns. Transitions between different invasion regimes can be interpreted as heteroclinic bifurcations. The results are a crucial building block for a conceptual understanding of Liesegang pattern formation.

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Bounds for the Nonlinear Schrödinger Approximation of the Fermi-Pasta-Ulam-System

Guido Schneider
Universität Stuttgart, Germany

We prove that the evolution of a slowly varying envelope of small amplitude of an underlying oscillating wave packet in the Fermi-Pasta-Ulam system can be described approximately by the Nonlinear Schrödinger equation. In contrast to other lattice equations for which this question has been addressed in the existing literature the Fermi-Pasta-Ulam system possesses a nontrivial quadratic resonance due to curves of eigenvalues which vanish at the wave number $k = 0$. The proof of the error estimates is based on normal form transforms and a wave number dependent scaling of the error function.

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Bifurcation of Resonances in Open Periodic Waveguides

Stephen Shipman
Louisiana State University, USA
(Natalia Ptitsyna)

We present a discrete model of resonant scattering of waves by an open periodic waveguide, in which the ambient space is modeled by a planar lattice and the waveguide by a linear periodic lattice coupled to the planar one along a line. The interaction of plane waves with embedded guided modes of the waveguide causes resonant scattering manifest by transmission anomalies and field amplification. Resonance is initiated at a critical coupling strength, where a tangent bifurcation of guided modes appears, beginning with a single standing wave and splitting into a pair of waves traveling in opposing directions. Joint work with Natalia Ptitsyna.

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Special Session 7: Boundary Value Problems for Differential Equations and Inclusions

Cristina Marcelli, Technical University of Marche – Ancona, Italy
Francesca Papalini, Technical University of Marche – Ancona, Italy

Introduction: The aim of this special session is to collect a wide outline about recent progresses on the field of boundary value problems, which still presents many open questions and it is involved in many modern applications in Physics, Biology, Engineering, Economy and Medicine. The problems included in the topic cover a broad class of differential equations and inclusions, such as: singular equations, delay equations, functional equations, equations governed by nonlinear differential operators, difference equations, considered in bounded or unbounded domains.

On Solutions and Their Discretizations of Stepanov Almost-Periodic Differential Equations and Inclusions

Jan Andres
Palacky University, Czech Republic

Caratheodory almost-periodic solutions and their discretizations will be considered in their mutual relationship for differential equations and inclusions. Unlike for functions, Stepanov and Bohr almost-periodic sequences coincide. A particular attention will be therefore paid to non-uniformly continuous Stepanov almost-periodic solutions.

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Branches of Harmonic Solutions for a Class of Periodic Differential-Algebraic Equations

Alessandro Calamai
Universitá Politecnica delle Marche, Italy

We study a class of T-periodic parametrized differential-algebraic equations (DAEs). Our approach is based on the equivalence of such equations with suitable ordinary differential equations on manifolds. By combining recent results on the degree of tangent vector fields with an argument on periodic solutions of ODEs on manifolds, we get a global continuation result for T-periodic solutions of the given DAEs.

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Periodic Solutions for φ-Laplacian Pendulum Equations

Jose Angel Cid
University of Jaen, Spain
(Pedro J. Torres)

We present some results about the multiplicity and stability of T-periodic solutions for the equation $(\varphi(x'))' + c\,x' + g(x) = e(t) + s$, where $\varphi :]-a, a[\longrightarrow]-b, b[$ is an increasing homeomorphism.

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Weakly Nonlinear and Symmetric Periodic Differential Systems

Nataliya Dilna
Slovak Academy of Sciences, Bratislava, Slovak R.
(Michal Fečkan)

We find a unique symmetric and periodic solution for weakly nonlinear ordinary differential equations and establish conditions, under which this solution is either stable or hyperbolic.

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Multiple Solutions for Nonlinear Dirichlet Problems with Concave Terms

Leszek Gasinski
Jagiellonian University, Poland
(N. S. Papageorgiou)

We consider a nonlinear parametric Dirichlet problem with parameter, driven by the p-Laplacian and with a concave term and a Caratheodory perturbation which is asymptotically $(p-1)$-linear at infinity. Using variational methods combined with Morse theory and truncation techniques, we show that there is a critical value of the parameter such that for all lower parameters the problem has five nontrivial smooth solutions, four of constant sign (two positive and two negative) and the fifth nodal.

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On a Class of Integral Inclusions

Shui Hung Hou
The Hong Kong Polytechnic University, Hong Kong

Sufficient conditions for the existence of solutions to a class of integral inclusions in Banach spaces are given. This is established using fixed-point theorem for set-valued maps.

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Positive Solutions for p-Laplacian Equations with Combined Nonlinearities

Sophia Kyritsi
Hellenic Naval Academy, Greece
(Nikolaos S. Papageorgiou)

We consider a nonlinear Dirichlet problem driven by the p-Laplacian differential operator, with a nonlinearity concave near the origin and a nonlinear perturbation of it. We look for multiple positive solutions. We consider two distinct cases. One when the perturbation is p-linear and resonant with respect to $\lambda_1 > 0$ (the principal eigenvalue of $\left(-\triangle_p, W_0^{1,p}(Z)\right)$) at infinity and the other when the perturbation is p-superlinear at infinity. In both cases we obtain two positive smooth solutions. The approach is variational, coupled with the method of upper–lower solutions and with suitable truncation techniques.

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On Second-Order Boundary Value Problems in Banach Spaces

Luisa Malaguti
University of Modena and Reggio Emilia, Italy
(Jan Andres, Martina Pavlačková)

The talk deals with the existence and localization of Carathéodory solutions of second-order Floquet boundary value problems in Banach spaces. The results are obtained by combining degree arguments and bounding (Liapunov-like) functions techniques. The problems concern both semilinear differential equations and inclusions. Some examples are supplied.

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Nonoscillatory Solutions for Differential Equations with Generalized Φ-Laplacian

Mauro Marini
Florence Unversity, Italy
(Mariella Cecchi, Zuzana Došlá)

Consider the differential equation

$$(a(t)\Phi(x'))' + b(t)F(x) = 0, \qquad (1)$$

where Φ is an increasing odd homeomorphismus, $\Phi : (-\rho, \rho) \to (-\sigma, \sigma)$, $\Phi(0) = 0$ and 01) and some new phenomena, due to the lack of the homogeneity property of Φ, are presented. Moreover, asymptotic properties of solutions x of (1) satisfying $\lim_{t\to\infty} x(t) = \infty$, $\lim_{t\to\infty} a(t)\Phi(x'(t)) = \lambda$, are considered and the boundedness of the solvability interval Λ ($\lambda \in \Lambda$) is compared with the one of $\operatorname{Im}\Phi$ and with the existence of solutions y such that $\lim_{t\to\infty} y'(t) = \infty$ (the so-called extremal solutions).

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Boundary Value Problems on the Half Line for Nonlinear Second Order Equations

Serena Matucci
University of Florence, Italy
(Zuzana Došlá, Mauro Marini)

The second order nonlinear differential equation with p-Laplacian

$$(a(t)\Phi_p(x'))' = b(t)F(x), \quad t \in I = [0, \infty),$$

is considered, where a, b are continuous positive functions on I, and F is a continuous function on $\mathbb{R}$ such that $uF(u) > 0$ for $u \neq 0$ and

$$\lim_{u\to 0} \frac{F(u)}{\Phi_p(u)} = L, \quad 0 \le L < \infty.$$

Necessary and sufficient conditions are given for the existence (and uniqueness) of positive decreasing solutions, satisfying

$$\text{(i) } x(0) = c, \lim_{t\to\infty} x(t) = 0,$$

or

$$\text{(ii) } x'(0) = d, \lim_{t\to\infty} x(t) = 0,$$

where $c > 0$ and $d < 0$ are constants. The results are applied to study the existence of globally positive solutions on $(0, \infty)$ for the BVP

$$\begin{cases} (a(t)|x'|^\alpha \operatorname{sgn} x')' = B(t)|x|^\beta \operatorname{sgn} x \\ x(0) = 0, \quad \lim_{t\to\infty} x(t) = 0, \end{cases}$$

where B has no constant sign on $(0, \infty)$. Our approach combines phase space analysis, properties of principal solutions in the half-linear case and fixed point theorems.

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Frictional Contact Boundary Conditions Via Hemivariational Inequalities

Anna Ochal
Jagiellonian University, Poland
(S. Migorski)

We consider a hemivariational inequality with Volterra integral term. It models the frictional contact between a deformable viscoelastic body and a foundation. We prove existence and uniqueness results of abstract differential inclusions. Then we use these results to prove the unique solvability of the frictional contact problem. We present some examples of subdifferential boundary conditions for which our results work.

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Semilinear Inclusions in Frechet Spaces

Valentina Taddei
University of Modena and Reggio Emilia, Italy
(I. Benedetti (Firenze) and L. Malaguti (Modena and Reggio Emilia))

We consider semilinear evolution inclusions in a reflexive separable Banach space E

$$x' \in A(t,x)x + F(t,x), \quad \text{for a.a. } t \in [a,b],$$

where $A(t,x)$ is a linear and bounded operator for any $t \in [a,b]$, $x \in E$. We assume the regularities on A and F with respect to the weak topology and we investigate the inclusion in the Sobolev space $W^{1,p}([a,b],E)$, with $1 < p < \infty$. We allow both A and F to have a superlinear growth in x. We give existence results for local and global solutions of the initial problem and for solutions of the two-points boundary value problem, even in the case of multiple solutions of the homogeneous problem. We use both Ky Fan fixed point theorem and a continuation principle in Frechet spaces, assuming the existence of a Lyapunov function in order to prove the usual pushing condition. Some examples complete the discussion.

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Special Session 8: New Trends in Mathematical Fluid Dynamics

Eduard Feireisl, Academy of Sciences, Prague, Czech Republic

Introduction: The section focusses on the recent development of the theory of partial differential equations and system related and/or inspired by problems arising in fluid mechanics. Special emphasis is put on theoretical aspects, in particular, the questions of existence, uniqueness, and qualitative properties of solutions will be discussed. Related problems are also included in the session programme.

Two-Phase Flows of Viscous Incompressible Fluids with Different Densities

Helmut Abels
University Regensburg, Germany

We discuss different models for a two-phase flow of two immiscible, incompressible fluids in the case when the densities of the fluids are different. In particular we will present a new thermodynamically consistent diffuse interface model and compare it with the known models. Such models were introduced to describe the flow when singularities in the interface, which separates the fluids, (droplet formation/coalescence) occur. The fluids are assumed to be macroscopically immiscible, but a partial mixing in a small interfacial region is assumed. We will present recent results on the mathematical analysis of these models.

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Boundary Layer Problem: Navier-Stokes and Euler Equations

Nikolai Chemetov
CMAF/University of Lisbon, Portugal

We consider the Navier-Stokes equations in a 2D-bounded domain, admitting flows through the boundary of the domain.

The main result of our work: Under the viscous parameter tends to zero we show that the solutions of the Navier-Stokes equations converge strongly to the solution of the Euler equations, satisfying the Navier slip boundary conditions on the part of the boundary, where there exists the influx of the fluid.

This result solved a famous problem of boundary layers.

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Asymptotic Behavior of Fluid Flow Around a Rotating Obstacle

Reinhard Farwig
Technische Universität Darmstadt, Germany
(Toshiaki Hishida)

Consider a viscous incompressible flow around a body in $\mathbb{R}^3$ rotating with constant angular velocity $\omega \,||\, e_3$. Using a coordinate system attached to the rotating body, the problem is reduced to a modified Navier-Stokes system in a fixed exterior domain including the new term $(\omega \wedge x) \cdot \nabla u$ which is not subordinate to the Laplacian. The talk will address the question of the asymptotic behavior of stationary solutions to the new system as $|x| \to \infty$.

In the linear Stokes case with rotation terms the third column vector of the classical Stokes fundamental solution determines the behavior of the fundamental solution for large $|x|$; the influence of the rotation is not felt until the second order term. As for the classical exterior Navier-Stokes problem the Stokes fundamental solution does not describe the asymptotic behavior in the nonlinear case. Indeed, under a suitable smallness assumption on the velocity field, u, and the net force on the boundary, $\mathcal{N}$, we prove that the leading term of u is the so-called *Landau solution* U, a singular explicit solution of the stationary Navier-Stokes system in $\mathbb{R}^3$ with external force $(\mathcal{N} \cdot e_3)\delta_0\, e_3$ and decaying as $1/|x|$; here δ_0 is the Dirac measure supported in the origin.

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Asymptotic Behavior of Cahn-Hilliard-Navier-Stokes Systems

Maurizio Grasselli
Politecnico di Milano, Italy
(Ciprian G. Gal)

Cahn-Hilliard-Navier-Stokes systems model phase separation processes in binary fluids through a diffuse interface approach. In the incompressible isothermal case, these systems reduce to the Navier-Stokes equations subject to a force depending on the order parameter (Korteweg force) whose dynamic is governed by a Cahn-Hilliard equation with convective term. The goal of this talk is to illustrate some recent work I have done jointly with Ciprian G. Gal (University of Missouri, USA). The results are mainly concerned with existence of attractors and their dimension.

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On Implicitly Constituted Incompressible Fluids

Piotr Gwiazda
University of Warsaw, Poland
(Miroslav Bulíček, Josef Málek, Agnieszka Świerczewska-Gwiazda)

We consider flows of incompressible fluids with rheology given by an implicit constitutive equation relating the Cauchy stress and the symmetric part of the velocity gradient in such a way that it leads to a maximal monotone (possibly multivalued) graph.

Such a framework includes standard Navier-Stokes and power-law fluids, Bingham fluids, Herschel- Bulkley fluids, and shear-rate dependent fluids with discontinuous viscosities as special cases. We will mention results for steady and unsteady flows and consider different boundary conditions. The case of power-law-like rheology will be presented and the extension for more general functions describing the behaviour of the Cauchy stress tensor.

Using tools such as the Young measures, properties of spatially dependent maximal monotone operators and Lipschitz approximations of Sobolev functions, we are able to extend the results concerning large data existence of weak solutions to those values of the power-law index that are of importance from the point of view of engineering and physical applications. For the more general formulation the framework of Orlicz spaces will be used.

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Mathematical and Numerical Modelling of Shear-Dependent Non-Newtonian Flow in Compliant Vessels

Anna Hundertmark
Hamburg University of Technology, Germany
(Maria Lukacova)

In this contribution we present our recent results on the existence and uniqueness of the fluid-structure iteration problem for shear-dependent non-Newtonian fluids in compliant vessels. Equations for non-Newtonian fluids are coupled with the generalized string equation describing the domain deformation. In order to study existence and uniqueness of the weak solution to a coupled problem the so-called global iteration method with respect to domain deformation, energy estimates and theory of monotone operators have been used. If time permints we also present results of numerical simulation obtained by the combined finite volume-finite difference method for a coupled fluid-structure problem.

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Existence and Stability of Viscoelastic Shock Profiles

Marta Lewicka
University of Minnesota, USA
(Blake Baker, Kevin Zumbrun)

In this talk, generalizing work of Antman and Malek–Madani in the incompressible shear flow case, we study the existence and stability of planar viscoelastic traveling waves in a three-dimensional solid, for a simple prototypical elastic energy density, both for the general compressible and the incompressible shear flow case.

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On Inhomogeneous Heat-Conducting Incompressible Fluids

Josef Málek
Charles University in Prague, Czech Republic
(J. Frehse and M. Ružička)

We consider unsteady flows of inhomogeneous, incompressible, shear-thickening and heat conducting fluids where the viscosity depends on the density, the temperature and the shear rate, and the heat conductivity depends on the temperature and the density. For any values of initial total mass and initial total energy we establish the long-time existence of weak solution to internal flows inside an arbitrary bounded domain with Lipschitz boundary. The result will be published in Communications in PDEs.

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Global-in-Time Solutions for the Isothermal Matovich-Pearson Equations

Andro Mikelić
Université Lyon 1, Institut Camille Jordan, France
(E. Feireisl, Ph. Laurençot)

In this talk we present the mathematical study of the equations of Matovich and Pearson describing the process of glass fiber drawing. These equations may be viewed as a 1D-reduction of the incompressible Navier-Stokes equations including free boundary, valid for the drawing of a long and thin glass fiber. We concentrate on the isothermal case without surface tension. Then the equations of Matovich and Pearson represent a nonlinearly coupled system of an elliptic equation for the axial velocity and a hyperbolic transport equation for the fluid cross-sectional area. We first prove existence of a local solution, and, after constructing appropriate barrier functions, we deduce that the fluid radius is always strictly positive and that the local solution remains in the same regularity class. To the best of our knowledge, this is the first global existence

and uniqueness result for this important system of equations.

This is a joint work with Eduard Feireisl (Prague) and Philippe Laurençot (Toulouse).

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Analysis of a Two-Scale Reaction-Diffusion System with Nonlinear Micro-Macro Transmission Condition

Adrian Muntean
TU Eindhoven, The Netherlands

We study a reaction-diffusion system posed at two different spatial scales which involves nonlinear reaction and mass-transfer terms. The system incorporates a non-standard component – a nonlinear transfer function connecting micro- and macro-transport of chemical species including thus in the model nonlinear deviations from local equilibrium configurations [usually modeled in the literature, as a *faute de mieux* solution, by a linear expression – the Henry's law]. We prove non-negativity and L^∞-bounds for the active concentrations, uniqueness, and obtain global-in-time existence of weak solutions via a two-scale Galerkin method. Additionally, we give an *a priori* estimate on the rate of convergence of the two-scale Galerkin scheme.
This is a joint work with Maria-Neuss Radu (Heidelberg).

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A Weak Solution of the Steady Navier-Stokes Equation in Domains with Non-Compact Boundaries

Jiri Neustupa
Academy of Sciences, Prague, Czech Republic

We deal with the steady Navier-Stokes problem in a 3D domain, whose boundary has M bounded components and $N - M$ unbounded components. We impose no restriction on the size of prescribed fluxes through the unbounded components of the boundary and we prove the existence of a weak solution.

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Weak and Variational Solutions to Steady Compressible Navier-Stokes-Fourier System

Milan Pokorny
Charles University, Prague, Czech Republic
(Antonin Novotny (University of Toulon))

We consider steady compressible Navier–Stokes–Fourier system with general pressure law satisfying asymptotics

$$p(\varrho, \vartheta) \sim \varrho^\gamma + \varrho\vartheta.$$

Under further not very restrictive assumptions on heat conductivity we show that for any $\gamma > 1$ there exists a variational solution (i.e. satisfying weak formulation of the continuity equation, balance of momentum and entropy inequality, together with global balance of total energy) which is a weak solution (i.e. satisfies weak formulation of the continuity equation, balance of momentum and total energy) for $\gamma > \frac{4}{3}$ with further restrictions on the heat conductivity.

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Stabilization of a Viscous, Compressible and Heat-Conducting Fluid Via Highly Oscillating External Forces

Dalibor Prazak
Charles University, Prague, Czech Republic
(E. Feireisl)

We consider thermally and mechanically isolated fluid in a bounded domain in $\mathbb{R}^3$, whose density, velocity and temperature are governed by the full Navier-Stokes-Fourier system. We will discuss both the necessary and sufficient conditions for the external force f that guarantee the asymptotic boundedness of the solutions. In particular, we will show that the solutions remain bounded provided that the time oscillations of f increase sufficiently fast.

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Vortex Dynamics Estimates by Structural Complexity Analysis

Renzo Ricca
Univerisity Milano-Bicocca, Italy

Structural complexity [1] deals with the analysis and interpretation of morphological and topological properties of coherent structures in relation to the energy and dynamical properties of the physical system. Here we review some results on vortex tangles and present new results on vortex knots and unknots. In the case of vortex tangles the kinetic energy of the system can be directly related to the average crossing number of the tangle, thus establishing a relationship between crossing numbers and energy localization. By using projected area information for vortex knots and unknots we can relate linear and angular momenta of the vortex to signed area information. This allows a predictable use of this information for possible applications in vortex dynamics and turbulence research. An example of this application based on an experiment on vortex rings collision will be presented.

[1] Ricca, R. L. (2005) Structural complexity. In Encyclopedia of Nonlinear Science (ed. A. Scott), pp.885-887. Routledge, New York and London.

[2] Ricca, R. L. (2009) Structural complexity and dynamical systems. In Lectures on Topological Fluid Mechanics (ed. R. L. Ricca), pp.169-188. Springer-CIME Lecture Notes in Mathematics 1973. Springer-Verlag.

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Volume-Preserving Non-Autonomous Flows and Lagrangian Trajectories Associated to the 3D Navier-Stokes Equations

James Robinson
University of Warwick, England

(Witold Sadowski (Warsaw; Warwick))

We give an elementary proof of a result due to Aizenman, that for a volume-preserving non-autonomous flow in $\mathbb{R}^n$, the solution through almost every initial condition avoids any set with box-counting dimension strictly less than $n-1$. We then apply this result to the show that if $u(x,t)$ is a suitable weak solution of the three-dimensional Navier-Stokes equations (in the sense of Caffarelli, Kohn, & Nirenberg) with initial condition u_0 sufficiently regular ($u_0 \in H^{1/2}$ will do), then the particle trajectories – the solutions of $\dot{X} = u(X,t)$ – are unique for almost every initial condition. This follows from the subsidiary result, of interest in its own right, that the box-counting dimension of the set of space-time singularities is bounded by 5/3.

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Generalized Stokes Equations – Existence of Weak Solutions

Agnieszka Świerczewska-Gwiazda
University of Warsaw, Poland

(Piotr Gwiazda, Aneta Wroblewska)

Our interest is directed to unsteady generalized Stokes equation

$$u_t - \operatorname{div} T(Du) + \nabla p = f, \ \operatorname{div} u = 0.$$

where u, p are the velocity and the pressure respectively, T is the Cauchy stress tensor. The growth and the coercivity of the tensor T are prescribed by an N–function. Since the case of fast growing (w.r.t. shear rate) tensors have been already considered by us in previous papers, we concentrate on slowly growing tensors. The generality of the growth conditions implies the formulation of the problem in Orlicz spaces. We will show the existence of weak solutions. The problems related to density of smooth functions in Orlicz spaces, embeddings and Korn inequality will be presented.

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Regularity of the Motion of a Rigid Body in a Inviscid Incompressible Fluid

Takéo Takahashi
INRIA Nancy, France

(Olivier GLASS, Franck SUEUR)

We study the motion of a rigid body immersed in an incompressible inviscid fluid which occupies a three-dimensional bounded domain. Assuming smoothness of the boundaries (of the body and of the domain), we show the smoothness of the motion of the rigid body. In particular for analytic boundaries we obtain that the motion of the rigid body is analytic (till the classical solutions exist and till it does not hit the boundary).

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Special Session 9: Mechanical Problems

Kuo-Chang Chen, National Tsing Hua University, Taiwan
Tianhong Li, Chinese Academy of Sciences, China

Introduction: The aim of this special session is to bring together researchers on various aspects of mechanical systems, to exchange ideas and stimulate collaborations. We will focus on recent progress related to the dynamics of various classical mechanical systems, such as celestial mechanics, fluid mechanics, magnetohydrodynamics, or related topics. Mathematicians and scientists working on either theoretical or numerical aspects of mechanical systems are welcomed to participate and send us abstracts.

Collisions and Singularities in n-Body Type Problems

Vivina Laura Barutello
University of Turin, Italy
(Davide L. Ferrario, Susanna Terracini)

This talk will investigate the occurrence af collisions in solutions of dynamical systems with singular forces. The validity of Sundman-type asymptotic estimates for such solutions is established for a wide class of dynamical systems, including the classical n-body problems with Newtonian, quasi-homogeneous and logarithmic potentials. The solutions are meant in the generalized sense of Morse (locally –in space and time– minimal trajectories with respect to compactly supported variations) and their uniform limits. The analysis includes the extension of the Von Zeipel's Theorem and the proof of isolatedness of collisions. Furthermore, such asymptotic analysis is applied to prove the absence of collisions for locally minimal trajectories.

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A Variational Approach to the Planar 2-Center Problem

Roberto Castelli
University of Paderborn, Germany
(Susanna Terracini)

The planar 2-center problem describes the motion of a point particle under the gravitational attraction of two fixed point masses in the plane. In this talk we present a variational argument useful to deal with the singularity of the system and we prove the existence of eight-shaped periodic solutions in the case of equal masses.

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Korteweg-de Vries-Burgers-Type Equation

Zhaosheng Feng
University of Texas-Pan American, USA

In this talk, we study a Korteweg-de Vries-Burgers-type equation which arise from fluid mechanics to describe the propagation of waves in an elastic tube filled with a viscous fluid. Some fundamental properties such as nonlinear wave propagation and the stability are presented.

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Evolution on Shape Variables for the Planar Three-Body Problem

Toshiaki Fujiwara
Kitasato University, Japan
(Hiroshi Fukuda, Hiroshi Ozaki and Tetsuya Taniguchi)

For planar three-body problem, we can decompose the motion into three components, size change, rotation and shape change. The shape change is described by shape variables $Q_k(t) \in \mathbb{C}$, $k = 1, 2, 3$, that has unit moment of inertia and vanishing angular momentum.

We consider the motion for the shape variables $Q_k(t)$. It will be shown that the time evolution of the vector $\Psi_k(t) = \text{transpose}(Q_k(t), \dot{Q}_k/\sqrt{\sum |\dot{Q}_k|^2})$ is driven by an unitary matrix $U(t, 0) \in U(2)$ as $\Psi_k(t) = U(t, 0)\Psi_k(0)$. The unitary matrix is common for all $k = 1, 2, 3$.

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Some Remarks on Convex and Concave Central Configurations

Chun Hsien Hsiao
National Tsing Hua University, Taiwan
(Kuo-Chang Chen)

It is well-known that the planar four-body problem has at least 6 convex central configurations and 14 concave central configurations. Not much is known about the case of five or more bodies. In this talk I will briefly discuss this problem and its analogy in the spatial problem, and show some examples of central configurations which are both convex and concave. This is a joint work with Kuo-Chang Chen.

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Influence of the Distribution of the Axial, Radial and Circumferential Component of the Magnetic Induction Vector Within the Space between Switches on Rupturing Capacity of Vacuum Switches

Bogdan Kwiatkowski
University of Rzeszow, Poland

(Gomolka Zbigniew, Koziorowska Anna, Krutys Pawel)

The paper presents experimental tests results of magnetic field dstribution for highcurrent switch. Provided tests show influence of switch shape on field dstribution to obtain sustained vitality of vacuum switch. The method shown in the paper let obtain all components of magnetic induction vector. There is aspired to gain the highest value of axial component designing switches shape. It warrants favorable influence on fixity and turn off ability of switch system. There are lower values of peripheral and radial components making bipolar field in space between switches. (The Measurement of magnetic field in highcurrent unipolar switches). Keywords: vacuum chamber, bipolar switch, magnetic field.

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The Planet Motion under Effect of Drag Force $F = -\varepsilon v$

Kamal Mamehrashi
Payame Noor University of Boukan, Iran

The dynamical evolution of a binary orbit under drag forces is the major subject of this paper. In this paper we consider the motion of a planet which will be collapsed. The initial equations don't make us analyze it quickly but we can simplify the system equations and study them in binary system with two order. At last we show it in configuration space with using Lie Transforms.

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Continuation of Gerver's Supereight Choreography

Francisco Javier Muñoz-Almaraz
Universidad CEU—Cardenal Herrera, Spain

(J. Galán-Vioque, E. Freire and A. Vanderbauwhede)

It is well known that periodic orbits in reversible systems are not isolated. The first integrals play an important role in the development of numerical continuation techniques for the families of symmetric periodic orbits. Because of the great amount of time–reversal symmetries and first integrals for the N-body problem, these developed schemes show their potentialities in celestial mechanics. In this talk we provide a complete description of the families of periodic orbits which can be obtain from the continuation of Gerver's supereight when one or several masses are varied.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Three-Body Choreographic Motion in Given Curves

Hiroshi Ozaki
Tokai University, Numazu, Japan

(Hiroshi Fukuda, Toshiaki Fujiwara)

In the two-body problem, as shown by Johannes Kepler 400 years ago, the motion of every planet is completely determined by its own elliptical orbit and its nonvanishing anugular momentum. How about the motion in the three-body problem? Is the motion determined by its own shape of the orbit for each body and the total angular momentum of the system? We show, by a geometrical method, that it is true in some sort of orbits in the three body problem. We exemplify the motion of equal mass three bodies in smooth point symmetric convex curves and in eight shaped curves. In the former case, the motion is uniquely determined by a given curve, and the total angular momentum. In the latter case, the motion is uniquely determined by the given eight-shaped curve, and the total energy of the system. The reality of the motion should be tested whether the equation of motion is satisfied or not. Extensions of the method for generic curves are shown. The extended methods are applicable to generic curves which does not have point symmetry. Each body may have its own curve and its own non-vanishing masses.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Physical Statistical Modelling of Bending Vibrations

Nils Raabe
TU Dortmund, Germany

(Claus Weihs)

One serious problem in deep-hole drilling is the formation of a dynamic disturbance called spiralling which causes holes with several lobes. One explanation for the occurrence of spiralling is the intersection of time varying bending eigenfrequencies with multiples of the rotational frequency of the boring bar leading to a regenerative effect. This effect results from the periodical tilt of the drillhead cutting in each lobe after each revolution and continues in a self exciting manner even when the original causing eigenfrequency keeps changing. In former work we proposed a physical-statistical model consisting of a system of coupled differential equations and al-

lowing the explicit Maximum Likelihood estimation of the modal parameters and by this the implicit estimation of the bending eigenfrequency courses. An extensive simulation for the evaluation of the properties of these estimators and fitted courses has now been conducted. It is shown that the results of the model can be improved by fitting polynomial local regressions frequency band wise and furthermore pointwise confidence bands can be constructed. With such confidence bands it is possible to set up the machining parameters in a way that intersections of specific eigenfrequencies with multiples of the rotational frequency and spiralling correspondingly get unlikely.

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Heteroclinic Connections between Triple Collisions and Relative Periodic Orbits in the Isosceles Three-Body Problem

Mitsuru Shibayama
Kyoto University, Japan
(Kazuyuki Yagasaki)

We study the isosceles three-body problem and show that there exist infinitely many families of relative periodic orbits converging to heteroclinic cycles between equilibria on the collision manifold in Devaney's blown-up coordinates. Towards this end, we prove that two types of heteroclinic orbits exist in much wider parameter ranges than previously detected, using self-validating interval arithmetic calculations, and we appeal to the previous results on heteroclinic orbits. Moreover, we give numerical computations for heteroclinic and relative periodic orbits to demonstrate our theoretical results. The numerical results also indicate that the two types of heteroclinic orbits and families of relative periodic orbits exist in wider parameter regions than detected in the theory and that some of them are related to Euler orbits.

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Symbolic Dynamics and Parabolic Trajectories in the n-Centre and n-Body Problems

Susanna Terracini
Universitá di Milano Bicocca, Italy
(Roberto Castelli)

We consider both the planar n-centre problem and the classical three-dimensional n-body problem with Coulombic potentials. We discuss the existence of noncollision periodic and parabolic trajectories featuring chaotic behaviour and the scattering problem. The proofs are based on variational methods.

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Stability for Solutions of Wave Equations

Peng-Fei Yao
Academy of Sciences, Beijing, PR China

We introduce some recent results on stable solutions for the quasilinear wave equation with a boundary feedback, for the quasilinear wave equation with an interior feedback and for the Cauchy problem with variable coefficients. Those results are derived by the Riemannian geometric method. Whether those results hold true depends the curvature of the metric, given by the coefficients of the wave equation. We show that the behavior of solutions for the wave equation are actually controlled by the curvature.

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Special Session 10: Delay Differential Equations

Hans-Otto Walther, Justus-Liebig-Universität Giessen, Germany
Bernhard Lani-Wayda, Justus-Liebig-Universität Giessen, Germany

Introduction: We hope that the session will feature recent developments in delay equations, such as state-dependent delays and laser dynamics, as well as more 'classical' problems like e.g. Wright's equation.

Distributed Versus Discrete Delays in the Stability of Delay Differential Equations

Fatihcan Atay
Max Planck Inst. for Math. in the Sci., Germany

Stability under distributed delays is studied with respect to the variance of the delay dstribution. Applications to coupled oscillator systems and traffic flow models are presented, where it is seen that distributed delays yield larger stability regions in the parameter space than discrete delays. The situation is systematically investigated for general delayed-feedback systems near a Hopf bifurcation. It is proved that, among delay dstributions having the same mean value, discrete delays indeed represent a local extremum with respect to stability. Indeed,

if the feedback is destabilizing (resp., stabilizing), then a discrete delay turns out to be locally the most destabilizing (resp., stabilizing) delay dstribution. In general, increasing the variance of the delay dstribution from a discrete delay decreases the effect of the feedback term, although the effect is not necessarily monotone with further increases in variance.

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Global Stability for a Class of Epidemic Models with Delays and a Nonlinear Incidence Rate

Yoichi Enatsu
Waseda University, Japan

(Yukihiko Nakata and Yoshiaki Muroya (Univ. Waseda, Japan))

Stability analysis on mathematical models which describe the dynamics of infectious diseases have played a crucial role in the disease control in epidemiological aspect. In this talk, we establish the global asymptotic stability of a disease-free equilibrium and a positive (endemic) equilibrium for a class of delayed epidemic models governed by a class of nonlinear incidence rates. Some applications of our generalized results are also presented concerning the global asymptotic stability of a positive equilibrium when it exists.

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Bifurcation of the Essential Spectrum for Neutral Delay Differential Equations

Thomas Erneux
Universite Libre de Bruxelles, Belgium

(Gregory Kozyreff)

We consider scalar neutral delay differential equations when the stability boundary is determined by the essential spectrum. By applying multiple timescale techniques we derive a partial differential equation for the amplitude of the bifurcating periodic solution. This equation allows the coexistence of multiple stable periodic solutions. Our analysis is motivated by the study of mathematical models for opto-electronic oscillators.

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Positive Travelling Waves for Multi-Dimensional Reaction-Diffusion Systems with Delays

Teresa Faria
University of Lisbon, Portugal

(Sergei Trofimchuk (Univ. Talca, Chile))

For a family of n-dimensional delayed reaction-diffusion equations with distributed delay, we prove the existence of positive travelling wave solutions connecting the equilibrium 0 to a positive equilibrium K. After a Liapunov-Schmidt reduction, these waves are obtained via the Banach contraction principle (in a suitable Banach space), as perturbations of a positive heteroclinic solution for the associated system without diffusion, whose existence is proven under some requirements. By a careful analysis of the exponential decay of the travelling wave profiles at $-\infty$, their positiveness is deduced. Applications to some biological models are presented.

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Dynamical Systems for Parabolic Equations with Hysteresis

Pavel Gurevich
Free University of Berlin, Germany

In the talk, we will be interested in mathematical models involving partial differential equations with hysteresis. Although such models have been actively investigated, many questions remain open, especially those related to periodicity and long-time behavior of solutions.

Considering an easily formulated parabolic problem with hysteresis, we will show how it reduces to an infinite-dimensional dynamical system generated by two "simple" flows and the nonlinear law of switching between them.

We will demonstrate that such dynamical systems may exhibit interesting phenomena such as existence and coexistence of periodic solutions of different types (attractors, repellers, saddles), while the structure of periodic solutions themselves can be quite complicated. In particular, a trajectory may consist of sequences, each of which contains a fast oscillating part followed by a slow excursion.

A part of results was obtained jointly with W. Jaeger, A. Skubachevskii, and S. Tikhomirov.

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Global Stability and Optimal Control in a Differential Delay Model

Anatoli Ivanov
Pennsylvania State University, USA

(Musa A. Mammadov)

We use and further develop recent results on global asymptotic stability in scalar essentially nonlinear differential delay equation of the form

$$x'(t) = f(x(t-\tau)) - g(x(t)).$$

One of the approaches is the use of global attractivity in the limiting ($\tau = +\infty$) difference equation

given implicitly by $g(x_{n+1}) = f(x_n)$. The results are applied to several other models including the following optimal control problem appearing in economics. Maximize the functional

$$C(t) = [1 - u(t)]f(x(t-\tau)),$$
$$J = \liminf_{t\to\infty} C(t) \Rightarrow \max$$

where $x(t)$ solves the differential delay equation subject to control $u(t)$:

$$x'(t) = u(t)\, f(x(t-\tau)) - g(x(t)).$$

$\longrightarrow \infty \diamond \infty \longleftarrow$

Stability and Dynamics Influenced by Delay Distribution

Gábor Kiss
University of Bristol, England

We compare the stability properties of the zero solution of some delay differential equations with distributed delays and associated equations with a discrete delay. We present numerical examples when delay dstributions not only lead to increased stability region in the corresponding parameter space, but also generate complicated solutions after the loss of the linear stability.

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The Period Function and Global Properties for Some Delay Differential Equations

Tibor Krisztin
University of Szeged, Hungary
(Ábel Garab, Gabriella Vas)

Consider the scalar delay differential equation $\dot{x}(t) = \tau[-x(t) + f(x(t-1))]$ with $f \in C(R, R)$, $f(0) = 0$ and a parameter $\tau > 0$. Assume that for all $\tau > \tau^*$, there is a unique periodic orbit with a prescribed oscillation frequency. Let $\omega(\tau)$ denote the minimal period. We study the period function $\tau \mapsto \omega(\tau)$. Monotone properties of $\omega(\tau)$ can be used to describe global properties for a system modeling a ring of identical neurons.

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Effects of Harvesting in a Delayed Population Model

Eduardo Liz
Universidad de Vigo, Spain

We analyze the effect of harvesting in usual delayed recruitment models of the form $x'(t) = -\delta x(t) + f(x(t-\tau))$. For constant effort harvesting, that is, when the cath is proportional to the stock density, we discuss the phenomenon of bubbling (characterized by the existence of two Hopf bifurcation points), and the *hydra effect*, which means that the stock size gets larger in average as harvesting rate increases. A constant yield harvesting strategy leads to different dynamics, including coexistence of two attractors due to the Allee effect.

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Nonnegative Solutions of Delay Differential Equations

Mihály Pituk
University of Pannonia, Hungary

In some applications only nonnegative solutions are of interest. In this talk we will discuss some properties of nonnegative solutions in the neighborhood of the zero equilibrium of a system of delay differential equations. In particular, we will describe the strict Liapunov exponents of the nonnegative solutions in terms of the real eigenvalues of the associated linearized equation.

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Study of Well-Posedness and Qualitative Properties of Solutions to Parabolic Partial Differential Equations with State-Dependent Delays

Alexander Rezounenko
Kharkiv National University, Ukraine

The main goal of the talk is to present and discuss results on the well-posedness and long-time behavior of non-linear models described by parabolic partial differential equations with discrete (concentrated) and distributed state-dependent delays. It is well-known that the presence of discrete state-dependent delays in the models causes that nonlinearities are not Lipschitz continuous on the space of continuous functions (even for solutions to ordinary differential equations). The arguments required for this case be more delicate. We study both local and non-local in space coordinates delay terms. In the cases when the studied initial value problem generates a dynamical system, we study its long-time asymptotic behavior and prove the existence of a compact global attractor. The results can be used to treat, for example, the diffusive Nicholson' s blowflies equation.

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Linearized Stability for State-Dependent Delay Equations

Wolfgang Ruess
Universität Duisburg-Essen, Germany

The object of study is a differential equation with

state-dependent delay of the form $\dot{u}(t) = f(u(t + r(u_t)), t \geq 0, u_0 = \varphi \in C(I;X)$, with f a continuous function from a Banach space X into X, $I = [-R, 0]$, some $R > 0$, and $r : C(I;X) \to I$ a continuous delay function. With $F : C(I;X) \to X$ defined by $F(\varphi) = f(\varphi(r(\varphi)))$, and given an equilibrium $\varphi_e \in C(I;X)$ with $F(\varphi_e) = 0$, we consider differentiability conditions on F at φ_e in order for exponential stability of the linearized equation to carry over to local exponential stability of the equilibrium for the original nonlinear equation.

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Existence of Many Stable Rapidly Oscillating Periodic Solutions of Ddes

Daniel Stoffer
ETH-Zürich, Switzerland

A class of stiff delay differential equations with piecewise constant nonlinearity modelling positive or negative feedback is considered. For positive feedback take an even $n \in \mathbb{N}$ for negative feedback an odd $n \in \mathbb{N}$. Then the following holds: if the stiffness parameter is sufficiently large then there are $2a(n)$ stable ROPSs (rapidly oscillating periodic solutions) with n humps per time unit (a hump is the part of the solution between two successive zeros). $a(n) = \sum_{d \text{ odd}, d|n} \varphi(d)2^{(n/d)}/(2n)$ is the the number of essentially different binary n-stage shift register sequences (here φ denotes the Euler function).

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A Periodic Orbit for a Differential Equation with State-Dependent Delay

Eugen Stumpf
University of Rostock, Germany

In this talk, we consider the scalar differential equation

$$\dot{x}(t) = a[x(t) - x(t - r)] - |x(t)|x(t)$$

with a positive parameter a and a delay $r > 0$. In the case of the constant delay $r = 1$ it is known that for parameters $1 > a > 0$ the trivial solution is asymptotically stable, whereas for $a > 1$ the trivial solution gets unstable, and a global center-unstable manifold connects the trivial solution to a slowly oscillating periodic orbit. Here, we consider a state-dependent delay $r = r(x(t)) > 0$ instead of the constant one, and generalize the result on the existence of slowly oscillating periodic orbits for parameters $a > 1$ under mild conditions on the delay function r.

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Spectral Analysis of Abstract Hyperbolic Equations with Delay

Victor Vlasov
Lomonosov State University, Moscow, Russia
(Nadezda A. Rautian)

We study the solvability of abstract hyperbolic equations with variable delay and integral Volterra terms. We consider several spectral problems in autonomous cases by considering the operator-valued functions as the symbols of the equations under investigations. We also present some applications of our results to integro-differential equations of Gurtin-Pipkin type arising from the theory of heat equations with memory.

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Scaling Properties of the Spectrum for ODEs with Large Delay

Matthias Wolfrum
WIAS-Berlin, Germany
(S. Yanchuk, M. Lichtner)

We study the spectrum of linear DDEs in the limit of large delay. In this singular limit, the spectrum decomposes into two different parts, corresponding to different timescales of the dynamics. One part, called strong spectrum converges to finitely many eigenvalues that can be approximated by suitable ODE problems. The second part, called pseudo-continuous spectrum, accumulates near criticality and converges after rescaling to a set of spectral curves. We present theoretical results proving that this provides a complete description of the spectrum for sufficiently large delay and demonstrate the application of our theoretical results to the Lang-Kobayashi system for semiconductor lasers with delayed optical feedback.

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Reappearance and Stability of Periodic Solutions in Systems with Time Delay

Serhiy Yanchuk
Humboldt University of Berlin, Germany

We describe generic properties of systems with time delay, which are related to the appearance and stability of periodic solutions. In particular, we show that delay systems generically have families of periodic solutions, which are reappearing for infinitely many delay times. As delay increases, the solution families overlap leading to increasing coexistence of multiple stable as well as unstable solutions. We also consider stability issue of periodic solutions with large delay by explaining asymptotic properties of the spectrum of characteristic multipliers. We

show that the spectrum of multipliers can be split into two parts: pseudocontinuous and strongly unstable. The pseudocontinuous part of the spectrum mediates destabilization of periodic solutions.

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Special Session 11: New Developments in Qualitative Behavior of Nonlinear Evolutionary PDEs

Irena Lasiecka, University of Virginia, USA
Grozdena Todorova, University of Tennessee, USA

Introduction: New developments in the dynamic and rapidly growing area of nonlinear evolutionary PDEs will be reported. Questions such as well-posedness, regularity, long time behavior of solutions of important nonlinear equations and systems, stability/instability problems, formation of singularities and their applications will be central to many presentations. In addition issues such as inverse problems and related control theory will also be discussed.

Existence and Asymptotic Behavior of Solutions to Quasilinear Thermoelastic Systems without Mechanical Dissipation

Albert Altarovici
University of Virginia, Switzerland
(Irena Lasiecka)

We consider a class of quasilinear thermoelastic systems. The model considered is characterized by supercritical nonlinearity and accounts for large displacements and large strains. Existence of finite energy (weak)global solutions and exponential decay rates for the energy of weak solutions will be shown. The methods of proof will involve compensated compactness along with the application of special PDO non-local multipliers that have been developed in the context of the proof.

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Parabolic Problems on Non-Smooth Domains

Herbert Amann
University of Zürich, Switzerland

We describe a maximal regularity theorem for general parabolic systems on non smooth manifolds having corners, edges, conical points, cracks etc., on the boundary and in the interior.

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Glimm Functional and Convergence Rate of Glimm Scheme for General Hyperbolic Systems

Fabio Ancona
Università di Padova, Italy
(Andrea Marson)

Consider the Cauchy problem for an N-dimensional, strictly hyperbolic, quasilinear system

$$u_t + A(u)u_x = 0, \qquad u(0,x) = \bar{u}(x), \tag{1}$$

where $u \mapsto A(u)$ is a smooth matrix-valued map, and the initial data $\overline{u}$ is assumed to have small total variation. We investigate the rate of convergence of approximate solutions of (1) constructed by the Glimm scheme.

We shall introduce a new wave interaction functional wich allows to obtain the same type of error estimates valid for Glimm approximate solutions of hyperbolic systems of conservation laws $u_t + F(u)_x = 0$ satisfying the classical Lax or Liu assumptions on the eigenvalues $\lambda_k(u)$ and on the eigenvectors $r_k(u)$ of the Jacobian matrix $A(u) = DF(u)$. Namely, letting $u(t,\cdot)$ be the (unique) vanishing viscosity solution of (1) with initial data $\overline{u}$, the following a-priori bound holds

$$\left\|u^\varepsilon(T,\cdot) - u(T,\cdot)\right\| = o(1)\cdot\sqrt{\varepsilon}\,|\log\varepsilon| \tag{2}$$

for an approximate solution u^ε of (1) constructed by the Glimm scheme, with mesh size $\Delta x = \Delta t = \varepsilon$, and with a suitable choice of the sampling sequence.

Joint work with Andrea Marson, University of Padova.

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Inverse Problems for Networks of Elastic Strings

Sergei Avdonin
University of Alaska, USA
(Guenter Leugering and Victor Mikhaylov)

We consider the in-plane motion of elastic strings on a tree-like network. The two-velocity wave equation for a two component vector displacement is assumed to hold on each edge of a tree. We investigate the inverse problem of recovering not only the physical properties, i.e. the velocities and lengths of the

strings, but also the topology of the tree and the angles between branching edges. It is shown that the inverse problem can be solved by applying measurements at all leaves, the root of the tree being fixed.

We use a new version of the Boundary Control method proposed in (S. Avdonin and P. Kurasov, IPI 2 (2008), 1–21; S. Avdonin, G. Leugering and V. Mikhaylov, ZAMM 90 (2010), 136–150), which combines the spectral and dynamical approaches to inverse problems for PDEs on graphs, and develop a constructive procedure for the recovery tree's parameters. This procedures is recursive — it allows recalculating efficiently the inverse data from the original tree to the smaller trees, 'removing' leaves step by step up to the rooted edge. Because of its recursive nature, this procedure may serve as a base for developing effective numerical algorithms.

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Controllability of Coupled Parabolic Equations

Assia Benabdallah
University of Marseille, France

(F. Ammar Khodja, C. Dupaix and M. Gonzales-Burgos)

Controllability of linear differential systems is well-known. In particular we have at our disposal the famous Kalman rank condition, that is to say, if A, B are matrices, $A \in \mathcal{L}(\mathbb{R}^n)$, $B \in \mathcal{L}(\mathbb{R}^m, \mathbb{R}^n)$, then the system $Y' = AY + Bu$ is controllable at time $T > 0$ if and only if rank $[A \,|\, B] =$ rank $\left[B, AB, \ldots, A^{n-1}B\right] = n$.
What is the situation for Coupled Parabolic Equations? Can we expect generalization of Kalman's condition? This is not yet a solved problem. Recently, some partial answers has been obtained. The main goal of this talk is to investigate this question by giving the most important results.

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Exponential Global Attractors for Semigroups in Metric Spaces and Applications

Alexandre Carvalho
Universidade de São Paulo, Brazil

(J. W. Cholewa)

In this work we consider semigroups in a general metric space V with pointwise exponentially attracting local unstable manifolds of compact invariant sets. Under suitable assumptions we show that these semigroups possess strong exponential dissipative properties. In particular, there exists a compact global attractor, which attracts exponentially each bounded subset of V. Applications of abstract results to ordinary and partial differential equations are given.

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Nonlinear Hyperbolic-Elliptic Systems

Nikolai Chemetov
CMAF/University of Lisbon, Portugal

We investigate a nonlinear hyperbolic-elliptic type system of PDEs in a bounded domain

$$\omega_t + \mathrm{div}(g(\omega)v) = 0$$

with $v = -\nabla h$,

$$-\Delta h + h = \omega,$$

which is closed by the condition $h = a(x,t)$ on the boundary of the domain and the initial condition $\omega|_{t=0} = \omega_0$.

Motivated by physics, on the influx part of the boundary we consider nonzero boundary condition

$$\omega = b(x, t, \frac{\partial h}{\partial n}).$$

Here n is the outside normal to the boundary.

We prove the solvability of this system, using a kinetic formulation of the problem.

The system can be used for different physical situations, such as:

a) the motion of superconducting vortices in the superconductor; b) the collective cell movement (the Keller-Segel model).

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Boundedness and Blowup for Nonlinear Degenerate Parabolic Equations

Shaohua Chen
Cape Breton University, Canada

We will discuss a quasilinear parabolic equation with lower order terms and show that, under suitable conditions, whether the solution is bounded or blows up in a finite time depends only on the first eigenvalue of the negative Laplace operator with Dirichlet boundary condition. For some special cases, the result is sharp.

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Global Well-Posedness for a Hyperbolic Equation with Nonlinear Acoustic Boundary Conditions

Philip Graber
University of Virginia, USA

(Irena Lasiecka)

We consider a structural acoustic wave equation with nonlinear acoustic boundary conditions. This

is a coupled system of second and first order in time partial differential equations, with boundary conditions on the interface. Both a linear and a nonlinear version of the problem are considered. In each case, we prove wellposedness in the Hadamard sense for strong and weak solutions. The main tool used in the proof is the theory of nonlinear semigroups. We present the system of partial differential equations as a suitable Cauchy problem $dw/dt = Aw$. Though the operator A is not maximally dissipative we are able to show that it is a translate of a maximally dissipative operator. The obtained semigroup solution is shown to satisfy a suitable variational equality, thus giving weak solutions to the system of PDEs. The results obtained (i) dispel the notion that the model does not generate semigroup solutions, (ii) provide treatment of nonlinear models, and (iii) provide existence of a correct state space which is invariant under the flow – thus showing that physical model under consideration is a dynamical system. The latter is obtained by eliminating compatibility conditions which have been assumed in previous work (on the linear case).

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Localization of Analytic Regularity Criteria on the Vorticity and Balance between the Vorticity Magnitude and Coherence of the Vorticity Direction in the 3D NSE

Rafaela Guberovic
ETHZürich, Switzerland
(Grujic, Zoran)

In '95 daVaiga has shown that $\|Du\|_q^{\frac{2q}{2q-3}} \in L^1(0,T)$ is a regularity class for the Navier-Stokes Equations for any $3 \leq q < \infty$. Previously, Beale-Kato-Majda proved the regularity when the time-integrability of the L^∞-norm of the vorticity holds. In this talk we will show the localized versions of the aforementioned conditions imply local enstrophy remains bounded. The geometric conditions on the vorticity direction field are important in studying the regularity and the localized version has been recently obtained by Grujic. The special scaling invariant regularity class of weighted L^pL^q type for the vorticity magnitude with the coherence factor as a weight will be demonstrated in a localized setting.

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Optimal Decay Rate of the Energy for Linear Wave Equations with a Critical Potential

Ryo Ikehata
Hiroshima University, Japan
(Grozdena Todorova and Borislav Yordanov)

We will present an (almost) optimal decay rate of the energy for damped wave equations in unbounded domains with a variable coefficient decaying critically near infinity. For this purpose we develop a new weighted energy method. We shall introduce two types of diffusive structure for the equation.

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Carleman Estimates with Two Large Parameters and Their Applications

Victor Isakov
Wichita State University, USA
(Nanhee Kim)

We derive weighted (with two large parameters) weak energy estimates for a general partial differential operator of second order. These estimates are used to derive Carleman type estimates for the classical (dynamical) elasticity system with residual stress under natural pseudo-convexity type assumptions. As consequences we obtain uniqueness and stability results for the continuation of solutions of this system in space-like directions and for identification of all 6 residual stress (space dependent) coefficients from one set of special boundary data. Proofs use differential quadratic forms and the Fourier analysis.

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Some Well-Posedness and Exponential Decay Results in Nonlinear Acoustics

Barbara Kaltenbacher
University of Graz, Austria
(Irena Lasiecka)

Nonlinear acoustics plays a role in a wide range of applications. We are especially motivated by high intensity ultrasound, which is made use of in medical therapy such as lithotripsy, but also in technical processes like ultrasound cleaning or welding. The most widely used PDE models of nonlinear acoustics are Westervelt and Kuznetsov's equation, which are evolutionary quasilinear, potentially degenerate damped wave equations typically defined on a bounded domain in R^n, $n = 1, 2, 3$. In this talk we address the issues of local and global in time well-posedness of these models with different boundary conditions. Here, the problem of appropriately extending nonhomogeneous boundary data to the interior plays a crucial role. Local analysis for small initial data is based on energy estimates together with Banach's fixed point theorem. For showing global existence we combine the energy estimates with barrier's method. Exponential decay of the boundary data are shown to yield exponential decay of the solution. We will discuss additional

results e.g., on Neumann boundary conditions, the full Kuznetsov model, as well as optimal control, see, and give an outlook on further interesting research topics in the context of nonlinear acoustics.

[1] C. Clason, B. Kaltenbacher, S. Veljovic, Boundary optimal control of the Westervelt and the Kuznetsov equation, *JMAA*, vo 356, pp 738-751, 2009.
[2] P. M. Jordan, An analytical study of Kuznetsov's equation: diffusive solitons, shock formation, and solution bifurcation *Physics Letters A* 326 (2004) 77–84.
[3] B. Kaltenbacher and I. Lasiecka, Global existence and exponential decay rates for the Westervelt equation, *Discrete and Continuous Dynamical Systems*, Series S, vol 2, pp 503-525, 2009.
[4] B. Kaltenbacher, I. Lasiecka, S. Veljović, Well-posedness and exponential decay for the Westervelt equation with inhomogeneous Dirichlet boundary data, submitted.
[5] B. Kaltenbacher, I. Lasiecka, S. Veljović, Some well-posedness results in nonlinear acoustics, Tech. Rep. IOC-21, International Doctorate Program *Identification, Optimization and Control with Applications in Modern Technologies* (October 2008). http://www2.am.uni-erlangen.de/elitenetzwerk-optimierung/preprintfiles/IOC21.pdf

$\longrightarrow \infty \diamond \infty \longleftarrow$

Scaling Laws and Dimension Reduction in Nonlinear Elasticity

Marta Lewicka
University of Minnesota, USA
(Reza Pakzad)

This talk will concern the analysis and the rigorous derivation of plate and shell models for thin films exhibiting residual stress at free equilibria. We analyze the scaling of the energy minimizers in terms of the reference plate's thickness and rigorously derive the corresponding limiting theories, as the vanishing thickness Γ-limits. The theories are differentiated by the embeddability properties of the target metrics – in the same spirit as different scalings of external forces lead to a hierarchy of nonlinear elastic plate theories as recently displayed by Friesecke, James and Muller.

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Smoothing Properties for the Magnetic Schödinger Operator with Singular Potential

Kiyoshi Mochizuki
Chuo University, Japan

Uniform resolvent estimates are proved for the magnetic Schrödinger operator in R^n $(n \geq 3)$ with singular potential behaving like $O(|x|^{-2})$ at origin. These estimates are then applied to show smoothing properties for related evolution equations.

[1] Uniform resolvent estimates for magnetic Schrödinger operators and smoothing effects for related evolution equations, Publ. Res. Inst. Math. Sci. to be published.
Resolvent estimates for magnetic Schrödinger operators and their applications to related evolution equations, Rendiconti dell'Universitá di Trieste, to be published.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Gradient System on Networks

Delio Mugnolo
University of Ulm, Germany

Several systems appearing in mechanics, neuroscience, quantum mechanics and further natural sciences can be modelled by a network formalism. One consideres an ensemble of intervals equipped with linear diffusion equations interacting in a few junction points, synapses, etc. A drawing of such systems strongly resembles a graph, and indeed the investigation of evolution equation on networks takes often place at the border between classical dynamical systems and graph theory.

In its easiest form, when the prescribed node conditions are linear, such interactions problems can be described by means of classical semigroup theory: their investigation goes back to Günther Lumer. In this talk I will discuss the nonlinear case by means of the theory of gradient systems in infinite dimensional Hilbert spaces. This results in a generalization of a nonlinear Robin boundary condition for an individual interval.

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Well-Posedness of the Cauchy Problem on Torus to Electromagnetoelastic System

Wladimir Neves
Federal University of Rio de Janeiro, Brazil
(Viatcheslav Priimenko, Mikhail Vishnevskii)

We prove the well-posedness of the Cauchy problem on torus to an electromagnetoelastic system. The physical model consists of three coupled partial differential equations, one of them is a hyperbolic equation describing the elastic medium and two other ones form a parabolic system, which comes from the Maxwell's equations. Experimental measurements suggest that the elastic medium has a periodic structure, moreover with finite number of discontinuities on the fundamental domain. Thus we study in this talk the problem which we have defined as periodically Cauchy diffraction problem.

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Decay Estimates for the Korteweg-de Vries Equation on the Half-Line

Ademir Pazoto
Federal University of Rio de Janeiro, Brazil
(Felipe Linares and Lionel Rosier)

We shall present some results concerning large-time behavior of solutions of the KdV equation posed on the positive half-line under the effect of a localized damping term. Assuming that the damping function is effective on a subset of the domain we prove that the total energy decays locally uniform at an exponential rate. Combining multiplier techniques and compactness arguments the problem is reduced to prove the unique continuation property of weak solutions. We also use a Lyapunov approach to establish the decay of the solutions in some weighted spaces introduced by T. Kato.

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Decay Estimates for Nonlinear Wave Equations with Variable Coefficients

Michael Roberts
University of Tennessee Knoxville, USA
(Michael Horning)

We studied the long time behavior of solutions of the nonlinear wave equation $u_{tt} - \mathrm{div}(b(x)\nabla u) + a(x)u_t + |u|^{p-1}u = 0$. Such an equation appears in models for traveling waves in a non-homogeneous gas with damping that changes with position. We established decay estimates of the energy, L^2, and L^{p+1} norm of solutions. We found three different regimes of decay of solutions depending on the exponent of the absorption term, $|u|^{p-1}u$. We show the existence of two critical exponents $p_1(n,\alpha,\beta) = 1 + (2-\beta)/(n-\alpha)$ and $p_2(n,\alpha) = (n+\alpha)/(n-\alpha)$. For $p > p_1(n,\alpha,\beta)$, the decay of solutions of the nonlinear equation coincides with that of the corresponding linear problem. For $p_1(n,\alpha,\beta) > p$, the solution decays much faster. The other critical exponent $p_2(n,\alpha)$ further divides this region into two subregions with different decay rates. Deriving the sharp decay of solutions even for the linear problem with potential $a(x)$ is a delicate task and requires serious strengthening of the multiplier method. Here we used a modification of an approach of Todorova and Yordanov to derive the exact decay of the nonlinear equation.

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The Antman-Seidman Viscoelastic Rod Model with Self-Contact

Thomas Seidman
University of Maryland, Baltimore County, USA

For large motion of a rod in 3-space one must consider the possibility of nonlocal self-contact — a nonsmooth constraint prohibiting any interpenetration (collocation) of distinct parts of the rod. This global constraint is the conjunction of infinitely many scalar constraints and justifying the associated forces by computing the normal (in function space) to this constraint set requires a new result in nonsmooth analysis.

[This involves collaborative work with S. S. Antman and with K. A. Hoffman.]

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Existence and Continuity of Strong Solutions of Partly Dissipative Reaction Diffusion Systems

Zhoude Shao
Millersville University of Pennsylvania, USA

We consider a partly dissipative reaction diffusion system of the FitzHugh-Nagumo type in the following form

$$u_t - \Delta u + f(u) + g(x,v) = 0. \;\; t > 0, \;\; x \in \Omega \subset R^n$$
$$v_t + \sigma(x)v + h(x,u) = 0.$$

Under appropriate assumptions, we first establish the existence of strong solutions of the system by the standard Galerkin type of argument. Then we prove the continuity of the strong solutions with respect to initial data in the space $V \times H^1(\Omega)$, where V is a subspace of $H^1(\Omega)$ defined according to the boundary condition chosen for the u-component. The last result is obtained by applying the Nirenberg-Gagliardo inequality and is independent of the spatial dimension n. The results are motivated by the investigation of issues related to the long time dynamics of the above system such as determining modes, regularity of the global attractor, and the existence of approximate inertial manifolds.

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Diffusion Phenomenon in Hilbert Spaces and Applications

Grozdena Todorova
University of Tennessee, USA
(Petronela Radu and Borislav Yordanov)

We prove the diffusion phenomenon in Hilbert spaces and derive as applications an optimal decay for some dissipative wave equations with variable coefficients in exterior domain.

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Positive Solutions and Blow-Up for Semilinear Wave Equations with Non-Compactly Supported Non-Zero Initial Position

Hiroshi Uesaka
Nihon University, Japan
(H. Takamura and K. Wakasa)

Recently we got some blow-up results for semilinear wave equations with non-compactly supported non-zero initial position f with semi-linear term $F(u) = A|u|^{p-1}u$ with $p > 1$ in all spatial dimensions, which will be published in JDEqs soon. These results extend and improve the result by Uesaka in 2 or 3 dimensions. They give a new framework for well-known Asakura's one. We consider here radially symmetric solutions.

In this talk we give some improvement in our former assumptions and in established Uesaka's result by Takamura's idea. The main assumption is as follows: We assume that there exist a positive continuous function $\varphi(r)$ satisfying $\varphi(r) \nearrow \infty$ $(r \to \infty)$ with $r = |x|$ and a positive constant R such that for $r \geq R$

$$f(r) > 0$$

and

$$f''(r) + \frac{n-1}{r}f'(r) + F(f(r)) \geq \frac{\varphi(r)}{(1+r)^l},$$

where $0 < l < \frac{2}{p-1} + 2$. Then we have a blow-up result.

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On the Wave Equation with Second Order Dynamical Boundary Conditions

Enzo Vitillaro
Università degli Studi di Perugia, Italy

The aim of the talk is to report on the author's recent results concerning the wave equation posed in a regular bounded domain, supplied with a second order dynamical boundary condition also involving a Laplace-Beltrami operator, a nonlinear damping term and a nonlinear term $f(u)$. First we are going to discuss local well–posedeness of the problem, then global existence or blow-up will be proved, depending on the behavior of $f(u)$.

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Qualitative Behaviour of Solutions for the Two-Phase Navier-Stokes Equations with Surface Tension

Mathias Wilke
University of Halle-Wittenberg, Germany
(M. Köhne, J. Prüss)

In this talk we are concerned with the two-phase free boundary value problem for the isothermal Navier-Stokes system in bounded geometries, in absence of phase transitions, external forces and boundary contacts. We prove well-posedness in an L_p-setting, and that the system generates a local semiflow on the induced phase manifold. Moreover, we show that each equilibrium is stable if the phases are connected, and it is shown that global solutions which do not develop singularities converge to an equilibrium as time goes to infinity. The latter is proved by means of an energy functional combined with the generalized principle of linearized stability.

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Carleman Estimates and the Applications to Inverse Source Problems for Equations of Fluid Dynamics

Masahiro Yamamoto
The University of Tokyo, Japan

First we show Carleman estimates for the Navier-Stokes equations and for the Euler equations which are governing equations in the fluid dynamics. Then we apply them to inverse source problems of determining spatially varying factors in the source terms in these equations and prove conditional stability estimates.

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Uniform Decay of Local Energy for the Wave Equation with Variable Coefficients

Peng-Fei Yao
Academy of Sciences, Beijing, PR China

We establish Morawetz's multipliers in a version of the Riemannian geometry to derive uniform decay of local energy for the wave equation with variable coefficients on an exterior domain for nonsmooth data. Instead of the assumption that the coefficient matrix is constant near infinity in the literature we assume that the coefficient matrix has a cone structure outside some ball and obeys some curvature condition. We show that the uniform local decay is controlled by the curvature of a Riemannian metric, given by the coefficient matrix. In particular, some criteria on curvature of the Riemannian metric are given to have uniform local decay.

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Special Session 12: Nonautonomous Dynamics

Russell Johnson, Universitá di Firenze, Italy
Arno Berger, University of Alberta, Canada
Sylvia Novo, Universidad de Valladolid, Spain

Introduction: This section was organized with the goal of presenting an overview of recent activity in the area of Nonautonomous Dynamical Systems. Such systems are defined by differential or difference equations whose coefficients vary with time. The time dependence may exhibit a wide range of behavior, from periodic to stochastic: thus the coefficients may be, for instance, Bohr almost periodic functions, almost automorphic functions, or chain recurrent functions.

The invited speakers will discuss issues related to stability theory, bifurcation theory, control theory, and spectral theory of nonautonomous systems, as well as chaotic dynamics, numerical methods, and finite-time phenomena. Frequently, techniques which are successful when used to study autonomous systems cannot be applied to nonautonomous systems. So new concepts and techniques become necessary; many of these will be taken up in the course of the session.

Sliding Homoclinic Orbits, Bifurcation and Chaos in Almost Periodic Discontinuous Systems

Flaviano Battelli
Marche Politechnic University, Ancona, Italy
(Michal Fečkan (Comenius University))

Chaotic behaviour of a time dependent perturbation of a discontinuous differential equation whose unperturbed part has a *sliding* homoclinic orbit (that is a solution homoclinic to a hyperbolic fixed point with a part belonging to a discontinuity surface) is studied. Following a functional analytic approach we construct a Melnikov-like function $\mathcal{M}(\alpha)$ in such a way that if $\mathcal{M}(\alpha)$ has a simple zero at some point and the time dependent perturbation satisfies a kind of recurrence condition, which is satisfied by almost periodic perturbations, then the system has solutions that behave chaotically. Applications of this result to quasi periodic systems are also given.

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More on Finite-Time Hyperbolicity

Arno Berger
University of Alberta, Canada

Motivated by applications, notably in fluid dynamics, finite-time dynamics aims at identifying dynamical properties of systems that are defined only over a bounded interval of time. Many classical asymptotic concepts do not apply in this situation and have to be modified or replaced altogether. Numerous modified or new concepts have been proposed in this regard, with attention being focussed on the development of finite-time stability, spectral and bifurcation theories that are both practicable and consistent with their classical counterparts. A key ingredient in all these areas is some form of hyperbolicity. After providing a brief overview of hyperbolicity in the classical and the finite-time setting, this talk will present several results that generalise and unify earlier work. In particular, the existence, non-uniqueness and robustness of finite-time (un)stable manifolds will be discussed, as well as the basic problem of detecting (Lagrangian) hyperbolicity from (Eulerian) data encoded in a dynamic partition of the extended phase space. Some of the fundamental challenges inherent to finite-time dynamics, both practical and conceptual, will become apparent.

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Global Attractor for a Non-Autonomous Integro-Differential Equation in Materials with Memory

Tomás Caraballo
Universidad de Sevilla, Spain

We analyze in this talk the long-time behavior of an integro-differential parabolic equation of diffusion type with memory terms, expressed by convolution integrals involving infinite delays and by a forcing term with bounded delay. The assumptions imposed on the coefficients are weak in the sense that uniqueness of solutions of the corresponding initial value problems cannot be guaranteed. Then, it is proved that the model generates a multivalued non–autonomous dynamical system which possesses a pullback attractor.

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State Space Decomposition for Nonautonomous Dynamical Systems

Xiaopeng Chen
Huazhong University, Peoples Rep. of China
(Jinqiao Duan)

Decomposition of state spaces into dynamically different components is helpful for understanding dynamics of complex systems. A Conley type decomposition theorem is proved for nonautonomous dynamical systems defined on a non-compact but sep-

arable state space. Namely, the state space can be decomposed into a chain recurrent part and a gradient-like part. This result applies to both nonautonomous ordinary differential equations on an Euclidean space (which is only locally compact), and nonautonomous partial differential equations on an infinite dimensional function space (which is not even locally compact). This decomposition result is demonstrated by discussing a few concrete examples, such as the Lorenz system and the Navier-Stokes system, under time-dependent forcing.

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Entropy-Like Concepts for Deterministic Control Systems

Fritz Colonius
University of Augsburg, Germany

For deterministic control systems described by ordinary differential equations, the difficulty of solving control tasks can be measured by concepts which are akin to topological entropy in the theory of dynamical systems. In this contribution, a number of results will be presented including controlled invariance for subsets of the state space and the behavior under time-varying perturbations.

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Rotation Number and Exponential Dichotomy for Linear Nonautonomous Hamiltonian Systems

Roberta Fabbri
Universitá di Firenze, Italy

In the talk the properties of the rotation number for a random family of linear nonautonomous Hamiltonian systems and its relation with the exponential dichotomy concept are considered. In particular, it is shown that the presence of an exponential dichotomy for certain perturbed Hamiltonian systems is equivalent to have real intervals in which the rotation number results to be constant.

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Coherent Sets for Nonautonomous Dynamical Systems with Geophysical Applications

Gary Froyland
University of New South Wales, Sydney, Australia
(Simon Lloyd, Naratip Santitissadeekorn)

We describe a mathematical formalism and numerical algorithms for identifying and tracking slowly mixing objects in nonautonomous dynamical systems. In the autonomous setting, such objects are variously known as almost-invariant sets, metastable sets, persistent patterns, or strange eigenmodes, and have proved to be important in a variety of applications. We explain how to extend existing autonomous approaches to the nonautonomous setting. We call the new time-dependent slowly mixing objects coherent sets as they represent regions of phase space that disperse very slowly and remain coherent. We illustrate the new methods of analysis on oceanic and atmospheric data.

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Dynamics of Periodic Point Free Toral Homeomorphisms – from Skew Products to Non-Fibred Maps

Tobias Jaeger
TU Dresden, Germany

The aim of the talk is to describe a link between one-dimensional non-autonomous and two-dimensional autonomous dynamical systems on the two-torus. We concentrate in particular on two results.

1. A conservative toral homeomorphism is semi-conjugate to an irrational rotation of the two-torus if and only if the iterates of its lift to the plane remain within a bounded C^0-distance to the iterates of the strict irrational translation (bounded mean motion). This can be considered as an extension of Poincare's celebrated result on the linearisation of circle homeomorphisms.

2. A non-wandering toral homeomorphism without periodic orbits is either topologically transitive, or it has at least two periodic circloids. In the latter case, there exists a decomposition of the torus into periodic open annuli with transitive dynamics, seperated by regions that are densely filled with invariant circloids.

The crucial concept in this theory is that of invariant or periodic circloids. These are compact periodic sets that 'cut the the torus open', such that their complement is an open annulus, and are minimal with this property. Circloids are a direct generalisation of periodic strips, which play a fundamental role for the dynamics of quasiperiodically forced circle homeomorphisms. The above-mentioned results both had precursors in this context that will equally be discussed.

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Coincidence of Lyapunov Exponents and Central Exponents of Linear Ito Stochastic Differential Equations with Nondegenerate Stochastic Term

Dinh Cong Nguyen
Vietnam Academy of Sci. & Tech., Vietnam
(Nguyen Thi Thuy Quynh)

We show that under a nondegeneracy condition Lya-

punov exponents and central exponents of linear Ito stochastic differential equation coincide. Furthermore, as the stochastic term is small and tends to zero the highest Lyapunov exponent tends to the highest central exponent of the ordinary differential equation which is the deterministic part of the system.

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Monotonone and Sublinear Skew-Product Semiflows I: The General Case

Carmen Nunez
Universidad de Valladolid, Spain
(Rafael Obaya, Ana M. Sanz)

We analyze the dynamics of a general monotone and sublinear skew-product semiflow, paying special attention to the long-term behavior of the strongly positive semiorbits and to the minimal sets. Four possibilities arise depending on the existence or absence of strongly positive minimal sets and bounded semiorbits, as well as on the coexistence or not of bounded and unbounded strongly positive semiorbits. The results extend and unify previously known properties, and allow us to describe scenarios which are impossible in the autonomous or periodic cases.

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Dynamical Theory for Monotone Neutral Functional Differential Equations with Nonautonomous Linear D-Operator

Rafael Obaya
University of Valladolid, Spain
(Victor M. Villarragut)

We apply the skew-product formalism to study neutral functional differential equations with infinite delay and non-autonomous linear D-operator. We obtain different conditions which assure the invertibility of the operator for the compact open topology. In these situations we introduce and investigate different monotone structures. Assuming convenient properties of stability we prove that the omega limit set of each relatively compact trajectory defines a 1-covering of the base.

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Probabilistic and Geometric Descriptions of Coherent Structures in Flows

Kathrin Padberg-Gehle
TU Dresden, Germany
(Gary Froyland)

In this contribution we focus on the numerical detection and approximation of coherent structures and barriers to particle transport in flows, connecting the classical geometrical approach via invariant manifolds with a probabilistic approach via transfer operators and almost-invariant sets. In particular, we analyse several example systems for different dynamical settings. Almost-invariant regions, i.e. regions that minimally mix with their surroundings, are identified via eigenvectors of a transfer operator and are ranked by the corresponding eigenvalues in order of the sets' invariance. While we demonstrate that the corresponding almost-invariant sets are typically bounded by segments of invariant manifolds of hyperbolic objects, without such a ranking it is not at all clear which intersections of invariant manifolds form the major barriers to mixing. Furthermore, we show that boundaries formed by segments of invariant manifolds do not necessarily bound sets of minimal leakage.

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A Finite-Time Condition for Exponential Dichotomy

Kenneth Palmer
National Taiwan University, Taiwan

We prove that if a difference equation on Z has an exponential dichotomy on sufficiently many intervals of sufficiently large fixed length and with uniform constants and exponents, then it has an exponential dichotomy on Z. We give applications of the result to slowly varying systems, almost periodic systems, a perturbation theorem, the variational equation along pseudo orbits lying in a hyperbolic set of a diffeomorphism, transversality of homoclinic orbits of a diffeomorphism and hyperbolicity of a compact invariant set of a diffeomorphism.

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Nonautonomous Bifurcations: Robustness and Imperfection

Christian Poetzsche
Munich University of Technology, Germany

In the framework of nonautonomous dynamical systems, we interpret a bifurcation as a topological change in the set of bounded entire solutions to a given evolutionary equation. A Fredholm theory based on exponential dichotomies enabled us to employ tools from analytical branching theory yielding nonautonomous versions of fold, transcritical and pitchfork patterns.

This approach has the serious drawback that precise quantitative information on the dichotomies is required — an assumption rarely satisfied in applications. In this talk, we therefore discuss persistence and changes in the above bifurcation scenarios under perturbation. This will be done on the basis of

abstract analytical techniques due to Shi (1999) and Liu, Shi & Wang (2007).

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Approximation of Nonautonomous Invariant Manifolds

Martin Rasmussen
Imperial College London, England
(C. Pötzsche, B. Aulbach, S. Siegmund)

In this talk, different approaches to the approximation of nonautonomous invariant manifolds are presented. The nonautonomous techniques under consideration will be illustrated by a discussion of the Lorenz system and the Hénon attractor. The obtained results are joint work with C. Pötzsche (2/3 of the talk) and B. Aulbach and S. Siegmund (1/3 of the talk).

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Dynamical Systems Approach to a Uniqueness Problem of a Nonautonomous Planar System

Eyal Ron
Free University, Berlin, Germany

We presents a dynamical systems approach for handling uniqueness problems in nonautonomous planar systems. The method is shown while being applied to a problem of uniqueness of a ground state of the continuum limit of a strictly anharmonic multidimensional lattice. Although this problem, and even a more general version of it, was already solved by Pucci and Serrin, we believe that our method sheds a new light on it.

Our solution to the uniqueness problem has two parts. In the first one we show, using classical phase plane analysis tools, that solutions in the first quadrant of the plane cannot cross each other even though the system is nonautonomous. The second part presents a local result, i.e. that is independent of the initial condition, when analyzing what happens near the origin. In addition we discuss Pucci and Serrin's solution to the problem, and present it while using some concepts and results from the dynamical systems approach.

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Monotonone and Sublinear Skew-Product Semiflows II: Two-Dimensional Systems of Differential Equations

Ana Sanz
University of Valladolid, Spain
(C. Núñez, R. Obaya)

As a continuation of the talk entitled 'Monotonone and sublinear skew-product semiflows I: the general case' presented in this session, we explore further the description of the long-term behavior of monotone and sublinear skew-product semiflows in the case that they are defined from recurrent nonautonomous differential equations. Our study includes ordinary, finite-delay and reaction-diffusion differential equations. We provide a complete description of all the minimal sets, and show the optimality of the results by means of suitable examples. Again, some significative differences with the autonomous case are established.

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Avoidance of Sets for Flow Solutions of Non-Autonomous ODEs

Nick Sharples
University of Warwick, England
(James Robinson)

A regular Lagrangian flow solution X to the ODE $\dot{\xi} = b(\xi)$ is said to avoid a subset $S \subset \mathbb{R}^n$ of the phase space if for every time interval $(-\tau, \tau)$ the set of initial conditions whose trajectories approach S during this interval, $\left\{x \middle| \inf_{t\in(-\tau,\tau)} \operatorname{dist}(X(t,x), S) = 0\right\}$, has zero n-dimensional Lebesgue measure. Aizenman showed in 1978 that for $b \in L^q(\mathbb{R}^n)$ avoidance is guaranteed if the upper box counting dimension of S is sufficiently small. For the non-autonomous case $b \in L^p(0,T; L^q(\mathbb{R}^n))$ we extend the definition of avoidance in a way consistent with embedding nonautonomous systems into autonomous systems of higher dimension. Cipriano & Cruzeiro generalised the avoidance result in 2005 to the case when $b \in L^1(0,T;L^q)$ and the subset of the phase space $S \subset [0,T] \times \mathbb{R}^n$ has a product structure $S = [0,T] \times S_x$ for some $S_x \subset \mathbb{R}^n$. An elementary proof of the same result for $q = \infty$ was given by Robinson & Sadowski in 2009.

In the general non-autonomous case, where $S \subset [0,T] \times \mathbb{R}^n$ is arbitrary, we can write S as a subset of a set with the above product structure which yields an avoidance criterion for the general case. We give an example which shows that the avoidance criterion obtained in this fashion is not optimal. For the general case we give the following criterion for avoidance: for $b \in L^p(0,T;L^q)$ a regular Lagrangian flow solution X avoids the set $S \subset [0,T] \times \mathbb{R}^n$ if the function $(t,x) \mapsto \operatorname{dist}((t,x), S)^{-1}$ is in $L^{p^*}(0,T;L^{q^*})$ where $\frac{1}{p} + \frac{1}{p^*} = \frac{1}{q} + \frac{1}{q^*} = 1$. We discuss how the integrability of $\operatorname{dist}((t,x), S)^{-1}$ characterises the fractal nature of S and how this relates to other notions of fractal dimension.

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On the Error in the Approximation of Evans Functions and Exponential Dichotomies

Erik Van Vleck
University of Kansas, USA

We consider the numerical approximation of Evans functions, a commonly used technique to determine linear stability of traveling wave solutions to nonlinear evolution equations. The error in the approximation of Evans functions and in general computational procedures to determine whether a not a given linear differential equation has exponential dichotomy involves a global error analysis. We outline the ingredients of such a global error analysis, local error bounds and ideas related to continuity of Lyapunov exponents, and provide some examples that show the efficacy of our techniques.

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The Liouville Transform and the Connection between the K-dV and the Camassa-Holm Hierarchies

Luca Zampogni
Universita di Perugia, Italy
(Russell Johnson)

We use the Liouville transform, which establishes a connection between the Schrödinger and the Sturm-Liouville operators on the line, to show that there is deep relation between the K-dV and the Camassa-Holm (CH) hierarchies. These hierarchies are obtained through the coefficients of the asymptotic expansions of certain Green's functions. New solutions of the (CH)-hierarchy are obtained from initial data which derive from generalized reflectionless Schrödinger potentials in the sense of Kotani.

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Special Session 13: Stability of Partial Differential Equations and Evolution Equations

Yuri Latushkin, University of Missouri-Columbia, USA
Roland Schnaubelt, Karlsruhe Institute of Technology, Germany

Introduction: The speakers will discuss recent results on stability methods for solutions of partial differential equations

Stabilization and Energy Decay Rates of Memory Damped PDE's

Fatiha Alabau-Boussouira
Metz Univ. and INRIA t.p. CORIDA, France

Evolutions equations may be subjected to the action of non local dissipative operators. This type of dissipation appears in the framework of stabilization of viscoelastic materials for which the damping involves a convolution operator in time with a decreasing positive kernel. We will present some recent results for energy decay rates and stabilization in this context.

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Stability of Rotating Patterns and the Freezing Method

Wolf-Jürgen Beyn
Bielefeld University, Germany
(Jens Lorenz, Vera Thümmler)

We consider semilinear parabolic systems in two space dimensions which exhibit localized rotating patterns. Sufficient conditions are given that guarantee stability with asymptotic phase in suitable Sobolev spaces. The result is related to a numerical approach, called the method of freezing. It uses the equivariance structure of the PDE and transforms the given system into a partial differential algebraic equation (PDAE) which is subsequently discretized by truncation to a bounded domain with finite boundary conditions. The approach allows to freeze the target pattern in a rotating frame while giving simultaneously information about the angular and translational speeds. We discuss how Lyapunov stability of the resulting PDAE and its numerical analog relate to the properties of the original equation.

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Decay of Orbits of Bounded C_0-Semigroups on Hilbert Spaces

Ralph Chill
Universite de Metz, France
(Yuri Tomilov)

Last years there was an increasing interest to the study of the rates of decay of bounded C_0-semigroups motivated by concrete applications to PDEs, e.g. to the study of energy decay for damped wave equations. We intend to review recent results and technique pertaining to decay of bounded C_0-semigroups on Banach spaces obtained by Batty & Duyckaerts (2008) and Borichev & Tomilov (2010) and to present certain improvements over the work by Borichev & Tomilov.

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Well-Posedness, Instabilities and Bifurcation for the Flow in a Rotating Hele-Shaw Cell

Mats Ehrnstrom
Leibniz University Hannover, Germany
(J. Escher and B.-V. Matioc)

We study the motion of an incompressible fluid located in a Hele-Shaw cell rotating at constant speed in the horizontal plane. This is a time-dependent moving-boundary problem, which we recast as an abstract Cauchy problem. Using elliptic theory in combination with theory for analytic semi-groups on suitable spaces, local existence and uniqueness of solutions is proved.

Another topic of interest is time-independent solutions. In this context, we show that for each given volume there is exactly one rotationally invariant equilibrium, and that it is unstable. There are, however, other time-independent solutions: With the aid of bifurcation theory we establish the existence of global branches of stationary fingering patterns. Further properties of the solutions along those branches can then be deduced.

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Asymptotic Stability in the Energy Space for Multi-Soliton Solutions in the Toda Lattice

Aaron Hoffman
Boston University, USA
(G. N. Benes, and C. E. Wayne)

We prove that, for the Toda Lattice, initial data which is close in ℓ^2 to an n-soliton solution, remains close to an n-soliton solution for all time. Furthermore, the residual decays in a weighted norm. For the linear estimates we make use of Backlund Transformations which relate n- and $n-1$-soliton solutions and an idea of Mizumachi and Pego.

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Stability and Interactions of BBM Solitary Waves

Henrik Kalisch
University of Bergen, Norway
(Hai Yen Nguyen, Nguyet Thanh Nguyen)

Stability and interactions of solitary waves of the regularized long-wave equation are studied. Both theoretical and numerical results will be presented. Particular attention will be given to solitary waves of depression and their stability properties.

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Evolution Equations for Simulation and Stability Analysis of Semiconductor Lasers

Mark Lichtner
WIAS, Berlin, Germany

Semiconductor lasers belong to the most efficient coherent light sources with electro-optical efficiencies of more than 70%. They are ubiquitous in everyday life as we face them e.g. in CD players, printers, displays and product scanners. However, laser diodes are characterized by a huge amount of structural and design parameters and are subject to dynamic instabilities. To provide an understanding of the complicated operation regimes observed in experiments, to get an understanding of the underlying physics, and to permit the exploration of novel design concepts mathematical modeling and numerical simulation tools are essential and are becoming more important nowadays. Realistic mathematical models consist of complicated systems of dissipative wave equations which are nonlinearly coupled to carrier and heat diffusion equations. We will show simulation and experimental results and perform a rigorous mathematical well posedness and linearized stability analysis based on semigroup techniques in suitable function spaces.

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On Parabolic Systems with Nonlinear Dynamic Boundary Conditions

Martin Meyries
Karlsruhe Institute of Technology, Germany

We present a theory for well-posedness of strong solutions for quasilinear parabolic systems with nonlinear dynamic (or Wentzell) boundary conditions, aiming at low initial regularity. The approach is

based on a maximal regularity result for the inhomogeneous linearized problem in L_p-spaces with temporal weights, which allows to require lower regularity for the initial values and incorporates a smoothing effect on the solutions. This effect is employed to obtain criteria for global existence. As an application we study the long-time behavior of solutions of reaction-diffusion systems with nonlinear boundary conditions of reactive-diffusive type.

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The Existence of Bifurcating Invariant Tori in a Spatially Extended Reaction-Diffusion-Convection System with Spatially Localized Amplification

Guido Schneider
Universität Stuttgart, Germany
(Andreas Kirchhoff)

We consider a spatially extended reaction-diffusion-convection system with marginal stable ground state and spatially localized amplification. We are interested in solutions bifurcating from the spatially homogeneous ground state in case that simultaneously pairs of imaginary eigenvalues cross the imaginary axis. For this system we prove the bifurcation of an invariant torus which possibly contains quasiperiodic solutions. There is a serious difficulty according to the fact that the linearization at the ground state possesses essential spectrum up to the imaginary axis for all values of the bifurcation parameter. The proof is based on the hard implicit function theorem and energy estimates. We discuss the stability of the bifurcating solutions, too.

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Decay of Orbits of Bounded C_0-Semigroups on Banach Spaces

Yuri Tomilov
Torun University and PAN, Warsaw, Poland
(A. Borichev, R. Chill)

Last years there was an increasing interest to the study of the rates of decay of bounded C_0-semigroups motivated by concrete applications to PDEs, e.g. to the study of energy decay for damped wave equations. We intend to review recent results and technique pertaining to decay of bounded C_0-semigroups on Banach spaces obtained by Batty&Duyckaerts(2008) and Borichev&Tomilov(2010) and to present certain improvements over the work by Borichev&Tomilov.

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Modulation Equations for the Evolution of Wavepackets in Vlasov-Poisson Plasmas

Hannes Uecker
Universität Oldenburg, Germany

The Vlasov-Poisson system from plasma physics shows some linear instabilities which yield a linear growth of small amplitude wave packets. For very large times these may lead to a well known plateau formation described by a quasilinear diffusion equation. However, the range of intermediate to large times seems to have been investigated less systematically. We derive a Nonlinear Schrödinger equation with some dissipative perturbation terms in the weakly nonlinear regime and compare its predictions with full Vlasov-Poisson simulations.

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On Convergence of Solutions to Equilibria for Quasilinear Parabolic Problems

Rico Zacher
University Halle-Wittenberg, Germany
(Jan Prüss, Gieri Simonett)

We show convergence of solutions to equilibria for quasilinear parabolic evolution equations in situations where the set of equilibria is non-discrete, but forms a finite-dimensional C^1-manifold which is normally stable. Our results do not depend on the presence of an appropriate Lyapunov functional as in the Lojasiewicz-Simon approach, but are of a local nature. As applications we consider the asymptotic behaviour of the solutions of the Mullins-Sekerka model and discuss the stability of travelling waves for a quasilinear parabolic equation.

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Special Session 14: Geometric Mechanics

Manuel de Leon, Instituto de Ciencias Matematicas, Spain
Juan-Pablo Ortega, CNRS, Universiti de Franche-Comti, UFR des Sciences et Techniques, France
Andrew Lewis, Queen's University, Canada

Introduction: The research on mechanical and dynamical systems has had a deep impact in other research areas as well as in the development of several technologies. A big part of its advances has been based on numerical and analytical techniques. In the sixties it has been introduced the most sophisticated and powerful techniques coming from Geometry and Topology, which have led, for instance, to the beginning of the modern Hamiltonian Mechanics. Geometric techniques have been also applied to diverse control problems in locomotion systems, robotics, etc. Most of these ideas have been developed in the last 30 years by Mathematicians of a high scientific level such as J. Marsden, A. Weinstein, R. Abraham, V. Arnold or R. Brockett among others. The goal of this Special Session is to bring together researchers working in the applications of geometric methods (in a broad sense) to mechanics and control theory, paying special attention to facilitate the interaction between theory and applications.

Lagrangian Submanifolds in Nonholonomic Mechanics

Manuel de Leon

The use of Lie algebroids has permitted a better understanding of the Hamilton-Jacobi theory for nonholonomic mechanical systems. The aim of this paper is to go further in the geometric setting of the theory, introducing a convenient notion of lagrangian submanifold.

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Noether-Like First Integrals in Nonholonomic Mechanics

Francesco Fassò
University of Padova, Italy
(Andrea Giacobbe and Nicola Sansonetto)

We review recent results on the link between symmetries and conservation laws in nonholonomic mechanics, with particular attention to linear first integrals ('gauge mechanism', 'momentum equation') and their 'weakly Notherian' character.

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Meromorphic and Hypergeometric Solutions of the Nonholonomic Suslov Problem

Yuri Fedorov
Politechnic University of Catalonia, Spain
(Andrzej Maciejewski, Maria Przybylska)

We consider the long standing problem of integrability of the Poisson equations describing spatial motion of the rigid body in the classical nonholonomic Suslov problem.

In the more general case, the Poisson equations are transformed into a generalized third order hypergeometric equation. A study of its monodromy group allows us also to calculate the "scattering" angle: the angle between the axes of limit permanent rotations of the body in space.

In the particular cases, when the solution is meromorphic, we present it in an explicit form and show that it corresponds to rather special motions of the body.

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The Modular Class of a Poisson Map

Rui Loja Fernandes
Instituto Superior Tecnico, Lisbon, Portugal
(Raquel Caseiro)

We introduce the modular class of a Poisson map. We look at several examples and we apply it to understand the behavior of the modular class of a Poisson manifold under different kinds of reduction, as well as its implications to Hamiltonian dynamics. We also discuss its symplectic groupoid version, which lives in groupoid cohomology.

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Stochastic Chaplygin Systems

Simon Hochgerner
EPFL, Switzerland

We mimic the stochastic Hamiltonian reduction of Lazaro-Cami and Ortega for the case of certain nonholonomic systems with symmetries. Using the nonholonomic connection it is shown that the drift of the stochastically perturbed n-dimensional Chaplygin ball is a certain gradient of the density of the preserved measure of the deterministic system.

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Canonical Vector Fields on Tangent Bundles

Demeter Krupka
University Brno & Univ. Melbourne, Czech Rep.

Complete classification of canonical vector fields on

tangent bundles of differential manifolds is obtained. It is shown that every canonical vector field is a linear combination with constant coefficients of three vector fields, a variational vector field, the Liouville vector field and the vertical lift of a vector field.

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Regular Variational Problems on Nonholonomic Manifolds

Olga Krupkova
Univ. of Ostrava & Univ. Melbourne, Czech Rep.

Variational mechanical systems on nonholonomic manifolds are modelled as exterior differential systems, and classified according to their dynamical properties. Regular constrained systems are described and studied in detail.

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Routh Reduction for Quasi-Invariant Lagrangians

Bavo Langerock
St-Lucas School of Architecture, Belgium
(F. Cantrijn and J. Vankerschaver)

We describe Routh reduction as a special case of standard symplectic reduction. We use this correspondence to present a generalization of Routh reduction which is valid for Lagrangians that are quasi-invariant under the action of a Lie group, i.e. Lagrangians invariant up to a total time derivative. We discuss the special case of systems with quasi-cyclic coordinates and we show how a reduction technique described in [1] called functional Routhian reduction can be seen as a particular instance of Routh reduction of a quasi-cyclic coordinate.

[1] Ames, A. D., Gregg, R. D. and Spong, M. W. “A geometric approach to three-dimensional hipped bipedal robotic walking” In 46th IEEE Conference on Decision and Control, pp.5123-5130 (2007).

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Hamiltonian Dynamics on Lie Algebroids, Unimodularity and Preservation of Volumes

Juan Carlos Marrero
University of La Laguna, Spain

In this talk, I will present some ideas about the relation between the unimodularity of a Lie algebroid and the existence of invariant volume forms for the hamiltonian dynamics on the dual bundle.

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Kinematic Reduction and Hamilton-Jacobi Theory

David Martín de Diego
ICMAT, Madrid, Spain
(M. de León, J.C. Marrero and M. Muõz-Lecanda)

For control purposes, it is very interesting to reduce a mechanical control system to a kinematic system. For instance, it would be able to plan the motion for the associated kinematic system instead of working with the initial mechanical control. Unfortunately, there is no a systematic procedure for finding such a kinematic reduction. In this talk, we will careful study the geometry behind the kinematic reductions showing the close relation with the classical Hamilton-Jacobi theory.new geometric integrators for optimal control problems for mechanical systems, in particular, for the numerical integration of systems with symmetry.

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Reduction in Nonholonomic Mechanics from a Variational Point of View

Eduardo Martínez
University of Zaragoza, Spain

Reduction for holonomic mechanical systems has been studied conveniently within the framework of mechanical systems on Lie algebroids, from the symplectic/Poisson perspective as well as the from the variational one. For nonholonomic systems a symplectic description has been already provided. I will complement the picture with the study of reduction from the point of view of variational calculus.

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Nonholonomic Systems as Restricted Euler-Lagrange Systems

Tom Mestdag
Ghent University, Belgium

For a mechanical system with linear nonholonomic constraints we present a Lagrangian formulation of the nonholonomic and vakonomic dynamics using the method of anholonomic frames. We use this approach to deal with the issue of when a nonholonomic system can be interpreted as the restriction of a special type of Euler-Lagrange system. We give an application of this result in the context of geometric integrators.

[1] M. Crampin and T. Mestdag, Anholonomic frames in constrained dynamics, accepted in Dynamical Systems, DOI: 10.1080/14689360903360888.

[2] A. M. Bloch, O. E. Fernandez and T. Mestdag, Hamiltonization of nonholonomic systems and the inverse problem of the calculus of variations, Rep. Math. Phys. 63 (2009) 225-249.

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The Hydrodynamic Chaplygin Sleigh

Luis Garcia Naranjo
EPFL, Switzerland
(Yu. Fedorov)

We consider the motion of rigid bodies in a potential fluid subject to certain nonholonomic constraints and show that it is described by Euler-Poincaré-Suslov equations. In the 2-dimensional case, when the constraint is realized by a blade attached to the body, the system provides a hydrodynamic generalization of the Chaplygin sleigh, whose dynamics are studied in detail. Namely, the equations of motion are integrated explicitly and the asymptotic behavior of the system is determined. It is shown how the presence of the fluid brings new features to such behavior.

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Superposition Rules and Stochastic Lie-Scheffers Systems

Juan-Pablo Ortega
CNRS, Universite de Franche-Comte, France
(Andreu Lazaro)

In this talk we will show a version of the Lie-Scheffers Theorem for stochastic differential equations. This result characterizes the existence of nonlinear superposition rules for the general solution of those equations in terms of the involution properties of the dstribution generated by the vector fields that define it. When stated in the particular case of standard deterministic systems, our main theorem improves various aspects of the classical Lie-Scheffers result. We show that the stochastic analog of the classical Lie-Scheffers systems can be reduced to the study of Lie group valued stochastic Lie-Scheffers systems.

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The Variational Structure of Complex Fluid Equations

Tudor Ratiu
Ecole Polytechnique Fd́érale de Lausanne, Switzerland

In this talk I will present the variational structure of complex fluid equations with special emphasis on liquid crystals. In order to get the equations of motion a new kind of Euler-Poincaré reduction is needed that involves group cocycles and affine actions. The Eringen equations for liquid crystals will be discussed in detail.

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The Role of Isotropy in the Dynamics of Relative Equilibria for Hamiltonian and Mechanical Systems

Miguel Rodriguez-Olmos
The University of Manchester, England

We will use the bundle equations of Roberts et al for symmetric Hamiltonian systems in order to study bifurcations of their relative equilibria. It will be shown how the existence of continuous isotropy groups for the symmetry action can affect these properties with respect to the free case. Notably, we will discuss how these isotropy groups can induce bifurcations from a formally stable branch of relative equilibria. Finally, we will show how to particularize these results to the case of mechanical systems of the form kinetic + potential energy and study some examples.

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On a Coulomb Three-Body System

Jürgen Scheurle
Technical University of Munich, Germany

Motivated by certain physical and chemical applications we study the motion of a positively and a negatively charged particle in the field of a fixed positive point charge. Depending on some relations between the charge and mass values various types of interesting dynamical behaviour occur. Using Geomteric Mechanics and Dynamical Systems methods we are going to analyze this behaviour. It turns out that there are some specific features of the present system as compared to other three-body systems investigated in the literature.

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Singularities of Poisson Structures and Hamiltonian Bifurcations

Jan-Cees van der Meer
Technische Universiteit Eindhoven, Netherlands

Consider a Poisson structure on $C^\infty(\mathbb{R}^3,\mathbb{R})$ with bracket $\{,\}$ and suppose that C is a Casimir function. Then $\{f,g\} =$ is a possible Poisson structure. This confirms earlier observations concerning the Poisson structure for Hamiltonian systems that are reduced to a one degree of freedom system and generalizes the Lie-Poisson structure on the dual of a Lie algebra and the KKS-symplectic form. The fact that the governing reduced Poisson structure

is described by one function makes it possible to find a representation, called the energy-momentum representation of the Poisson structure, describing both the singularity of the Poisson structure and the singularity of the energy-momentum mapping and hence the bifurcation of relative equilibria for such systems. It is shown that Hamiltonian Hopf bifurcations are directly related to singularities of Poisson structures of type $\mathfrak{sl}(2)$.

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Port-Hamiltonian Systems on Open Graphs

Arjan van der Schaft
University of Groningen, Netherlands

A topic of great current interest, motivated by diverse applications, is the subject of dynamics on networks. In this talk we discuss how one can define generalized Hamiltonian dynamics (possibly including resistive elements, algebraic constraints, and external ports) on graphs in an intrinsic way. Main tool in this endeavor is the definition of two Dirac structures determined by the graph. (A Dirac structure is a geometric object generalizing at the same time symplectic forms and Poisson brackets.)

The first Dirac structure, dating back to the classical work by Kirchhoff, is the appropriate Dirac structure for e.g. defining the dynamics of RLC electrical circuits in a port-Hamiltonian way. In this case the dynamics is associated to the edges of the graph. The second Dirac structure allows to associate dynamics to every vertex of the graph, and is the natural Dirac structure to formulate e.g. the dynamics of consensus algorithms for multi-agent systems, or the dynamics resulting from coordination control strategies. We will also discuss the possibility to extend this framework to one-complexes, thereby also extending to chemical reaction networks.

Next to the modeling and analysis of port-Hamiltonian dynamics on graphs we will discuss ways to synthesize or control the port-Hamiltonian dynamics on graphs, and to reduce the complexity of the dynamics in a structure-preserving manner.

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Modeling Fluid-Structure Interactions Using Symplectic Reduction

Joris Vankerschaver
Ghent University, Belgium
(E. Kanso, J. E. Marsden)

In this talk, we study the dynamics of rigid bodies immersed in a perfect flow. We devote particular attention to rigid bodies with circulation. I exhibit some of the geometric structures underlying this problem by reformulating it as a geodesic motion on the product of a space of embeddings with the euclidian group. After identifying the symmetry groups of this problem, comprising the particle relabeling symmetry and global translations and rotations, I use symplectic reduction by stages to obtain a finite-dimensional Hamiltonian description for this system. In this way, several different systems of equations found in the literature on fluid-structure interactions can be given a unified description.

In particular, when the system has circulation, we obtain in this way the equations of Chaplygin and Lamb. Time permitting, I will show how these equations are geodesic on a central extension of the special Euclidian group, and I will show how this description can be derived through reduction using some concepts from symplectic geometry such as the flux homomorphism.

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Quasivelocities and Symmetries in Nonholonomic Systems and Control

Dmitry Zenkov
North Carolina State University, USA
(Anthony M. Bloch, Jerrold E. Marsden)

Quasivelocities are the components of a mechanical system's velocity relative to a set of vector fields that are not associated with configuration coordinates. This talk discusses how quasivelocities may be used in the formulation of nonholonomic systems with symmetry. In particular, the use of quasivelocities in understanding unusual momentum conservation laws is investigated, as is the applications of these conservation laws and discrete symmetries to the qualitative analysis and control of nonholonomic dynamics.

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Special Session 15: Magnetohydrodynamics: Mathematical Problems and Astrophysical Applications

Antonio Ferriz-Mas, Universidad de Vigo and IAA/CSIC, Spain
Rainer Hollerbach, University of Leeds, United Kingdom
Manuel Nunez, Universidad de Valladolid, Spain
Frank Stefani, Forschungszentrum Dresden-Rossendorf, Germany
Andreas Tilgner, University of Göttingen, Germany

Introduction: The session is devoted to several aspects of astrophysical magnetohydrodynamics where the mathematics of the problem plays an essential role in the present research. These problems include among others the topology of the magnetic field and its evolution invariants, magnetic instabilities, relativistic magnetohydrodynamics and homogeneous dynamos. While the mathematical and numerical aspects are emphasized, experimental results and their theoretical interpretation are also welcome.

Braid Theory and Self-Organized Criticality in MHD

Mitchell Berger
University of Exeter, England

Two great puzzles in solar astrophysics concern the source of coronal heating and the distribution of solar flares. The atmosphere of the sun is heated to one million degrees or more, possibly by swarms of tiny flares. These tiny flares could be consequences of the braiding of magnetic field lines. Reconnection between braided threads of magnetic flux can release energy stored in the braid. The larger flares exhibit a power law energy distribution. Several authors have suggested that a self-organization process in the solar magnetic field could lead to such a distribution. Here we show how reconnection of braided lines can organize the small scale structure of the field, leading to power law energy release.

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Shear Flow Instability and Turbulence at Low Magnetic Reynolds Number

Thomas Boeck
TU Ilmenau, Germany
(Dmitry Krasnov, Oleg Zikanov, Maurice Rossi)

Experiments with liquid metals can be used to study applied as well as fundamental problems of MHD. Such experiments are inevitably performed in the presence of bounding walls, which give rise to specific MHD boundary layers that may carry considerable fractions of the induced currents. For detailed understanding of transition and turbulence in liquid-metal MHD by simulation and theory it is therefore necessary to consider wall-bounded flows even if one is mainly interested in the bulk phenomena.

We report recent numerical investigations of transition and turbulence in the generic channel and duct geometries with a homogeneous magnetic field in the approximation of low magnetic Reynolds number. We focus on nonlinear transition mechanisms involving transient linear growth of non-modal perturbations, which we compute by an optimization method. The nonlinear transition to turbulence and the turbulent dynamics itself are investigated by direct simulation with parallel codes based on spectral and finite-difference discretizations. We also discuss mathematical challenges posed by the use of the full induction equation for future studies at finite magnetic Reynolds number.

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On Some Conservation Theorems for Vorticity and for the Magnetic Field

Antonio Ferriz-Mas
Universidad de Vigo and IAA/CSIC, Spain

The analogy between the magnetic induction equation for the magnetic field in Magnetohydrodynamics (MHD) and the vorticity equation in Hydrodynamics (HD) for a fluid with uniform density is well known. A number of conservation theorems – holding when the respective diffusivities vanish – can be deduced (e.g., Kelvin's, Alfvén's, Helmholtz' and Walén's theorems, as well as some conservation theorems involving the kinematic and the magnetic helicities).

In the HD case, the restriction of uniform density may be replaced by a weaker restriction of thermodynamic nature, while in the MHD case the conservation theorems remain unaffected by the compressibility of the plasma or by any consideration of thermodynamic nature.

In this contribution I point out the similarities and differences between the conservation theorems for vorticity and for the magnetic field in the HD and the MHD cases. Although the Mathematics involved in both cases is practically the same, the Physics behind these conservation theorems is quite different.

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MHD Simulations of Relativistic Stars

Jose Font
University of Valencia, Spain
(Pablo Cerda-Duran, Nikolaos Stergioulas, and Ewald Müller)

We present MHD simulations of relativistic stars performed with a numerical code which solves the general relativistic magneto-hydrodynamics equations coupled to the Einstein equations (in the conformally flat approximation) for the evolution of a dynamical spacetime. This code has been developed to study astrophysical scenarios in which both, high magnetic fields and strong gravitational fields appear, such as the magneto-rotational collapse of stellar cores and the evolution of highly magnetized neutron stars, or magnetars. In the talk we first discuss magneto-rotational core collapse simulations of a realistic iron core progenitor. Next we discuss simulations of Alfvén oscillations in magnetars, a likely outcome of gravitational collapse, which we model as a relativistic star with a dipolar magnetic field. Mechanisms affecting the amplification of the magnetic field during the collapse and the presence of quasi-periodic oscillations in the resulting magnetars are briefly discussed.

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Fully Developed MHD Turbulence under Strong Input of Cross-Helicity and Magnetic Helicity

Peter Frick
Institute of Cont. Media Mechanics, Perm, Russia
(Irina Mizeva, Rodion Stepanov)

Fully developed small-scale MHD turbulence is an essential part of any astrophysical dynamo, playing the key role in the energy balance regulation. It provides the energy transfer to dissipation range, the effective diffusion of large-scale magnetic field (beta effect) as well as the magnetic field generation in large (i.e. alpha effect) and small (small-scale dynamo) scales. The MHD equations in the dissipationless limit conserve three quadratic quantities: energy, cross-helicity and magnetic helicity, both of helicities can strongly affect the cascade processes. We study the cascade of magnetic helicity and the role of the cross-helicity, which characterizes the correlation of the velocity field and the magnetic field. It is known that high level of cross-helicity suppresses the nonlinear interactions and therefore blocks the spectral energy transfer. We study in details what happens if the external force, which maintains the turbulence, permanently injects small or moderate quantity of cross-helicity. Firstly, we introduce a shell model of MHD-turbulence which differs from conventional shell models by the definition of helicities (the model allows us to define the helicity of any sign in each shell, while previous models attribute the sign of helicity to the balance of odd and even shells). Secondly, we consider the influence of the cross-helicity input rate on the properties of developed MHD turbulence.

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Mathematical Background of the Riga Dynamo Experiment

Agris Gailitis
Institute of Physics, University of Latvia

Ponomarenko was the first (1973) who considered magnetic field generation by an endless helical stream moving as a solid cylinder and maintaining in full electrical contact with its immobile surroundings. Solving the induction equation at very high magnetic Reynolds number by means of Bessel function asymptotic he discovered a temporary growing magnetic field. As the growing solution exists even at moderate magnetic Reynolds number this flow was chosen for the first liquid Na laboratory dynamo experiment. Unfortunately the solution belongs to so called convective instability, hence the original Ponomarenko flow can serve only as a field amplifier but not as a real generator. The flow modification necessary for genuine generation is a topic for the present talk.

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Dynamo Action in Heterogenous Domains

Andre Giesecke
Forschungszentrum Dresden, Germany
(Frank Stefani, Gunter Gerbeth)

From the technically point of view the realisation of dynamo action in the laboratory is a demanding task because it requires magnetic Reynolds numbers of the order of 10…100. In order to reach such values in experiments, materials with high relative magnetic permeabilities have been utilized (Lowes & Wilkinson, von Karman Sodium dynamo).

The modification of the induction process by material properties is examined by means of simulations of the kinematic induction equation in hetrogenous domains where disk like assemblies with high conductivity and/or high permeability are introduced in a cylindrical volume filled with liquid sodium.

Both material properties not only lead to a decrease of the effective magnetic Reynolds number but also result in a quite distinct geometric structure of the final eigenmode. Furthermore, high permeability material even if localized in a small volume like the soft iron impellers in the VKS dynamo, essentially determines the field generation process

and is reponsible for the selection of the dominating azimuthal dynamo mode.

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A New Model for Reversals

Christophe Gissinger
University of Princeton, NJ., USA
(S. Fauve, E. Dormy)

One of the most intriguing problems in geophysics is the ability of the Earth's magnetic field to reverse its polarity. Recently, the VKS experiment was able to reproduce for the first time in laboratory such chaotic inversions of a dynamo-generated dipolar field. Because reversals occur in different problems, like thermal convection or fluids dynamos, several dynamical systems have been proposed to model chaotic reversals using low dimensional dynamics. Using symmetry arguments and comparison with numerical simulations, we will present a new model for chaotic reversals, based on the interaction between 3 modes. We will show that the deterministic chaos occurring in this simple system yields a behavior similar to the one observed in turbulent numerical simulations. Finally, we will compare the model with reversals of the magnetic field of the Earth obtained from recent observations.

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Magnetohydrodynamic Spherical Couette Flow

Rainer Hollerbach
University of Leeds, UK, England

Numerical solutions are presented of the flow of an electrically conducting fluid confined in a spherical shell, with the outer sphere stationary, the inner sphere rotating, and a magnetic field imposed parallel to the axis of rotation. Both axisymmetric basic states and their non-axisymmetric instabilities are computed. Two distinct instability classes emerge, both arising from the basic state's meridional circulation, but otherwise very different from one another, and separated by a region of stability that persists even for large Reynolds numbers. We suggest that the experimental results of Sisan et al. (doi:10.1103/PhysRev Lett. 93.114502) may perhaps be turbulent analogs of some of these instabilities.

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Turbulent Relaxation of Braided Magnetic Fields

Gunnar Hornig
University of Dundee, United Kingdom, Scotland
(Anthony Yeates, Antonia Wilmot-Smith, David Pontin)

Magnetic braiding of coronal loops due to the motion of their photospheric footpoints has long been discussed as a possible mechanism for the heating of the solar corona (E. Parker, 1972). This motivated a series of numerical experiments (Wilmot-Smith et al. 2009, 2010) on the turbulent relaxation of braided magnetic fields. These experiments have produced relaxed states which differ drastically from the predictions of the Taylor hypothesis.

We present a new topological measure, a type of generalised flux function, which allows us to analyse the relaxation process and which shows that there are further constraints on the relaxation process, beyond the conservation of the total helicity, which prevent the system from relaxing to a Taylor state.

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Initial-Value Problems for the Poloidal Magnetic Field

Ralf Kaiser
University of Bayreuth, Germany
(Hannes Uecker)

The kinematic dynamo equation constitutes a system of parabolic equations for the magnetic field components with coefficients provided by a prescribed flow field. In general, the flow field couples these components in a nontrivial way which makes the question for non-decaying solutions difficult to answer. Only in special situations a field component or a related scalar quantity decouples, and a general decay result may be obtained. Examples are the case of an axisymmetric magnetic field or the case of a non-radial flow field. Here, a scalar quantity describing the poloidal magnetic field decouples and satisfies a sourceless parabolic equation in the conductor. At the boundary it has to C^1-match with some harmonic function in the unbounded vacuum region vanishing at spatial infinity. As is well-known describing axisymmetry in cylindrical coordinates introduces singular coefficients. These can be avoided, however, by a five-dimensional formulation. So, the above problem should be considered not only in three dimensions. The focus of dynamo theory is less on existence theorems than on decay results for the magnetic field under certain restrictions on the magnetic field and/or the flow field, thus excluding dynamo action under these restrictions. However, proving decay results requires sometimes the solution of an auxiliary problem. For

instance, in proving a "non-radial velocity theorem" one needs positive solutions of an auxiliary problem which differs from the problem above only in the asymptotic condition at spatial infinity. Similarly in the axisymmetric problem, Backus made use of solutions of an auxiliary problem of the above type without having established them rigorously. So, there is some need to examine the above problem more closely. The basic idea of our treatment is to consider the above initial-value problem in all space as a parabolic problem in a bounded domain (the conductor) with non-local boundary condition, and to carry over the well-established methods for linear parabolic equations with standard boundary conditions such as Dirichlet's or Neumann's boundary condition to our situation: First, we solve weakly a Poisson-type problem in all space. The regularity of the solution in the conductor and in the vacuum region are standard, only at the boundary we need special considerations. Next, we consider the corresponding eigenvalue problem and the associated Fourier series. As a by-product we prove the completeness of the well-known poloidal free decay modes where the conductor is a ball in three dimensions. Finally, the initial-value problem is solved by a Galerkin procedure based on the above described eigenfunctions.

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Relativistic MHD: A Full Wave Decomposition Riemann Solver

Juan Miralles
Universidad de Alicante, Spain

(L. Anton, J. M. Marti, J. M. Ibanez, M. A. Aloy, P. Mimica)

Relativistic flows in which the magnetic fields plays an important dynamical roll are common scenarios in astrophyisics. To study these systems a relativistic magnetohydrodynamic description is necessary. In this talk we present the Relativistic Magnetohydrodynamics System of Equations as well as its characteristic structure, pointing out the main differences with respect to the classical MHD. Based on the full wave decomposition of the Relativistic MHD system, provided by the normalized eigenvectors of the flux Jacobians, we propose a linearized Riemann Solver well suited for numerical applications.

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The Unexpected Energetic Structure of Anisotropic Magnetohydrodynamic Turbulence

Wolf-Christian Müller
Max-Planck-Institut für Plasmaphysik, Germany

(Roland Grappin)

A new approach toward the analysis of high-Reynolds-number direct numerical pseudospectral simulations of incompressible magnetohydrodynamic (MHD) turbulence subject to a strong mean field has led to unexpected results: the scaling of one dimensional spectra taken along rays passing through the origin of Fourier space is independent of the rays' orientation with respect to the mean magnetic field direction; the spectral amplitude variation with respect to the angle between a ray and the mean magnetic field can be eliminated by normalization with an angle-dependent dissipation wavenumber. Thus the anisotropy of MHD turblence permeated by a strong mean magnetic field does *not* appear as direction dependent scaling but rather as a direction dependent Reynolds number. These findings challenge present phenomenologies which are based on the critical balance argument.

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A Unified Mechanism for Reversing and Hemispherical Dynamos

Francois Petrelis
Ecole Normale Superieure, France

(B. Gallet, S. Fauve, E. Dormy, J-P Valet)

I discuss the behaviour of the magnetic field generated by dynamo effect in the vicinity of the dynamo threshold. Taking into account two modes of magnetic field, random reversals are predicted when the modes are close to achieve a saddle-node bifurcation. If the two modes are of dipolar and quadrupolar symmetry, the reversals occur when the flow breaks equatorial symmetry. Hemispherical dynamos also find a simple explanation within this scenario. This is illustrated with measurements of the Von Karman Sodium experiment and with the solution of a kinematic α^2 dynamo.

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Inviscid Helical Magnetorotational Instability in a Taylor-Couette Flow

Janis Priede
Coventry University, England

The magnetorotational instability (MRI) is a mechanism by which the magnetic field can destabilise a hydrodynamically stable flow of conducting fluid without altering the flow itself. This instability is

thought to be active in the accretion discs where the magnetic field drives a turbulent transport of angular momentum necessary for the fast formation of stars. The standard MRI (SMRI) involves a strong coupling between the flow and magnetic field perturbations that requires the magnetic Reynold number $Rm \sim 10$. For liquid metals this translates into hydrodynamic Reynolds number $Re \sim 10^6 - 10^7$. Hollerbach and Rüdiger [Phys. Rev. Lett. 95, 124501, 2005] found that MRI can take place in the cylindrical Taylor-Couette flow at $Re \sim 10^3$ when the imposed magnetic field is helical rather than purely axial as in the classical case. Although an instability resembling the helical MRI (HMRI) has been reproduced in the laboratory by Stefani et al., [Phys. Rev. Lett 97, 184502, 2006], its astrophysical relevance is still unclear.

We revisit the problem of HMRI in the limit of a vanishing viscosity that is pertinent to the astrophysical conditions. The inductionless approximation defined by a zero magnetic Prandtl number ($Pm = 0$), which supposes the liquid to be poorly conducting, allows us to focus exclusively on the HMRI. We analyse numerically the effect of helical magnetic field on the thresholds of convective and absolute instabilities in the cylindrical Taylor-Couette flow. In this case, the stability threshold is defined in the terms of so-called MHD interaction parameter N, whose dependence is studied on the helicity of the magnetic field and the velocity profile.

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New Lower Bounds on the Energy of Knots and Braids

Renzo Ricca
Univerisity Milano-Bicocca, Italy

In this talk we present new results on the minimal energy of magnetic knots and braids in ideal magnetohydrodynamics. That topology bounds the magnetic energy of complex structures has been established by the classical results of Arnold (1974), Moffatt (1990), Freedman and He (1991) and, more recently, by Ricca [1]. By relying on standard relaxation techniques of the magnetic field, the groundstate energy of tight knots is now determined analytically [2], by finding the relationship between minimal energy and geometric topology, through ropelength information and internal twist of the magnetic flux tube. By applying these results to relaxed magnetic fields of complex topology, we determine a new lower bound on the energy in terms of topological crossing number. This result is easily extended to magnetic braids, thus providing a useful information in topological fluid mechanics [3] and, in the context of astrophysical flows, for solar coronal loops and large-scale, twisted magnetic fields.

[1] Ricca, R. L. (2008) Proc. R. Soc. A 464, 293-300.
[2] Maggioni F. & Ricca, R. L. (2009) Proc. R. Soc. A 465, 2761-2783.
[3] Ricca, R. L. (Ed.) (2009) Lectures on Topological Fluid Mechanics. Lecture Notes in Mathematics 1973, Springer-Verlag, Heidelberg.

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Broken Ergodicity in MHD Turbulence

John Shebalin
NASA Johnson Space Center, USA

Ideal magnetohydrodynamic (MHD) turbulence may be represented by finite Fourier series, where the inherent periodic box serves as a surrogate for a bounded astrophysical plasma. Independent Fourier coefficients form a canonical ensemble described by a Gaussian probability density function containing modal Hermitian covariance matrices with positive eigenvalues. The eigenvalues at lowest wave number can be very small, resulting in a large-scale coherent structure: a turbulent dynamo. This is seen in computations and a theoretical explanation in terms of 'broken ergodicity' contains Taylor's theory of forcefree states. An important problem for future work is the case of real, i.e., dissipative flows. In real flows, broken ergodicity and coherent structure are still expected to occur in MHD turbulence at the largest scale, as suggested by low resolution simulations. One challenge is to incorporate coherent structure at the largest scale into the theory of turbulent fluctuations at smaller scales.

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Dynamo in Context of Riemannian Geometry: A Mathematical Tool and Cosmological Applications

Dmitry Sokoloff
Moscow State University, Russia
(D. Sokoloff, D. Tomin, A. Rubashny)

MHD dynamo is a way to generate magnetic fields by flows of condictive media in celestial bodies. Mathematical problems in dynamo theory are usualy based in context of the Euclidean geometry. It looks however instructive to choose a more general viewpoint and consider corresponding problems in a more broad context of Riemannian geometry. An obvious option here is magnetic field generation in the Early Universe which is a (pseudo)Riemannian space. We consider as well less straightforward options which use Riemannian geometry to clarify various aspects of dynamos which are difficult for consideration in Euclidean spaces only.

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Helical Magnetorotational Instability in Theory and Experiment

Frank Stefani
Forschungszentrum Dresden-Rossendorf, Germany

(Gunter Gerbeth, Thomas Gundrum, Rainer Hollerbach, Oleg Kirillov, Janis Priede, Guenther Ruediger, Jacek Szklarski)

The magnetorotational instability (MRI) is widely believed to play a key role in cosmic structure formation by maintaining turbulence and enabling angular momentum transport in accretion disks. The helical version of MRI (HMRI) was recently shown to have a scaling behaviour that is quite different to that of the standard version of MRI. Yet both versions are continuously connected. We solve this apparent paradox by showing the emergence of an exceptional spectral point at small but finite magnetic Prandtl number at which the slow magneto-Coriolis wave and one inertial wave coalesce and exchange their branches. Further, we compare the results of the PROMISE experiment on HMRI with various numerical predictions.

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Dynamo Action with Wave Motion

Andreas Tilgner
University of Göttingen, Germany

Time dependent velocity fields can act as dynamos even when the same velocity fields frozen in at any particular time cannot. This effect is observed in propagating waves in which the time dependence is simply a steady drift of a fixed velocity pattern. The effect contributes to magnetic field generation in numerical models of planetary dynamos and relies on the property that eigenmodes of the induction equation are not all orthogonal to each other, but is at first independent of helicity. Even though helicity is believed to play an important role in planetary dynamos, recent simulations of rotating Rayleigh-Benard convection suggest that helicity is negligible in the Earth's core.

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Special Session 16: Traveling Waves in Reaction-Diffusion Equations: Theory and Applications

Francois Hamel, Aix-Marseille University and Helmholtz Centre Munich, France
Masaharu Taniguchi, Tokyo Institute of Technology, Japan

Introduction: This special session is devoted to study qualitative properties of traveling waves or stationary states in reaction-diffusion equations. Our topics include multi-dimensional traveling waves, propagating phenomena, and formation of patterns.

Sharp Interface Limit of the Fisher-KPP Equation

Matthieu Alfaro
University Montpellier 2, France

(Arnaud Ducrot (Bordeaux))

We investigate the behavior, as a parameter related to the thickness of a diffuse interfacial layer tends to zero, of a rescaled Fisher-KPP equation in the whole space. It is known that, for compactly supported initial data, the sharp interface limit moves by a constant speed, which is the minimal speed of some related one-dimensional travelling waves.

On the one hand, we allow initial data with compact support plus, possibly, small perturbations at infinity. If the perturbations are small enough (exponential decay) the interface phenomena occurs. We also exhibit initial data "not so small" at infinity (only polynomial decay) which do not allow the interface phenomena.

On the other hand we provide a new estimate of the thickness of the transition layers of the solutions of the Fisher equation.

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Traveling Fronts of Reaction-Diffusion Equations with Stefan-Like Boundary Conditions

Chiun-Chuan Chen
National Taiwan University, Taiwan

Reaction-diffusion equations with Stefan-like boundary conditions are investigated. We obtain criteria for the existence of traveling front solutions.

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The Minimal Speed for a Two Species Competition System

Jong-Sheng Guo
National Taiwan Normal University, Taiwan

In this talk, we shall discuss the minimal speed for a two species competition system. In particular, we are interested in the linear determinacy for the minimal speed in the sense defined by Lewis, Li, and Weinberger in 2002. It is well-known that the minimal speed is always bigger than or equal to the constant defined through the linearization system related to the competition system. We are interested in determining parameters so that the minimal speed of the corresponding system obey the linear determinacy property. For this, we shall study the minimal speed for the corresponding lattice dynamical system.

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Singular Limit of a Degenerate Parabolic Allen-Cahn Equation

Danielle Hilhorst
University of Paris-Sud 11, France
(Matthieu Alfaro, Reiner Schaetzle)

We study the singular limit of a degenerate partial differential equation arising in population dynamics. More precisely, we consider the porous medium equation with a bistable reaction term, and study its limiting behavior as the reaction coefficient tends to infinity.

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Reflection of a Traveling Wave in a Combustion Model

Kota Ikeda
Meiji Inst. for Adv. Study of Math. Sci., Japan
(M. Mimura)

It is shown in some combustion experiment that thin solid, for an example, paper, cellulose dialysis bags and polyethylene sheets, burning against oxidizing wind develops finger-like patterns or fingering patterns. To investigate these phenomena, we proposed a reaction-diffusion model. We carry out numerical simulation and show that this model can generate finger-like pattern. Also, our model has reflection phenomena of a traveling wave solution which changes the direction at the boundary for some time. In this talk, as the first step to study this phenomenon, we construct two traveling wave solution which have positive and negative wave speeds, respectively.

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Dynamics of a Front Solution for a Bistable Reaction-Diffusion Equation with a Degenerate Spatial Heterogeneity

Hiroshi Matsuzawa
Numazu National College of Technology, Japan
(Shin-Ichiro Ei)

In this talk, we study a dynamics of a front solution with a sharp transition layer of a bistable reaction diffusion equation with a spatial heterogeneity in one space dimension. In particular, we consider the case where this spatial heterogeneity degenerates on an interval and study the dynamics of the transition layer on this interval. In this case the dynamics of the transition layer on the interval becomes so-called very slow dynamics. In order to analyze such a dynamics, we construct an attractive local invariant manifold which gives the law of motion of the transition layer. We will also give an example for competition-diffusion equations.

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Global Stability of Monostable Traveling Waves for Nonlocal Time-Delayed Reaction-Diffusion Equations

Ming Mei
Champlain College and McGill University, Canada
(Chunhua Ou and Xiao-Qiang Zhao)

For a class of nonlocal time-delayed reaction-diffusion equations, we prove that all noncritical wavefronts are globally exponentially, and critical wavefronts are globally algebraically stable, when the initial perturbations around the wavefront decay to zero exponentially near the negative infinity regardless of the magnitude of time delay. This work also improves and develops the existing stability results for local and nonlocal reaction-diffusion equations with delays. Our approach is based on the combination of the weighted energy method and the Green function technique.

This is a joint work with Chunhua Ou and Xiao-Qiang Zhao.

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Asymptotic Spreading in General Heterogeneous Media

Gregoire Nadin
CNRS and Paris 6, France
(H. Berestycki)

We will establish spreading properties for monostable reaction-diffusion equations with general space-time heterogeneous coefficients. That is, we will construct two non-empty bounded open sets $\overline{\mathcal{S}} \subset \underline{\mathcal{S}} \subset \mathbb{R}^N$ such that for all compact set

$K \subset \underline{\mathcal{S}}$ (resp. all closed set $F \subset \mathbb{R}^N \backslash cl(\underline{\mathcal{S}})$), one has $\lim_{t\to+\infty} \inf_{x\in tK} |u(t,x) - 1| = 0$ (resp. $\lim_{t\to+\infty} \sup_{x\in tF} |u(t,x)| = 0$), where 0 and 1 are two steady states of the equation. These sets are characterized in terms of two new notions of generalized principal eigenvalues for linear parabolic operators in unbounded domains. Several examples will be discussed.

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Homogenization Limit of Recurrent Traveling Waves in a 2D Saw-Toothed Cylinder

Ken-Ichi Nakamura
Univ. of Electro-Communications, Tokyo, Japan
(Bendong Lou, Hiroshi Matano)

We consider a curvature-driven motion of plane curves in a two-dimensional cylinder with spatially undulating boundary. The function which determines the boundary undulation is assumed to be almost periodic, or more generally, recurrent. In this talk we study the homogenization problem as the boundary undulation becomes finer and finer, and give sharp estimates of the average speed of the traveling wave for our problem. In the special case where the spatial undulation is periodic, the optimal rate of convergence to the homogenization limit of the average speed is derived from the estimates.

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Large Time Behavior of Disturbed Planar Fronts in the Allen-Cahn Equation

Mitsunori Nara
University of Tokyo, Japan
(Hiroshi Matano)

We study how a planar front in the Allen-Cahn equation in R^n (with $n > 1$) behaves when it receives an arbitrarily large (but bounded) perturbation near the front region. We first show that the behavior of the disturbed front can be approximated by that of the mean curvature flow up to $t = +\infty$. Using this observation, we then show that the planar front is asymptotically stable in $L^\infty(R^n)$ under spatially ergodic perturbations, which include quasi-periodic and almost periodic ones as a special case.

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Traveling Waves in the Various Shapes

Hirokazu Ninomiya
Meiji University, Japan

In this talk we will discuss traveling waves and entire solutions of the reaction-diffusion equations in the multi-dimensional space. On the multi-dimensional space, the planar waves and V-shaped/pyramidal/conical traveling waves are already known. We will present several new types of traveling waves of reaction-diffusion equations: (i) zipping waves of Allen-Cahn equations and (ii) traveling spots of interface equation derived from the FitzHugh-Nagumo equations. We will also discuss the relationship between the traveling waves and entire solutions.

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Rotating and Standing Waves on Sphere

Toshiyuki Ogawa
Osaka University, Japan

We consider a system of 3-component reaction-diffusion equations on a sphere. It is known that the wave instability can be observed in a 3-component RD system, namely, spatial non-trivial oscillations may bifurcate earlier than the spatial uniform oscillation. We calculate the normal form with SO(3) symmetry when the instability occurs in the lowest modes in the spherical harmonics and study the stabilities of rotating and standing waves.

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Increasing Rates of Spread in KPP Equations

Lionel Roques
INRA - BIOSP, France
(Hamel, Fady, Fayard, Klein)

We study the spreading properties of solutions of KPP reaction-diffusion equations, starting from exponentially unbounded initial conditions: that is functions which decrease to 0 more slowly than any exponentially decaying function. We prove that the level sets of the solutions move infinitely fast as time goes to infinity, and we give estimates of the position of these level sets. Our results are in sharp contrast with the classical case of exponentially bounded initial conditions.

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Bound States of Coherent Structures on Circles

Bjorn Sandstede
Brown University, USA

The existence and stability of bound states that are composed of transmission and contact defects will be considered on large circles. I will also discuss rigorous validity results for modulation equations near dissipative wave trains, and illustrate what these results, taken together, imply for the interaction of defects on unbounded domains.

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Existence of Dislocations in a Swift-Hohenberg Equation

Arnd Scheel
University of Minnesota, USA
(Mariana Haragus)

We show the existence of dislocations in a modified Swift-Hohenberg equation near onset of instability. The analysis is based on a local center-manifold reduction for spatial dynamics and normal form theory. We will also discuss extensions to different models and limitations of our approach.

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Traveling Waves for a Reaction-Diffusion Model for Tumour Growth with Contact Inhibition

Tohru Wakasa
Meiji University, Japan
(Michiel Bertsch, Masayasu Mimura and Yusaku Nagata)

In this talk we will introduce a reaction diffusion model proposed by M. Bertsch, R. Dal Passo and M. Mimura, which describes the spatio-temporal dynamics between normal cells and abnormal cells taking account of the effect of contact inhibition. Their previous study have shown that under suitable segregated conditions on initial data, the 1-dimensional boundary value problem of the system can be reduced to a free boundary problem. That is, contact inhibition occurs between normal cells and abnormal cells. In this talk we are interested in behavior of the solutions of the reaction-diffusion system. In a special case it is expected that "segregated traveling wave solution" plays an essential role for understanding the behavior of the solutions. We will focus on the traveling wave solutions, and will show existence and uniqueness of the segregated traveling wave solution. Moreover, some fundamental properties on the segregated traveling wave solution and other traveling wave solutions will be investigated.

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Special Session 17: Dynamical Networks and Their Applications in Neuroscience

Yuri Maistrenko, Ukranian Academy of Science, Ukraine
Georgi Medvedev, Drexel University, USA

Introduction: One of the main challenges in the theory for dynamical networks is how to translate the information about the dynamics of the local systems and the information about the types and topology of the connections between them into an effective description of the network dynamics. Successful analysis of dynamical networks requires the compilation of analytical and combinatorial approaches. The talks in this special session review recent progress in the theory and applications of dynamical networks with an emphasis on the problems in experimental and theoretical neuroscience.

Developing Networks and Changing Dynamics

Janet Best
Ohio State University, USA
(Deena Schmidt, Boris Pittel)

A fundamental question in many neural systems is the extent to which the neuronal network architecture may contribute to the observed dynamics. This question can be particularly interesting during development, when both the neural circuitry and the behavior are changing and the growing network substrate may itself alter the dynamics. Motivated by our models of sleep-wake regulation during development, we have investigated how the structure of a network may relate to the duration of events on the network. We consider stochastic processes on random networks and present results relating the structure of the network with the dstribution of event durations on the network. In particular, we describe how properties of the network may change during development, changing the dstribution of the event durations, as observed with sleep and wake bouts during postnatal development.

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Role of Dendrites in Noise-Induced Synchronization

Alla Borisyuk
University of Utah, USA
(Darci D. Taylor, Paul Bressloff)

Many types of epilepsy have been traced to mutations in somatic and dendritic ion channels. At the same time, seizures have long been associated with synchronization in networks of cells. In this project we are investigating how changes in the dendrite affect tendency of the cells towards synchronization. We focus on synchronization of uncoupled neurons driven to synchrony by a common noisy input; as may occur when neighboring tissue is recruited by the seizure focus. We use Lyapunov exponents (introduced in this context by Teramae and Tanaka) as a measure for noise-induced synchronization. We extend the theory to include dendrites via two different approaches: first, treating the soma and the dendrite as a single oscillator described by the dendritic phase-resetting-curve; and second, treating the somatic oscillator as receiving input filtered by the dendrite. We demonstrate that either approach can be used in the case of passive dendrites and some of the active currents, including non-uniform spatial channel dstribution. We find that some of the epilepsy-implicated currents can have either synchronizing or de-synchronizing effect and that distal dendrites can have a stronger synchronizing effect than proximal ones if "synaptic democracy" is included.

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Dynamics of Morris-Lecar Networks

Stephen Coombes
University of Nottingham, England

The Morris-Lecar (ML) neuron model is a two dimensional conductance based model that is often used as an idealised fast-spiking pyramidal neuron. Its planar nature has encouraged much analysis of the single neuron model using tools from phase-plane analysis and the "geometry of excitability". When treating synaptic or gap junction coupled networks of oscillating ML neurons these techniques are the natural basis for developing a weakly-coupled oscillator theory. However, to probe network dynamics in the strong coupling regime requires an alternative approach. I will show how results in this area can be obtained by using a piece-wise linear caricature of the ML model. In illustration of the usefulness of such an approach I will first consider gap junction coupling and show how to analyse emergent fluctuations in the mean membrane potential (as instabilities of an asynchronous network state). Next I will treat synaptically coupled networks with a phenomenological form of retrograde (cannabinoid) second messenger signalling that can support depolarisation induced suppression of excitation. In this case I will describe a mechanism for the emergence of ultra-low frequency (0.01–0.1 Hz) synchronised oscillations - a hallmark rhythm of the resting brain.

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Reliable Sequence Generation Via Symmetry Breaking and Adaptation in a Continuous Attractor Network

Carina Curto
University of Nebraska-Lincoln, USA
(Vladimir Itskov)

The hippocampal place cell network can generate reliable, long-lasting ($\sim$ 20 secs) sequences of neural activity even in the absence of changing external inputs (Pastalkova et al., Science 2008). The network mechanisms underlying long-lasting sequence generation, however, are poorly understood. We propose a simple model of the hippocampal network that exhibits a continuous family of "bump" attractors in the short-timescale dynamics. Threshold-adaptation on a slower, biologically realistic timescale has the effect of continuously moving the bump of neural activity, producing long-lasting sequential activity. Interestingly, these sequences are not reliable unless heterogeneities are introduced in the synaptic weights to break the symmetry of the network connectivity.

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Dynamics of Neuronal Networks with Plasticity

Lee Deville
University of Illinois, USA

We consider neuronal network models with plasticity and randomness; we show that complicated global structures can evolve even in the presence of simple local update rules. Specifically, we propose a discrete-time stochastic model of the evolution of a layered neuronal network which is capable of learning through plasticity of the connections between layers. The structure of the network is thus modulated by the conductances of the connecting neurons, which evolve according a model mimicking long-term potentiation. We demonstrate that the network is capable of rich properties (e.g. bifurcation, various forms of stability, etc.) that depend on maximum possible values of conductance, the inputs to the network, and the number of levels in the integrate-and-fire output neuron model. We will also remark on the Lyapunov exponents and information-theoretic properties of such networks.

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Synchronization in Excitatory Networks of Square-Wave Bursters

Martin Hasler
Ecole Polytechnique Féd., Lausanne, Switzerland
(Igor Belykh)

We study the influence of coupling strength and network topology on synchronization behavior in excitatory networks of bursting Hindmarsh-Rose neurons with fast threshold modulation coupling. We derive rigorous conditions on the coupling threshold required to synchronize square-wave bursters. We also show that the onset of synchrony in a network with any coupling topology admitting complete synchronization is ensured by one single condition.

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The Geometry of Memory Patterns in Recurrent Networks

Vladimir Itskov
University of Nebraska-Lincoln, USA
(Carina Curto and Anda Degeratu)

The allowed patterns of activity among neurons in a recurrent network are constrained by both the structure of inputs and the structure of recurrent connections. One can think of a recurrent network as a gating device that allows only certain patterns of activity in response to feedforward input. In this framework, allowed memory patterns correspond to stable fixed points of the recurrent network dynamics. The set of neurons co-activated at a stable fixed point is called a stable clique.

We study the relationship between coarse network properties and the combinatorics of stable cliques (memory patterns) associated to recurrent networks. This requires a geometric study of stable submatrices, and our results make unexpected connections to discrete and convex geometry.

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Synchronization of Coupled Dynamics

Jürgen Jost
Max Planck Inst. for Math. in the Sci., Germany

I shall present a general framework, developed in collaboration with Fatihcan Atay and Frank Bauer, on the stability of synchronized solutions of coupled dynamics, under a wide range of coupling schemes and dynamical rules, on a network of coupled oscillators. The conditions will involve both the network geometry, as encoded in its spectrum, and certain generalized Lyapunov exponents of the dynamics. In particular, we shall discuss examples where the isolated dynamics is simple, but the synchronized one is chaotic, or conversely.

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Cascade-Induced Synchrony in Complex Neuronal Networks

Peter Kramer
Rensselaer Polytechnic Institute, USA
(Katherine Newhall, Maxim Shkarayev, Gregor Kovacic, David Cai)

Kinetic theory provides a coarse-grained alternative to the integrate-and-fire neuronal network description. In the limit of infinitely short conductance responses, a Boltzmann-type differential-difference equation can be derived for the probability density function of the neuronal voltage. A Fokker-Planck equation can be derived in the limit of small conductance fluctuations. The talk will focus on the solutions of this equation in the case of all-excitatory networks, both with all-to-all and complex, such as scale-free, coupling architecture. The likelihood and temporal period of synchronous network oscillations, in which all the neurons fire in unison, will be described. The likelihood of synchrony is computed combinatorially using the network oscillation period, the voltage probability dstribution, and detailed local information about the network architecture and clustering. The oscillation period is found from a first-passage-time problem described by a Fokker-Planck equation, which is solved analyticaly via an eigenfunction expansion. The voltage probability dstribution is found using a Central-Limit-Theorem-type argument via a calculation of the voltage cumulants. Differences between oscillations in all-to-all coupled and scale-free networks will be discussed.

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Layered Neural Oscillator Networks and Their Reliability

Kevin Lin
University of Arizona, USA
(Eric Shea-Brown, Lai-Sang Young)

This talk concerns layered networks of coupled oscillators and the reliability of such networks. Reliability means that upon repeated presentations of a given stimulus, the network gives essentially the same response each time. I will present an analysis of how conditions within such layered networks affect their reliability. This is joint work with Eric Shea-Brown and Lai-Sang Young.

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Chimera States at the Transition from Coherence to Incoherence for Coupled Chaotic Oscillators

Yuri Maistrenko
National Academy of Sciences, Ukraine
(Iryna Omelchenko and Eckehard Schoell)

When identical chaotic oscillators interact, a state of space coherence, i.e. locking, may be attained in which the oscillators create a smooth profile with periodic dynamics in time. We find parameter regions of the periodic coherence which lie between the local and global coupling and conclude that they are arranged in accordance with the period-adding cascade, while in-region dynamics yield to the period-doubing cascade. Between the regions the coherence is also preserved but the dynamics are chaotic. Transition from the coherence to incoherence happens when decreasing radius or/and strength of coupling. We unfold the coherence-incoherence bifurcation and uncover the appearance of numerous chimera states, - hybrid states in which some group of oscillators remains phase locked but the others not.

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Synchronization of Coupled Limit Cycles

Georgi Medvedev
Drexel University, USA

For a class of coupled limit cycle oscillators, we give a condition on a linear coupling operator that guarantees exponential stability of the synchronous solution. Our method applies to networks with partial, time-dependent, and nonlinear coupling schemes, as well as to ensembles of local systems with nonperiodic attractors. We also study robustness of synchrony to noise. To this end, we analytically estimate the degree of coherence of the network oscillations in the presence of noise. The estimate of coherence highlights the main ingredients of stochastic stability of the synchronous regime. In particular, it quantifies the contribution of the network topology. Finally, we discuss the applications of these results to the analysis of the compartmental model of a neuron and gap-junctionally coupled neuronal population.

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On the Dynamical Nature of Chimera States

Oleh Omel'chenko
WIAS Berlin, Germany
(Matthias Wolfrum, Yuri Maistrenko)

Chimera states are a recently new discovered dynamical phenomenon that appears in arrays of nonlocally coupled phase oscillators and displays a spatial pattern of coherent and incoherent regions. In this talk, I will present an analysis of a complex high-dimensional chaotic behavior concerned with this dynamical regime and describe its scaling properties with respect to the system size.

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Robustness of Collective Decision Dynamics

Luca Scardovi
Technische Universität München, Germany
(N. Leonard, I. Poulakakis, G. Young)

I will describe methodology to investigate the role of directed interconnection topology on robustness of collective decision making in dynamical networks. In particular I will discuss robustness in agreement dynamics and coupled drift diffusion models and I will show how it can be quantified by H_2 norms and L_2 stability conditions. Our study reveals how some directed graphs outperform undirected graphs and certain nodes can be identified as better (lower variance) decision makers. I will also show how these measures are being used to evaluate performance of natural groups from observational data in an effort to identify underlying mechanisms of robust behavior.

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Neural Network Design as an Inverse Problem: Spatio-Temporal Patterns and Beyond

Marc Timme
Max Planck Inst. Dynamics & Self-Org., Germany
(Raoul-Martin Memmesheimer, Sven Jahnke)

We suggest a new perspective of research $[1-3]$ towards understanding the relations between the structure and dynamics of a complex network: can we design a network, e.g. by modifying the features of its units or interactions, such that it exhibits a desired dynamics? Here we present a case study where we positively answer this question analytically for networks of spiking neural oscillators.

First, we present a method of finding the set of all networks (defined by all mutual coupling strengths) that exhibit an arbitrary given periodic pattern of spikes as an invariant solution. The method is general as it covers networks of different types of neurons, excitatory and inhibitory couplings, interaction delays that may be heterogeneously distributed, and arbitrary network connectivities. Second, we show how to design networks if further restrictions are imposed, for instance by predefining the detailed network connectivity. Third, the method can be used to design networks that opti-

mize network properties. To illustrate this idea, we design networks that exhibit a predefined pattern dynamics while at the same time minimizing the networks' wiring costs.

[1] Phys. Rev. Lett. 97:188101 (2006).
[2] Physica D 224:182-201 (2006).
[3] Phys. Rev. Lett. 100:048102 (2008).

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Emergence of Patterns in Rings of Delayed Coupled Oscillators

Serhiy Yanchuk
Humboldt University of Berlin, Germany

In this talk, we describe the appearance and stability of time and space periodic patterns (rotating waves) in rings of delay coupled oscillators. We show how the inclusion of additional delays in the coupling leads to the splitting of each rotating wave into several new ones. The appearance of rotating waves is mediated by Hopf bifurcations of a symmetric equilibrium. The whole family of rotating waves admits an asymptotic description in the case when the number of oscillators is large. We also conclude that the coupling delays can be effectively replaced by increasing the number of oscillators in the chain.

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Special Session 18: Scaling and Liouville Theorems in Studies of PDEs

Peter Polacik, University of Minnesota, USA
Pavol Quittner, Comenius University, Bratislava, Slovakia

Introduction: Liouville-type theorems belong among important tools of qualitative studies of PDEs. They are often used in combination with scaling arguments for a priori estimates of solutions and other results. The aim of this session is to discuss existing Liouville theorems in various types of PDEs and explore their applications.

Fundamental Solutions and Liouville Results for Fully Nonlinear Elliptic Equations

Scott Armstrong
Louisiana State University, USA
(Boyan Sirakov and Charles Smart)

We construct fundamental solutions of fully nonlinear, positively homogeneous, uniformly elliptic equations. Examples of such equations include Hamilton-Jacobi-Bellman and Bellman-Isaacs equations. We justify the use of the term "fundamental solution" by showing that up to multiplication and addition of constants, these solutions are the unique solutions in $\mathbb{R}^n \setminus \{0\}$ which are bounded on one side both in a neighborhood of the origin and a neighborhood of infinity. Isolated singularities of solutions which are bounded on one side are completely characterized. Finally, we show how the homogeneity of the fundamental solution provides information about the long-time behavior of the corresponding controlled diffusion processes.

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Bifurcating Branches of Positive Solutions for a Nonlinear Elliptic System

Thomas Bartsch
Universität Giessen, Germany
(Norman Dancer, Zhi-Qiang Wang)

We present results concerning the bifurcation of positive solutions of a nonlinear Schrödinger type system. Local and global bifurcations in terms of the nonlinear coupling parameter of the system are investigated by using spectral analysis and by establishing a new Liouville type theorem for nonlinear elliptic systems which provides a-priori bounds of solution branches. If the domain is radial, possibly unbounded, then we also control the nodal structure of a certain weighted difference of the components of the solutions along the bifurcating branches.

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Non-Existence of Positive Solutions to the Lane-Emden System and a System of Integral Equations

Wenxiong Chen
Yeshiva University, USA
(Congming Li)

Consider the system of semilinear elliptic PDEs in R^n:

$$(-\Delta)^{\alpha/2}u = v^q \text{ and } (-\Delta)^{\alpha/2}v = u^p. \qquad (1)$$

In the special case when $\alpha = 2$, it is the well-known Lane-Emden system. We first prove that (1) is equivalent to a system of integral equations associated to the Hardy-Littlewood-Sobolev inequalities. Using the method of moving planes in integral forms, we obtain radial symmetry for positive solutions of the integral system and hence for the PDE system (1) as well. In the subcritical case, Our symmetry results, together with the non-existence of radial solutions by Mitidieri, implies that, under suitable integrability conditions, the Lane-Emden system possesses no positive solutions. This adds more results to the well-known Lane-Emden conjecture.

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An Elliptic Equation with Negative Exponent: Stability and Critical Power

Yihong Du
University of New England, Australia
(Zongming Guo)

We consider positive solutions of the equation

$$\Delta u = |x|^\alpha u^{-p} \text{ in } \Omega \subset \mathbb{R}^N \ \ (N \geq 2),$$

where $p > 0$, $\alpha > -2$, and Ω is a bounded or unbounded domain. We show that there is a critical power $p = p_c(\alpha)$ such that this equation with $\Omega = \mathbb{R}^N$ has no stable positive solution for $p > p_c(\alpha)$ but it admits a family of stable positive solutions when $0p_c(\alpha^-)$ $(\alpha^- = \min\{\alpha, 0\})$, we further show that this equation with $\Omega = B_r \setminus \{0\}$ has no positive solution with finite Morse index that has an isolated rupture at 0, and analogously it has no positive solution with finite Morse index when $\Omega = \mathbb{R}^N \setminus B_R$. Among other results, we also classify the positive solutions over $B_r \setminus \{0\}$ which are not bounded near 0.

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Homoclinic and Heteroclinic Orbits for a Semilinear Parabolic Equation: Part II

Marek Fila
Comenius University, Slovak Republic
(Eiji Yanagida)

This will be a continuation of Eiji Yanagida's talk.

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Blow-Up Rates for Solutions of Indefinite Superlinear Parabolic Problems

Juraj Foldes
Vanderbilt University, USA

We establish new nonlinear Liouville theorems for parabolic problems on half spaces. Based on these theorems, we derive estimates for the blow-up of positive solutions of indefinite parabolic problems with the nonlinearity possibly vanishing on the boundary. We also discuss optimality of the blow-up rates and we derive a priori estimates for positive solutions of indefinite elliptic problems.

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Extinction Versus Persistence in Strong Oscillating Flows

Francois Hamel
Aix-Marseille University, France

This talk, which is based on a joint work with N. Nadirashvili, will be devoted to the qualitative behavior of the solutions of some diffusion-advection equations in bounded domains, in the case of strong and oscillating flows. Some conditions for finite-time extinction or persistence will be discussed. The enhancement of the diffusion rate depends on the interplay between strong advection and time-homogenization, and in particular on the ratio between the strength of the flow and its frequency parameter. Quantitative estimates of this ratio, which depend on the geometry of the domain, will be given in the case of a uniform flow. In the general time-space dependent case, the finite-time behavior of the solutions is related to the existence of first integrals of the flow.

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The Application of Scaling and Liouville Theorems in Studies of Blowup Theories

Bei Hu
University of Notre Dame, Indiana, USA

In this talk I will some applications of Scaling and Liouville theorems in establishing the blowup rate.

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An Alternative Approach to Regularity for the Navier-Stokes Equations in Critical Spaces

Gabriel Koch
Oxford University, England

(Carlos Kenig)

In an important recent paper, L. Escauriaza, G. Seregin and V. Sverak [ESS] show that solutions to the Navier-Stokes equations (NSE) which remain bounded in $L^3(\mathbb{R}^3)$ (a "critical" space) cannot become singular in finite time (alternatively, that the L^3 norm must become infinite at a finite-time singularity). This, coupled with a decay result for global solutions by I. Gallgher, D. Iftimie and F. Planchon, is the same type of result which has been proved recently for "critical" hyperbolic/dispersive equations by C. Kenig and F. Merle. Their method, known as "concentration-compactness" + "rigidity theorem" (which relies on "profile decompositions" for bounded sequences), is quite general in nature and thus it is natural to ask whether it can be applied in the NSE setting to give an alternative proof of [ESS]. In collaboration with C. Kenig this has been achieved in a special case (solutions which remain bounded in $\dot{H}^{1/2}$) due to the profile decomposition of I. Gallagher which is currently available for NSE in that setting. Similar decompositions appear to be well within reach in L^3 as well, and we therefore expect to be able to generalize our result soon to that setting.

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Some Liouville Theorems for Conformally Invariant Elliptic Equations and Related Topics

Yanyan Li
Rutgers University, USA

I will discuss some Liouville theorems for conformally invariant elliptic and degenearate elliptic equations. Results on strong maximum principle, the Hopf Lemma, and removable singularities for viscosity solutions will also be presented. Some of these are joint works with Luis Caffarelli and Louis Nirenberg.

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Threshold and Generic Type I Behaviors for a Supercritical Nonlinear Heat Equation

Hiroshi Matano
University of Tokyo, Japan

(Frank Merle)

We study blow-up of radially symmetric solutions of the nonlinear heat equation $u_t = \Delta u + |u|^{p-1}u$ either on R^N or on a finite ball under the Dirichlet boundary conditions. We assume that $N \geq 3$ and $p > \frac{N+2}{N-2}$. Our first goal is to analyze a threshold behavior for solutions with initial data $u_0 = \lambda v$, where $v \in L^\infty \cap H^1$ and $v \geq 0$. It is known that there exists $\lambda^* > 0$ such that the solution converges to 0 if $0 < \lambda < \lambda^*$ while it blows up in finite time if $\lambda \geq \lambda^*$. We show that, for all $\lambda > \lambda^*$ with at most finitely many exceptional values of λ, the blow-up is complete and of type I with a flat local profile κ. The proof is based on a combination of the zero-number principle and energy estimates. Next we employ the very same idea to show that the constant solution κ attains the smallest rescaled energy among all nonzero stationary solutions of the rescaled equation. Using this result, we derive a sharp criterion for no blow-up.

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A Liouville Theorem for a Semilinear Heat Equation and Applications for Quenching

Nejla Nouaili
CEREMADE-Université Paris Dauphine, France

We prove a Liouville Theorem for a semilinear heat equation with absorption term in one dimension. We then derive from this theorem uniform estimates for quenching solutions of that equation.

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Singular Solutions of the Nonlinear Heat Equation

Dominika Pilarczyk
Wrocław University, Poland

We consider the Cauchy problem

$$\begin{aligned} u_t &= \Delta u + u^p, \quad (x,t) \in \mathbb{R}^n \times (0,\infty), \\ u(x,0) &= u_0(x), \end{aligned}$$

where p is supercritical in the sense that $n \geqslant 11$ and $p > p_{JL} = \frac{n-2\sqrt{n-1}}{n-4-2\sqrt{n-1}}$. The goal of this talk is to discuss the asymptotic stability of a singular steady state

$$v_\infty(x) = \left(\frac{2}{p-1}\Big(n-2-\frac{2}{p-1}\Big)\right)^{\frac{1}{p-1}} |x|^{-\frac{2}{p-1}}$$

in the weighted L^r– norm for $1 \leqslant r \leqslant \infty$. In our reasoning we use estimates of the fundamental solution of the equation

$$u_t = \Delta u + \frac{\lambda}{|x|^2} u$$

obtained recently by Milman and Semenov.

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Positive Liouville Theorems and Asymptotic Behavior for p-Laplacian Type Elliptic Equations with a Fuchsian Potential

Yehuda Pinchover
Technion – Israel Institute of Technology, Israel

We study positive Liouville theorems and the asymptotic behavior of positive solutions of p-Laplacian type elliptic equations of the form

$$Q'(u) := -\Delta_p(u) + V|u|^{p-2}u = 0 \qquad \text{in } X,$$

where X is a domain in $\mathbb{R}^d$, $d \geq 2$, and $1 < p < \infty$. We assume that the potential V has a Fuchsian type singularity at a point ζ, where either $\zeta = \infty$ and X is a truncated C^2-cone, or $\zeta = 0$ and ζ is either an isolated point of ∂X or belongs to a C^2-portion of ∂X. This is a joint work with Martin Fraas.

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Symmetry and Monotonicity Properties of Nonnegative Solutions of Elliptic Equation on Smooth Domains

Peter Polacik
University of Minnesota, USA

We consider a class of nonlinear elliptic equations on smooth reflectionally symmetric domains. We show how a well-known result of Serrin concerning overdetermined problems can be used to establish reflectional symmetry and monotonicity of nonnegative solutions.

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Parabolic Liouville-Type Theorems Via Their Elliptic Counterparts

Pavol Quittner
Comenius University, Bratislava, Slovakia

We study several problems with gradient structure (inclucing nonlinear heat equations, semilinear parabolic systems and problems with nonlinear boundary conditions) and show how the nonexistence of entire stationary solutions can be used in the proof of nonexistence of entire time-dependent solutions. We also mention some applications and open problems.

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Liouville Theorems and A-Priori Bounds for Nonlinear Second-Order Finite Difference Equations

Wolfgang Reichel
Karlsruhe Institute of Technology (KIT), Germany
(P. J. McKenna)

We consider nonlinear finite-difference boundary value problems of the type

$$-\Delta_h u = f(x, u) \text{ in } \Omega_h, \quad u = 0 \text{ on } \partial\Omega_h,$$

where Ω_h is a bounded discretized domain and $h > 0$ is the size of the uniform mesh. The nonlinearity $f(x, u)$ is assumed to have a power-like growth as $u \to \infty$. An interesting information on the set of positive solutions is, whether or not a-priori bounds exist, which are uniform with respect to the mesh-size h. As expected, a-priori bounds and nonlinear Liouville theorems for the finite-difference Laplacian are connected. We present some recent Liouville theorems and discuss the range of exponents for which these Liouville theorems are valid.

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Liouville Type Results for Linear Elliptic and Parabolic Operators

Luca Rossi
University of Padova, Italy

This talk is concerned with some extensions of the classical Liouville theorem for harmonic functions to solutions of more general linear PDEs. We start with presenting a positive result in the case of parabolic operators with periodic coefficients. We further exhibit an explicit counterexample showing that the periodicity assumption cannot be replaced by almost periodicity. Some sufficient conditions guaranteeing the almost periodicity of bounded solutions are also discussed.

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Liouville Theorems for Elliptic Inequalities and Systems

Boyan Sirakov
University Paris Ouest, France
(Scott N. Armstrong)

We review some recent results concerning the nonexistence of positive solutions of elliptic equations like $-Q[u] \geq f(x, u)$, where Q is a linear, quasilinear or fully nonlinear operator, and f is a function which behaves like a power of u, at zero or at infinity. We introduce a new method for proving such Liouville theorems, which, thanks to its simplicity and robustness, applies to various operators and notions of solutions (classical, weak-L^p, viscosity), permits us to consider systems of inequalities, and gives essentially optimal results.

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Existence and Nonexistence Results for Entire Solutions to Non-Cooperative Elliptic Systems

Tobias Weth
Goethe-Universität Frankfurt, Germany

We discuss a class of systems of nonlinear stationary Schrödinger equations which, for certain parameters, arises in the Hartree-Fock theory of a mixture of Bose-Einstein condensates in different hyperfine states. We focus on the case where the underlying domain is either the entire space or a half space (with Dirichlet boundary conditions in the latter case). In the case where the system is non-cooperative, existence and nonexistence results depend in a subtle way on parameters, and standard methods like the moving plane method cannot be used in the analysis of this problem.

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Homoclinic and Heteroclinic Orbits for a Semilinear Parabolic Equation: Part I

Eiji Yanagida
Tokyo Institute of Technology, Japan
(Marek Fila)

We study the existence of connecting orbits for the Fujita equation with a critical or supercritical exponent. For certain ranges of the exponent we prove the existence of heteroclinic connections from positive steady states to zero and a homoclinic orbit with respect to zero.

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A Liouville Theorem for a Semilinear Wave Equation and Applications to the Regularity of the Blow-Up Set

Hatem Zaag
CNRS Université Paris 13, France
(Frank Merle)

We prove a Liouville Theorem for the semilinear wave equation with power nonlinearity in one space dimension. This theorem allows us to derive regularity results on the singular set for blow-up solutions. Namely, we prove that the set of non-characteristic solutions is open and that the set of characteristic points has an empty interior. Furthermore, we show that the singular set is of class C^1 near a non-characteristic point and corner-shaped near a characteristic point.

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Special Session 19: Quasilinear Degenerate and Singular Elliptic and Parabolic Problems

Peter Takac, University of Rostock, Germany
Ratnasingham Shivaji, Mississippi State University, United States

Introduction: Most speakers will present the latest results on various kinds of quasilinear partial differential equations of elliptic and parabolic types. A few speakers plan to present overview lectures concerning an interesting hot topic. Among a number of topics are included: results on equations with the p-Laplace or related operators, regularity of solutions, maximum and anti-maximum principles, and nodal domains and lines of "nonlinear" eigenfunctions.

Intrinsic Ultra-Contractivity of a Schrödinger Semigroup in $\mathbb{R}^N$

Bénédicte Alziary
Université Toulouse 1 Capitole, France
(Peter Takáč)

The talk will summarize some recent results about the Schrödinger operator on $L^2(\mathbb{R}^N)$, with potential which is assumed growing somewhat faster than $|x|^2$ as $|x| \to \infty$. We give a sufficient condition on the electric potential q in the Schrödinger operator $\mathcal{A} = -\Delta + q(x)$ on $L^2(\mathbb{R}^N)$ that guarantees that the Schrödinger heat semigroup $\{e^{-\mathcal{A}t} : t \geq 0\}$ on $L^2(\mathbb{R}^N)$ generated by $-\mathcal{A}$ is intrinsically ultracontractive.

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Sturm Comparison Principle and Fucik Spectrum for Quarter-Linear Differential Equations

Yuanji Cheng
Malmo University, Sweden

In this talk we study the quarter-linear differential equation

$$-(\varphi(x,u'))' + q(x,u) = \lambda\psi(x,u), -T \leq x \leq T$$

which is invariant only under scaling of positive number, i.e., $u \to ku, k > 0$. We extend the classical Sturm-Liouville theory to this new class of equations and establish a comparison principle and furthurmore obtain sequences of eigenvalues / branches of Fucik spectrum for Dirichlet boundary.

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Variational Formulation of the Convection-Diffusion Equation

Jacky Cresson
University of Pau, France
(Pierre Inizan)

The convection-diffusion equation does not admit a standard variational formulation. We prove that the solutions of the convection-diffusion equation correspond to critical points of an explicit fractional Lagrangian functional. The method is related to non-resersibility of the underlying dynamics and a way to take into account this property in a fractional variational framework. We discuss other applications of this formalism to PDEs.

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Eigenvalue Problems for the p-Laplacian and Population Dynamics

Ann Derlet
Université Libre de Bruxelles, Belgium
(Jean-Pierre Gossez and Peter Takac)

We consider a weighted eigenvalue problem for the p-Laplacian with Neumann boundary conditions. We discuss the minimization of the positive principal eigenvalue over some class of sign-changing weights. It is proved that minimizers exist and satisfy a bang-bang type property. In dimension one, we obtain a complete description of the minimizers. Remarks on how such problems arise in population dynamics are also given.

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A Priori Estimates for the p-Laplacian

Pavel Drabek
University of West Bohemia, Pilsen, Czech Rep.
(Daniel Daners, University of Sydney)

The talk will summarize the results of the paper published in Trans. Amer. Math. Soc. 361(2009), 6475-6500. In this paper we prove a priori estimates for a class of quasi-linear elliptic equations. To make the proofs clear and transparent we concentrate on the p-Laplacian. We focus on L_p-estimates for weak solutions of the problem with all standard boundary conditions on non-smooth domains. As an application we prove existence, continuity and compactness of the resolvent operator.

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A Review of Results About Quasilinear and Singular Equations

Jacques Giacomoni
Université de Pau et des Pays de l'Adour, France
(Kaushik Bal and Mehdi Badra)

In this talk, I will present recent and joint contributions about quasilinear and singular elliptic and parabolic equations (existence, multiplicity of solutions, regularity). In particular, I will focus on the following problem:

$$\begin{cases} u_t - \Delta_p u = \frac{1}{u^\delta} + f(u), \\ \quad \text{in } Q = (0,T] \times \Omega \\ u > 0 \text{ in } Q, \\ u = 0 \text{ on } \Gamma = [0,T] \times \partial\Omega, \\ u(0,x) = u_0(x) \text{ in } \Omega, \end{cases} \tag{P$_t$}$$

where Ω is an open bounded domain with smooth boundary, $1 < p < \infty$ and 00, f a locally Lipschitz function in $\mathbb{R}^+$ and $u_0 \in L^2(\Omega) \cap W_0^{1,p}(\Omega)$.

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On the Fredholm Alternative for the p-Laplacian at Higher Eigenvalues in One Dimension

Petr Girg
University of West Bohemia, Czech Republic
(Jiří Benedikt, Peter Takáč)

We investigate the existence of a weak solution u to the quasilinear two-point boundary value problem

$$\begin{aligned} -(|u'|^{p-2}u')' = \lambda_k\, |u|^{p-2}u + f(x)\,, \\ 0 < x < a\,; \\ \text{Âă}u(0) = u(a) = 0\,. \end{aligned} \tag{P}$$

We assume $1 < p < 2$, $0 < a < \infty$, and $f \in L^\infty(0,a)$ is a given function, $f \not\equiv 0$. The number λ_k stands for the k-th eigenvalue of the one-dimensional p-Laplacian. Let $\sin_p(\pi_p x/a)$ denote the eigenfunction associated with λ_1; then $\sin_p(k\pi_p x/a)$ is the eigenfunction associated with λ_k. It is well-known that problem (P) has a solution when $k = 1$ and f satisfies the orthogonality condition

$$\int_0^a f(x) \sin_p(k\pi_p x/a)\, \mathrm{d}x = 0\,.$$

We focus on the case $k \geq 2$.Assuming the orthogonality condition above, we obtain new existence results for (P) which significantly generalize those already known. We are able to handle more degenerate sufficient conditions on f than any previous work. We present a simple example of f to which previous results and methods are not applicable. Our approach is based on the third- and second-order Taylor expansions of the functions $\sin_p$ and $\cos_p = \sin_p'$, respectively, in the L^1-norm (rather than in the standard L^∞-norm). We use computer algebra systems in our treatment of the problem. Computer assistance enters into our methods of proofs in two distinct manners at two points: We begin with symbolic computations in asymptotic Taylor expansions of third and fourth orders, followed by implementation of interval arithmetics in numerical computations for quadrature formulas that appear in these expansions.

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Principal Eigenvalues for Some Non Selfadjoint Elliptic Problems and Applications

Jean-Pierre Gossez
Universite Libre de Bruxelles, Belgium
(T. Godoy and S. Paczka (Cordoba, Argentina))

It is our purpose to present a minimax formula for the principal eigenvalues of a non selfadjoint elliptic problem involving an indefinite weight function. Several applications can be considered, which concern the antimaximum principle or some inverse problems. In this talk we will concentrate on the use of that formula to study the asymptotic behavior of the principal eigenvalues when the first order coefficient (drift term) becomes larger and larger. Studying such an asymptotic behavior is partly motivated by some questions from nonlinear propagation.

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Positive Solutions to Some Singular Quasilinear Elliptic Systems

Jesus Hernandez
Universidad Autonoma de Madrid, Spain

We present some recent work on existence of positive solutions to a class of quasilinear singular elliptic systems. Existence is proved by using sub- and supersolutions. We also study some semilinear systems. This is joint work with J. Giacomoni and A. Moussaoui and also with F. Mancebo and J. M. Vega.

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Functional Reaction-Diffusion Problems from Daisy-World Climate Models

Georg Hetzer
Auburn University, USA

Motivated by coupling an energy balance climate model and a two-species competition model (white

and black daisies) representing the bio-sphere, one is led to study functional reaction-diffusion equations with a p-Laplacian-type diffusion and memory and nonlocal reaction terms. The nonlocal term has the Volterra property. The talk will summarize some recent results about existence and long-term behavior.

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Existence of Compactons for Elliptic Equations with Autonomous Nonlinearity

Yavdat Ilyasov
Insitute of Mathematics RAS, Ufa SC, Russia

We present a class of elliptic equations with autonomous non-Lipschitz nonlinearity which possesses compactons: weak nonnegative solutions with compact support on $\mathbb{R}^n$. A new approach to finding the compactons will be introduced. The basic idea consists in using Pohozaev's identity and the spectral analysis with respect to the fibering method. This work is in collaboration with Y. Egorov.

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Existence of Positive Solutions for Infinite Semipositone Systems

Eun Kyoung Lee
Pusan National University, Korea
(R. Shivaji and Jinglong Ye)

In this talk, we consider positive solutions for infinite semipositone systems of the form:

$$\begin{cases} -\Delta_p u = \lambda \frac{f_1(v)}{u^{\alpha_1}} & \text{in } \Omega \\ -\Delta_p v = \lambda \frac{f_2(u)}{v^{\alpha_2}} & \text{in } \Omega \\ u = 0 = v & \text{on } \partial\Omega, \end{cases} \tag{P_1}$$

and

$$\begin{cases} -\Delta_p u = \lambda \frac{f_1(v)}{v^{\alpha}} & \text{in } \Omega \\ -\Delta_p v = \lambda \frac{f_2(u)}{u^{\alpha}} & \text{in } \Omega \\ u = 0 = v & \text{on } \partial\Omega, \end{cases} \tag{P_2}$$

where Ω is a bounded domain in $\mathbb{R}^n$ with smooth boundary, $\Delta_p u = \operatorname{div}(|\nabla u|^{p-2}\nabla u)$, α, $\alpha_i \in (0,1)$, $f_i \in C([0,\infty))$, $f_i(0) < 0$ for $i = 1,2$ and satisfying a combined sublinear condition at ∞. We use the method of sub-supersolutions to prove the existence of positive solutions of (P_1) and (P_2) for $\lambda \gg 1$.

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Strong Solutions of Doubly Nonlinear Navier-Stokes Equations

Jochen Merker
University of Rostock, Germany
(Ales Matas)

Strong solutions of a doubly nonlinear evolution equation $\frac{\partial Bu}{\partial t} + Au = f$ can be obtained by testing the approximate equation with $\frac{\partial u}{\partial t}$. In doing so, four different situations can occur, and correspondingly four different types of strong solution can be distinguished.

For example, if $Y \subset H \cong H^* \subset Y^*$ is a Gelfand triple and the monotone potential operator $B : Y \to Y^*$ has a continuously differentiable inverse such that $\langle v, dB^{-1}(u^*)v\rangle \geq c\|v\|_H^2$ for all $u^* \in Y^*$, $v \in H$, with a constant $c > 0$, then under standard assumptions on $A : X \to X^*$ and $f \in L^2(0,T;H)$ there exists a weak solution $u \in L^p(0,T;X)$ with $Bu \in W^{1,\infty,p'}(0,T;Y^*,X^*)$, which additionally has the properties $u \in L^\infty(0,T;X)$ and $\frac{\partial Bu}{\partial t} \in L^2(0,T;H)$. Thus, u should be called a strong solution. Especially, in this situation $Au(t) \in H$ for a.e. $t \in [0,T]$, and the doubly nonlinear equation is satisfied as an equation in $L^2(0,T;H)$.

As an application, parameters (m,p) are determined such that the doubly nonlinear Navier-Stokes equation

$$\frac{\partial Bu}{\partial t} + \operatorname{div}(Bu \otimes u) - \Delta_p u = -dP\,,\ \operatorname{div}(u) = 0$$

admits a strong solution for a kinetic energy, which behaves like $|u|^m$ as $|u| \to \infty$ and is a potential of B.

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Existence and Estimate of the Free-Boundary for a Non Local Inverse Elliptic-Parabolic Problem

Juan Francisco Padial
Universidad Politécnica de Madrid, Spain

We introduce an inverse 2D free-boundary elliptic-parabolic problem, modelling the transient regime of a magnetically confined plasma in a non ideal Stellerator device. The inverse nature of the problem comes from the fact that the associated Grad-Shafranov equation involves some unknown nonlinear terms which must be determined by the current-carrying Stellarator condition. We present some result about the existence and estimation of the free-boundary for a non local weak solution. To do that, we introduce a radial auxiliary problem in order to obtain a sub and supersolution of our original free-boundary problem. Finally, by applying the comparison principle for quasi-linear problems (Gilbarg-Trudinger 1983), we obtain some appropriate estimates of the free-boundary.

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Weighted Eigenvalue Problems in Whole Space

Mythily Ramaswamy
Centre for Applicable Mathematics, India
(T. V. Anoop and M. Lucia)

The weighted eigenvalue problem for the Laplacian in the whole space is considered searching for a large class of weight functions, which would ensure the existence of positive principal eigenvalue, its simplicity and the positivity of the first eigenfunction. The largest subspace of weight functions is determined in the Lorentz space set up. The results are extended to p-Laplacian.

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Continua of High Energy Local Minimizers in a Non Smooth Model of Phase Transitions

Stephen Robinson
Wake Forest University, USA
(Pavel Drabek)

We study critical points of the functional

$$J_\varepsilon(u) := \frac{\varepsilon^2}{2}\int_0^1 |u_x|^2 dx + \int_0^1 F(u)dx, u \in W, \quad (0.1)$$

where $F : \Re \to \Re$ is a double-well potential. This functional represents the total free energy in models of phase transition and allows for the study of interesting phenomena such as *slow dynamics*. In particular we consider a non-classical choice for F modeled on $F(u) = (1-u^2)^\alpha$ where $1 < \alpha < 2$.

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Multiplicity for a φ-Laplacian Operator with a Conormal Boundary Condition

Ian Schindler
University of Toulouse 1, France
(Bhatia Sumit Kaur and K. Sreenadh)

Let Ω be a bounded domain in $I\!R^N$ with smooth boundary $\partial\Omega, N \geq 2$. We study the existence of multiple positive solutions of the following problem:

$$\begin{cases} \left.\begin{aligned} -\mathrm{div}\,\varphi(|\nabla u|)\nabla u) + \varphi(|u|)u &= u^p e^{|u|^\alpha} \\ u &> 0 \end{aligned}\right\} \text{ in } \Omega, \\ \varphi(|\nabla u|)\dfrac{\partial u}{\partial n} = \lambda u^q \text{ on } \partial\Omega, \end{cases}$$

where $p > \max\{N-1,\ p_0-1\}, \alpha \in (0, \frac{N}{N-1}]$ and φ satisfies the asymptotic conditions $\varphi(t) \sim t^{p_0-2}$, $p_0 > 1$ as $t \to 0^+$ and $\varphi(t) \sim t^{N-2}$ as $t \to \infty$. The value of q in the boundary term satisfies $0 < q < \min\{p_0-1,\ N-1\}$. We show that there exists $\Lambda \in (0,\infty)$ such that (P_λ) admits a solution for $\lambda \in (0,\Lambda)$ and no solution for $\lambda > \Lambda$. In case of $\alpha < \frac{N}{N-1}$, we show that (P_λ) admits a second solution for $\lambda < \Lambda$.

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Polyharmonic PDEs and ODE Dynamics

Paul Schmidt
Auburn University, USA
(Monica Lazzo)

Radial solutions of $\Delta^m u = f(u)$ in space dimension n can be identified with solutions of the asymptotically autonomous ODE $\left(\partial_r^2 + \frac{n-1}{r}\partial_r\right)^m u = f(u)$. Many questions regarding the nature of such solutions can be answered by studying the dynamics of the autonomous limit equation $u^{(2m)} = f(u)$. We explain and illustrate this observation and present a number of conclusions for the case of a power nonlinearity.

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Spectrum for 1-Dim p-Laplace Eigenvalue Problems with Singular Sign-Changing Weights

Inbo Sim
University of Ulsan, Korea
(Ryuji Kajikiya and Yong-Hoon Lee)

In this talk, we study the existence of spectrum for one-dimensional p-Laplace eigenvalue problems with singular weights subject to Dirichlet boundary condition. For certain class of singular sign-changing weights, we prove the existence of discrete spectrum and present an example of a weight for which the spectrum is continuous.

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The Dirichlet Problem and Landesman-Lazer Type Results for Hamilton-Jacobi-Bellman Operators

Boyan Sirakov
University Paris Ouest, France
(Patricio Felmer, Alexander Quaas)

We study the Dirichlet problem in a bounded domain for a Hamilton-Jacobi-Bellman operator, that is, an arbitrary supremum of linear uniformly elliptic operators. We show that the solvability of this problem and the structure of its solution set depend on the signs of the two "half"-principal eigenvalues of the operator.

Further we consider the problem $F[u] + \lambda u = f(x,u)$ in Ω with a Dirichlet boundary condition,

where F is a Hamilton-Jacobi-Bellman operator, under Landesman-Lazer type hypotheses on the nonlinear function f. We establish various results about the existence of continua of solutions of this equation, and about their properties.

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Biharmonic Equation and Relevant Boundary Conditions on Polygonal Domains

Guido Sweers
Universität Köln, Germany
(Serguei Nazarov, Athanassis Stylianou)

The Kirchhoff-Love model for the vertical deformation of a thin plate consists of the fourth order biharmonic equation supplemented with appropriate boundary conditions. Typical situations are so-called clamped, hinged, respectively supported plate. We will present the corresponding boundary conditions and show some surprising behaviour near corners.

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The Effect of the Deep Ocean on a Climate Energy Balance Model

Lourdes Tello
Universidad Politecnica de Madrid, Spain
(A. Hidalgo)

We are concerned with a parabolic problem involving the coupling of the mean surface temperature with the ocean temperature. One of the difficulties of this problem is the dynamic and diffusive boundary condition. The purpose of this work is to compare the solution of such a parabolic problem with the solution of an energy balance model (studied by G. Hetzer [1990] and J. I. Diaz [1993]) where the deep ocean effect was not explicitly included. We have approximated the solutions by a finite volume scheme.

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Special Session 20: Topological Dynamics

Piotr Oprocha, Universidad de Murcia, Spain
Francisco Balibrea Gallego, Universidad de Murcia, Spain
Victor Jimenez Lopez, Universidad de Murcia, Spain
Lluis Alseda i Soler, Universitat Autonoma de Barcelona, Spain

Introduction: This session is focused in the topological aspects of dynamical systems (entropy, chaos, limit sets and the like), mainly in the setting of low dimensional discrete dynamics. Nevertheless, related parts of dynamical systems theory and ergodic theory are not excluded.

Chain Transitivity, ω-Limit Sets and Symbolic Dynamics

Andrew Barwell
University of Birmingham, England

ω-limit sets are important and interesting objects in dynamical systems, since they give us information pertaining to the asymptotic behaviour of the system at specific points. They are known to have the property of internal chain transitivity (ICT), which states that between any two points x and y in the set we can find a sequence of points in the set, starting at x and ending at y, which approximate an orbit segment to within arbitrary degrees of accuracy. However the cases when ICT sets are necessarily ω-limit sets are not fully understood. We use symbolic dynamics to identify a class of functions for which ICT sets are always ω-limit sets, and cite the property of pseudo-orbit shadowing to explain why the symbolic dynamics works in this case.

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Code and Order Equivalence in Polygonal Billiards

Jozef Bobok
Technical University in Prague, Czech Republic
(Serge Troubetzkoy)

We compare two equivalence relations on polygonal billiards. We show when code/order equivalent billiards have the same angles, resp. are similar, resp. affinely similar.

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Combinatorics of Quadrtic Julia Sets, Admissibility, Biaccessibility and Hausdorff Dimension

Henk Bruin
University of Surrey, England
(Dierk Schleicher)

Hubbard trees are the back-bone of Julia sets of

(quadratic) polynomials. Together with the dynamics on the circle of external angles, they can be used to construct kneading theory in the complex setting. In this talk I want to present results (joint with Dierk Schleicher, Bremen) about which 0-1-sequences occur as kneading sequences, which are biaccessible, and what is the Hausdorff dimension of biaccessible external angles.

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On the Relationship between the Asymptotic Behavior of a Convergent Non-Autonomous System and Its Limit Map

Jose Cánovas
Universidad Politécnica de Cartagena, Spain

Let $f_n : X \to X$ be a sequence of continuous maps on a compact metric space, which converges uniformly to a map $f : X \to X$. The sequence $f_{1,\infty} = (f_n)$ defines a non–autonomous discrete system $(X, f_{1,\infty})$. In general, the limit properties of $(X, f_{1,\infty})$ may be different from that of its limit system (X, f). However, when the limit map f has the shadowing property, some relationship between the asymptotic behavior of the non–autonomous and the autonomous system can be established. In this talk some of them will be presented.

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On Almost Totally Disconnected Minimal Systems

Roman Hric
Matej Bel University, Slovak Republic

I will present new results on almost totally disconnected minimal systems.

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Pseudoarcs and Generalized Inverse Limits

Judy Kennedy
Lamar University, USA
(Sina Greenwood)

We show that for surjective upper semicontinuous bonding functions f on an interval I whose graphs are pseudoarcs, the generalized inverse limit space is a Cantor set.

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An Example of a Volume Preserving Flow

Krystyna Kuperberg
Auburn University, USA

In 1996, G. Kuperberg gave an example of a volume preserving C^1 flow on S^3 with no compact orbits. The construction is based on Schweitzer's counterexample to the Seifert conjecture, and thus there are two minimal sets, each homeomorphic to the Denjoy minimal set.

We will show another example of a volume preserving flow on S^3 with no compact orbits. There is only one minimal set, but the flow is only C^0.

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Devaney Chaotic Fuzzy Dynamical Systems

Jiri Kupka
IRAFM, University of Ostrava, Czech Republic

It is well known that any given discrete dynamical system uniquely induces its fuzzified counterpart, i.e. a discrete dynamical system on the space of fuzzy sets. In this talk we present some relations between dynamical properties of the original and fuzzified dynamical system. Especially, we study conditions used in the definition of Devaney chaotic maps, i.e. periodic density and transitivity. Among other things we show that dynamical behavior of the set-valued and fuzzy extensions of the original system mutually inherits some global characteristics. Within this contribution we provide a solution of the problem that was introduced and partially solved in [H. Román-Flores, Y. Chalco-Cano, *Some chaotic properties of Zadeh's extension*, Chaos, Solitons & Fractals, Volume 35, Issue 3, February 2008, Pages 452-459].

$\longrightarrow \infty \diamond \infty \longleftarrow$

Dynamics of Iterative Systems

Petr Kurka
Center for Theoretical Study, Czech Republic

Iterative system is an action of the free monoid A^* of a finite alphabet A on a compact metric space X, or a system of continuous maps $(F_u : X \to X)_{u \in A^*}$ satisfying $F_{uv} = F_u F_v$ and $F_\lambda = Id_X$. We investigate minimality, transitivity and attractors in iterative systems.

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Finite Rank Bratteli Diagrams and Their Invariant Measures

Jan Kwiatkowski
Univ. of Warmia and Mazury in Olsztyn, Poland
(S. Bezuglyi, K. Medynets and B. Solomyak)

The simplex of invariant measures on Bratteli diagrams which have the same number of vertices on each level is study. It is shown that every ergodic (finite or infinite) is an extension of a finite ergodic measure from a simple subdiagram. Several structural properties of uniquely ergodic diagrams are

also established. It is also proved that Vershik maps on unique diagrams cannot be mixing.

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Topological Entropy for Set Valued Maps

Marek Lampart
Technical University of Ostrava, Czech Republic
(Peter Raith)

Any continuous map T on a compact metric space $\mathbb{X}$ induces in a natural way a continuous map $\overline{T}$ on the space $\mathcal{K}(\mathbb{X})$ of all nonempty compact subsets of $\mathbb{X}$. Let T be a homeomorphism on the interval or on the circle. It is proved that the topological entropy of the induced set valued map $\overline{T}$ is zero or infinity. Moreover, the topological entropy of $\overline{T}|_{\mathcal{C}(\mathbb{X})}$ is zero, where $\mathcal{C}(\mathbb{X})$ denotes the space of all nonempty compact and connected subsets of $\mathbb{X}$. For general continuous maps on compact metric spaces these results are not valid.

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On Li-Yorke Pairs for Graph, Dendrite and Dendroid Map

Issam Naghmouchi
Faculty of Science of Bizerte, Tunisia

We show that there is no Li-York pair for one to one finite graph and dendrite map. However, there exists a homeomorphisms on a dendroid with a scrambled set having nonempty interior.

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Recurrence in Pairs

William Ott
University of Houston, USA
(Kamel Haddad)

A point x in a topological dynamical system (X,T) is said to be weakly product recurrent if for every topological dynamical system (Y,S) and for every uniformly recurrent point $y \in Y$, the pair (x,y) is recurrent under the product action (T,S). In 1994, while studying the relationship between product recurrence and distality in the context of semigroup actions, Auslander and Furstenberg asked the following question: Does weak product recurrence imply product recurrence? We answer this question in the negative, discuss extensions of this result due to Piotr Oprocha, and formulate a new question in the spirit of Auslander and Furstenberg.

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Distributional Chaos and Completely Irregular Operators

Alfred Peris
Universitat Politècnica de València, Spain
(F. Martínez-Giménez, P. Oprocha)

Schweizer-Smital notion of dstributional chaos measures how frequently the orbits of two different points in a scrambled set separate far apart from certain distance, and stay closer than an arbitrary small ε. The purpose in this talk is to analyze examples of dstributionally chaotic shifts with the full space X as a scrambled set, which at the same time are topologically transitive too. These phenomena are related with the concept of completely irregular operators introduced by Beauzamy.

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ω-Limit Sets in Cylindrical Transformations Over Odometes

Artur Siemaszko
Univ. of Warmia and Mazury in Olsztyn, Poland
(Jan Kwiatkowski)

Let X be a compact metric space and $T : X \longrightarrow X$ be a homeomorphism of X. Let $\varphi : X \longrightarrow \mathbb{R}$ be a continuous function (called *a cocycle*). By a *cylinder transformation* we mean a homeomorphism $T_\varphi : X \times \mathbb{R} \longrightarrow X \times \mathbb{R}$ (or rather a $\mathbb{Z}$–action generated by it) given by the formula

$$T_\varphi(x,r) = (Tx, \varphi(x) + r).$$

We will also consider the case $\mathbb{R}^m$ instead of $\mathbb{R}$.

As such a transformation cannot be itself minimal (see [1] or [2]) we will focus on the problem of minimal subset. The problem of the minimal subsets of a cylinder transformation turns out to be related to the problem of possible forms of ω-limit sets. H. Poincaré was the first to consider flows (generated by differential equations) on $\mathbb{R}^3$ that had time one homeomorphisms topologically isomorphic to cylinder cocycle extensions over irrational rotations and to investigate vertical cross-sections of their ω-limit sets ([3]).

So far the only described ω-limits sets of cylindrical transformations are those of dense or discrete orbits, i.e. trivial in some sense.

We show how to construct topologically transitive cylindrical transformations over odometers with non-trivial ω-limit sets.

[1] A. S. Besicovitch, *A problem on topological transformations of the plane. II*, Proc. Cambridge Philos. Soc. 47, (1951), 38-45.
[2] P. Le Calvez, J.-Ch. Yoccoz, *Un théoreme d'indice pour les homéomorphismes du plan au*

voisinage d'un point fixe. (French. English summary) [*An index theorem for homeomorphisms of the plane near a fixed point*], Ann. of Math. (2) 146 (1997), no. 2, 241–293.
[3] H. Poincaré, *O krivykh opred'eliayemykh different'ialnymi uravneniami* (Russian) [On curves defined by differential equations], Moskva, Ogiz, 1947.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Distributional Chaos on Compact Metric Spaces – Recent Results and Open Problems

Jaroslav Smital
Silesian University, Opava, Czech Republic

16 years ago the first results concerning dstributional chaos were published. They concern continuous maps of the interval. This results were later extended to other one-dimensional systems like topological graphs or dendrites. However, there are only few results concerning general compact metric space. In the talk we point out some recent development and main open problems.

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Distributional Chaos and the Size of Scrambled Sets

Marta Stefankova
Silesian University, Opava, Czech Republic

Recently many papers appeared concerning dstributional chaos on compact metric spaces. In most of these papers only two-point scrambled sets were considered.

In my talk I would like to introduce three new types of dstributional chaos (DC) distinguished by certain properties of their scrambled sets and discuss their relations. First I will introduce *small DC* with two-point scrambled set and *large DC* with uncountable scrambled set. Finally I will introduce *uniformly large DC* as large DC such that the lower dstribution functions of any two different points from the scrambled set are the same.

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On Ingram's Conjecture

Sonja Štimac
University of Zagreb, Croatia
(Marcy Barge, Henk Bruin)

In recent years there has been intensive research of topological properties of inverse limit spaces of tent maps with classification of these spaces as ultimate goal. The Ingram conjecture claims that two inverse limits of tent maps with different slopes are not homeomorphic. I will discuss recent progress on Ingram's conjecture emerging from joint work with Barge and Bruin.

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Distributional Chaos in Planar Nonautonomous ODEs

Pawel Wilczynski
Jagiellonian University, Cracow, Poland

Our goal is to discuss the mechanism of generating dstributional chaos in planar nonautonomous ODE given by (in complex number notation) $\dot{z} = e^{i\kappa t}(\overline{z}^2 - 1)$, where $\kappa > 0$ is small enough.

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Special Session 21: Intermingled Dynamics and Structure in Complex Systems

Regino Criado, Universidad Rey Juan Carlos, Spain
Juan A. Almendral, Universidad Rey Juan Carlos, Spain

Introduction: The so-called Complex Network Theory is the result of merging concepts and tools from two fields, one from Physics (Statistical Physics) and the other from Mathematics (Graph Theory). From one side, Statistical Physics is an attempt to explain the macroscopic behavior of matter from a large ensemble of atomic components and molecular interactions. On the other side, Graph Theory provides us with the perfect tool to model pairwise relations between the components of a given system. Nowadays, there are many evidences proving that Complex Network Theory can be used to explain phenomena from areas as different as Sociology, Biology, or Engineering. One of its open problems is to elucidate the interplay between the structural and functional properties in a complex system. In general, the evolution of a network topology is intimately related to the dynamical processes taking over the net, and vice versa, the emergent dynamics is highly influenced by the underlying structure. In this Special Session we pretend to survey on this problem with both the introduction and analysis of new topological concepts from a theoretical viewpoint, and several analytical, numerical and experimental reports on real-world networks.

Evolutionary Dynamics, Topology and Complexity on RNA Neutral Networks

Jacobo Aguirre
Centro de Astrobio. CSIC-INTA, Madrid, Spain
(Javier M. Buldú, Michael Stich, Susanna C. Manrubia)

An RNA neutral network is a connected ensemble of all RNA sequences in the genome space folding into the same minimum free energy secondary structure. The secondary structure of an RNA molecule can be understood as a proxy for its function. In this kind of networks, each sequence occupies a node and two nodes are connected if the corresponding sequences differ in only one nucleotide. Therefore, a population of sequences can move, through mutations, on such networks without seeing its functionality affected.

In this work we study analytically and numerically the properties of generic RNA neutral networks that affect robustness: its areas of maximal neutrality against mutations and the minimum free energy associated to the folded state of each sequence. We have analyzed how topology determines the time that the population takes to reach the equilibrium state. When information on the energy associated to each sequence is included, topology also affects the survivability of the populations under temperature fluctuations.

As a present and future work, we have also characterized the topology of all “real” RNA networks of length 12 with the use of the Vienna package, paying special attention to the dependence of the properties of the sequences (such as their nucleotide composition or the folding energy) on the topological properties of the nodes.

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[2] E. van Nimwegen, J. P. Crutchfield and M. Huynen, Neutral evolution of mutational robustness, PNAS 96, 9716 (1999).
[3] P. Schuster, Prediction of RNA secondary structures: from theory to models and real molecules, Rep. Prog. Phys. 69:1419 (2006).
[4] I. L. Hofacker, W. Fontana, P. F. Stadler, L. S. Bonhoeffer, M. Tacker, and P. Schuster, Fast folding and comparison of RNA secondary structures, Monatsh. Chem.125: 167–188.1994.

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Detection of Overlapping Communities by Synchronization

Juan Almendral
Rey Juan Carlos University, Spain
(I. Leyva; I. Sendiña-Nadal; J. M. Buldú; S. Boccaletti)

We report evidence that, under the presence of different functional (synchronized) clusters, interfaces may appear and show a specific dynamical behavior, consisting in an almost periodical switching between the coherent evolutions of the clusters. This evidence let us derive an analytic treatment of an abstracted system to give further insight and, based on our findings, we develop an algorithm that is able to detect overlapping structure in both artificially constructed and real world modular network. Specifically, when we consider networks consisting of several domains of interacting phase oscillators, we find that, at each time, most of the oscillators will contribute to the synchronous behavior of the main cluster in which is contained, whereas a few nodes will find themselves in a frustrating situation of having to decide how to behave as a consequence of the contrasting inputs received by the other clusters.

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Overview and Application of Heterogeneous Preferential Attachment Networks

Rosa Benito
Universidad Politecnica de Madrid, Spain
(A. Santiago, J. P. Cardenas)

We present results concerning heterogeneous evolving networks based on preferential attachment (PA) schemes. We define a general class of heterogeneous PA models, where node intrinsic properties are described by fixed states in an arbitrary space, and introduce an affinity function that biases the attachment probabilities of links in the attachment kernel. We show how the introduction of heterogeneity in PA schemes yields a richer behavior in important network metrics, such as connectivity degrees, clustering coefficients and robustness to errors. We also define alternative model classes where the attachment kernel assumes different functional forms, and show the influence of such variations on the connectivity degrees of the networks. Finally we show an application example of these models to geophysics, where heterogenous evolving networks are used to model the structural properties of porous soils.

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Some Relationships between Structural Properties of a Network and the Associated Line Graph

Regino Criado
Rey Juan Carlos University, Spain
(Julio Flores, Alejandro Garcia del Amo and Miguel Romance)

The centrality and effciency measures of a network G are strongly related to the respective measures on the associated line graph G' and bipartite graph $B(G)$. We show some relationships between the Bonacich centralities $c(G)$, $c(G')$ and $c(B(G))$ and between the efficiencies $E(G)$ and $E(G')$. We also compute the behaviour of these parameters in some examples.

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Annealed Formulation of the Dynamics on Static and of Adaptive Complex Networks

Jesus Gomez-Gardenes
Universidad Rey Juan Carlos, Spain
(Beniamino Guerra)

We propose an annealed formulation of complex networks to study the dynamical behavior of both static and adaptive networked systems. The formulation relies on an annealed adjacency matrix, representing one network ensemble, and allows to solve the dynamical evolution of the network all at once. We particularize on the disease spreading dynamics on top of static and growing adaptive networks. Our results accurately reproduce those obtained by numerical simulations thus showing that the formalism, apart from fast and effective, constitutes an unifying framework for the study of dynamical processes on networks and complex adaptive systems.

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Different Notions of Chaos in Discrete Dynamical Systems and Relations between Them

Juan L. G. Guirao
Polythecnical University of Cartagena, Spain
(Marek Lampart)

The aim of the talk is to introduce different notions of the concept of chaos in the setting of Discrete Dynamical Systems generated by the iteration of a continuous self-map defined on a compact metric space and to state the relations between them. New results on this trend are stated.

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***k*-Cores in Complex Networks**

Jose Mendes
University of Aveiro, Portugal
(S. N. Dorogovtsev and A. V. Goltsev)

The k-core of a graph is its maximum subgraph in which each node has at least k neighbours within this subgraph. The k-core may be obtained in the following way (the pruning algorithm). Remove from a network all nodes with at least k links. (In graph theory, the number of connections of a node is called its degree.) Some of the remaining vertices may remain with less than k edges. Then remove these vertices, and so on until no further removal is possible. The result, if it exists, is the k-core. Thus, a network is organized as a set of successively enclosed k-cores. The highest k-cores represent most connected and robust areas of a network. Importantly, the notion of the k-core is applicable to a wide range of systems in physics, including disordered lattices. One can see that the k-core is a direct generalization of the percolation cluster (giant connected component in graph theory). Moreover, the k-core problem is among fundamental issues in condensed matter physics and network science. In particular, this problem is directly related to the nature of congestion and jamming phenomena in various systems, to the evolution of magnetization in ferromagnetic materials with random field disorder after the magnetic field is applied, etc. Generally, this notion turns out to be relevant in numerous situations, in which the state of a node changes after

a given function of the states of the neighbouring nodes exceeds some threshold.

The question is: what is the nature of the k-cores in complex networks which are particularly heterogeneous systems? One should also ask: how do these k-cores emerge? We have developed the theory of k-cores in most representative networks having complex architectures. These are uncorrelated random graphs with an arbitrary degree dstribution. An important feature of these networks is absence of short loops, which makes possible to find an exact solution of the problem. We have applied our theory to complex networks with power-law degree dstributions (scale-free networks). The Internet is among these networks.

In conclusion, we indicate that the k-core problem is generically related with so-called bootstrap percolation. This is an activation process on a network, in which a node becomes active if at least k of its nearest neighbours are active. Bootstrap percolation is an extensively studied yet rather poorly understood problem in condensed matter physics. Our future work is aimed at the better understanding of relations between these two fundamental problems.

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Modelling of Social Skills on Social Network Sites

Francisco Pedroche
Universitat Politecnica de Valencia, Spain

In recent years Social Network Sites (SNSs) on the Internet have caught the attention of researchers from different disciplines. The number of users of SNSs has grown enormously. The same has happened with the time the users spend visiting these sites. SNSs have a great variability; the owners of the SNSs introduce novelties in the characteristics of the sites. Therefore is interesting to develop general models to analyze SNSs. In this talk a method to classify the users of SNSs is presented. The method is based on the computation of the biased PageRank associated to the graph. The personalization vector is used to incorporate specific features of the users into the model. The concept of social competence is defined as a weighted vector to incorporate some features of the users related to social skills. The model can be used to analyze the dynamical behavior of a SNS. The model is aimed to be a practical tool for the managers of SNSs like Facebook, Myspace, Twitter, etc. Some of the concepts introduced can be applied to general complex networks. Some numerical examples are shown.

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Spectral Centralities: Global vs Local Approach

Miguel Romance
Rey Juan Carlos University, Spain

We will analyze several centrality measure by giving a general framework that includes the Bonacich centrality, PageRank centrality or in-degree vector among others. We will get some local scale estimators for such a global measures by giving some geometrical characterizations and some deviation results that helps to quantify the error of approximating a spectral centrality by a local estimator.

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Controlling Synchrony by Delay Coupling in Networks: From In-Phase to Splay and Cluster States

Eckehard Schöll
TU Berlin, Germany

(Chol-Ung Choe, Thomas Dahms, Philipp Hövel)

We study synchronization in delay-coupled oscillator networks, using a master stability function approach. Within a generic model of Stuart-Landau oscillators (normal form of super- or subcritical Hopf bifurcation) we derive analytical stability conditions and demonstrate that by tuning the coupling phase one can easily control the stability of synchronous periodic states. We propose the coupling phase as a crucial control parameter to switch between inphase synchronization or desynchronization for general network topologies, or between in-phase, cluster, or splay states in unidirectional rings. Our results are robust even for slightly nonidentical elements of the network.

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Bifurcation Analysis in Two Chaotic Coupled Systems Drived by an External Force

Gerard Vidal
Institute of Physics, Spain

(G. Vidal, H. Mancini)

Quasi-periodic oscillations arise frequently in preturbulent states of thermo-convective flows. In many experiments, different oscillation sources (like boundary layer modulation, "thermals" and bubbles) coupled by the convective flow are in the origin of the turbulent state in high Prandtl number fluids [1]. Coupled non linear oscillators has been proposed to model dynamical states in convective systems [2]. More recently, bifurcations appearing in the synchronization state of quasiperiodic oscillations by an external driving periodic force has been

studied numerically [3]. In this work we extend the analysis from two coupled Van der Pol oscillators studied in [3], to a new dynamical system composed by two coupled Takens-Bogdanov equations under an external driving force. The influence of a complex signal in the synchronization of the dynamical system is analized through time-histogram instead the phase information of the system used in [3].

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[2] T. Vadivasova, G. Strelkova, V. Anishenko, PRE 63, (2001) 036225.
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$\longrightarrow \infty \diamond \infty \longleftarrow$

The Complex World of the Aeronautical Transportation System

Massimiliano Zanin
Innaxis Foundation & Research Institute, Spain

The Air Transportation System is an extremely complex, yet critical, infrastructure of our societies. It is composed of a plethora of different elements that interact in a non-linear fashion; giving birth to unexpected emergent behaviours and affecting the quality of service experienced by users. In this context, the tools of Complexity Science, and especially those of Complex Networks, can be of utmost importance when trying to model and understand the aeronautical system; nevertheless, most of these tools have to be adapted to this specific problem, in order to obtain significant and realistic results. In this talk we will review some applications of Complex Networks where the evolution of internal dynamics (passenger's movements) is entangled with the external structure (topology of the network); all of this, under the presence of time restrictions imposed by a scheduling, and of external unexpected events – which add uncertainty to the system. Those models can be used to understand the present and future behaviour of the Air Transportation Network, thus allowing more efficient policies definition that would be of advantage to all users of the system.

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Recurrence Networks for Nonlinear Time Series Analysis

Yong Zou
Potsdam Inst. for Climate Impact Res., Germany
(Reik V. Donner, Jonathan F. Donges, Norbert Marwan, Juergen Kurths)

We present a new approach for analysing structural properties of time series from complex systems. Starting from the concept of recurrences in phase space, the recurrence matrix of a time series is interpreted as the adjacency matrix of an associated complex network which links different points in time if the considered states are closely neighboured in phase space. In comparison with similar network-based techniques, the new approach has important conceptual advantages and can be considered as a unifying framework for transforming time series into complex networks that also includes other existing methods as special cases. It is demonstrated that there are fundamental relationships between many topological properties of recurrence networks and different non-trivial statistical properties of the phase space density of the underlying dynamical system. Hence, this novel interpretation of the recurrence matrix yields new quantitative characteristics (such as average path length, clustering coefficient, or centrality measures of the recurrence network) related with the dynamical complexity of a time series, most of which are not yet provided by other existing methods of nonlinear time series analysis. We demonstrate that nonlinear measures based on recurrence plots obtained from such trajectories provide a practicable alternative for numerically detecting shrimps. Traditional diagonal line-based measures of recurrence quantification analysis (RQA) as well as measures from complex network theory are shown to allow an excellent classification of periodic and chaotic behavior in parameter space. Average path length and clustering coefficient of the resulting recurrence networks (RNs) are found to be particularly powerful discriminatory statistics for the identification of shrimps in the Roessler system. Finally we apply the proposed approach to a marine palaeo-climate record and are able to identify subtle changes of the climate regime.

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Special Session 22: Spatially Structured Oscillations and Waves in Neural Media

Stephen Coombes, University of Nottingham, UK
Paul Bressloff, University of Oxford, UK

Introduction: Analysis of the dynamical mechanisms underlying spatially structured activity states in neural tissue is crucially important for understanding a wide range of neurobiological phenomena, both naturally occurring and pathological. For example, neurological disorders such as epilepsy and migraine are characterized by waves propagating across the surface of the brain. Spatially coherent activity states are also prevalent during the normal healthy functioning of the brain, encoding local properties of visual and auditory stimuli, encoding head direction and spatial location, and maintaining persistent activity states in short-term working memory. In this minisymposium the existence and stability of coherent activity states in non-local (integro-differential) neural models and their subsequent spreading will be analyzed, and the dependence on various biologically relevant parameters determined.

On the Local and Global Dynamics of Neural Field Models with Time Delays

Fatihcan Atay
Max Planck Inst. for Math. in the Sci., Germany

We consider integro-differential field models of neural activity that take into account the various sources of time delays present in the system, including both fixed and distance-dependent delays as well as delay dstributions. We give a local stability analysis and obtain oscillatory patterns and waves as bifurcations from equilibria that arise as some system or input parameter is changed. Relations are indicated to the so-called chimera states, where synchronous and incoherent behaviour co-exist in the same system at different spatial locations.

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Connectivity, Activity Propagation and Patterns in Neural Mass Models

Ingo Bojak
Radboud University Nijmegen, Netherlands

One way to deal with the complexity of cortex is to investigate mesoscopic neural units instead of individual neurons. Then the ubiquitous cortical connectivity must be modelled accordingly. After an introduction from the perspective of non-invasive neuroimaging, we will discuss here two complementary approaches. First, one can approximate cortex as a continuous neural medium within which a pulse of activity can spread. Usually a single conduction velocity is assumed, but experimentally axons have a broad dstribution of speeds. We present here continuum propagators matching data from human and rat, respectively. We show that rat velocity dstributions do not scale to human ones, and that our novel propagators allow emergent pattern formation for smaller changes in firing rates. Second, we discuss a model tessellating human cortex with 32 408 triangles. Each triangle vertex then represents roughly 11 mm^2 of cortical tissue. Activity propagation is handled by delay-buffering all inputs at each vertex. While conceptually simple, this is technically involved due to the great number of connections (ca. 30 million) and the nature of signals (continuous rates). However, this method is very flexible, and we demonstrate this by simulating the changes in EEG and fMRI BOLD patterns upon switching on effective visual connectivity.

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Neural Fields with Sigmoidal Firing Rates: Approximate Solutions

Stephen Coombes
University of Nottingham, England
(H- Schmidt)

Many tissue level models of neural networks are written in the language of nonlinear integro-differential equations. Analytical solutions have only been obtained for the special case that the nonlinearity is a Heaviside function. Thus the pursuit of even approximate solutions to such models is of interest to the broad mathematical neuroscience community. Here we develop one such scheme, for stationary and travelling wave solutions, that can deal with a certain class of smoothed Heaviside functions. The dstribution that smoothes the Heaviside is viewed as a fundamental object, and all expressions describing the scheme are constructed in terms of integrals over this dstribution. The comparison of our scheme and results from direct numerical simulations is used to highlight the very good levels of approximation that can be achieved by iterating the process only a small number of times.

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Neural Masses and Neural Fields for Texture Perception

Olivier Faugeras
INRIA Sophia-Antipolis/ENS Paris, France
(Pascal Chossat and Gregory Faye)

We propose to use bifurcation theory and pattern formation as theoretical probes for various hypotheses about the neural organization of the brain. This

allows us to make predictions about the kinds of patterns that should be observed in the activity of real brains through, e.g. optical imaging, and opens the door to the design of experiments to test these hypotheses. We study the specific problem of visual edges and textures perception and suggest that these features may be represented at the population level in the visual cortex as a special second-order tensor, the structure tensor, perhaps within a hypercolumn. We then extend the classical ring model to this case and show that its natural framework is the non-Euclidean hyperbolic geometry. This brings in the beautiful structure of its group of isometries and certain of its subgroups which have a direct interpretation in terms of the organization of the neural populations that are assumed to encode the structure tensor. By studying the bifurcations of the solutions of the structure tensor equations, the analog of the classical Wilson and Cowan equations, under the assumption of invariance with respect to the action of these subgroups, we predict the appearance of characteristic patterns. These patterns can be described by what we call hyperbolic or H-planforms that are reminiscent of Euclidean planar waves and of the planforms that were used in previous work to account for some visual hallucinations. If these patterns could be observed through brain imaging techniques they would reveal the built-in or acquired invariance of the neural organization to the action of the corresponding subgroups.

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Symmetry and Time-Periodic Geometric Visual Hallucinations

Martin Golubitsky
MBI / Ohio State University, USA

Thirty years ago Ermentrout and Cowan observated that drug induced visual hallucinations could be modeled by spontaneous pattern formation in the primary visual cortex. They assumed that the drug uniformly excites the cortex (modeled as a planar patch of infinitesimal neurons with periodic boundary conditions) causing nonuniform patterns in the activity variable of cortical neurons through spontaneous symmetry breaking. Bressloff and Cowan improved on this model by using the orientation tuning of neurons in the visual cortex and the Hubel-Wiesel notion of hypercolumn to model the visual cortex as a patch of hypercolumns in $R^2 \times S^1$ rather than neurons in R^2. In addition, their model uses the fact that global connections of cortical neurons are anisotropic, which changes the symmetry in model equations and the patterns that form. In this talk I will describe joint work with LieJune Shiau and Andrew Torok that shows if the connections are weakly anisotropic then time-periodic solutions appear from a steady-state bifurcation. The associated time-periodic patterns can also be associated with (standard) geometric visual hallucinations.

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The Power Spectrum of a Neural Population During General Anaesthesia

Axel Hutt
INRIA Nancy, France

The neuronal mechanisms of general anesthesia are still poorly understood. Besides several characteristic features of anesthesia observed in experiments, a prominent effect is the bi-phasic change of power in the observed electroencephalogram (EEG). The work derives analytical conditions for this biphasic spectral behavior by the study of a spatially-extended neural population model. This model describes mathematically the effective membrane potential and involves excitatory and inhibitory synapses, excitatory and inhibitory cells, nonlocal spatial interactions and a finite axonal conduction speed. The power spectrum of Local Field Potentials and EEG generated by the neural activity is derived analytically and allow for the detailed study of bi-spectral power changes.

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Bumps and Chimeras

Carlo Laing
Massey University, New Zealand

In past few years several authors have studied “chimera” states in networks of coupled oscillators, in which some fraction of the oscillators are synchronised while the remainder are incoherent. Similar behaviour is seen in neural networks which support “bump” states, in which there is a spatially-localised group of active neurons surrounded by quiescent neurons. I will show one network for which chimera states can be explicitly constructed, and discuss the similarities and differences between chimera states and bump states.

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Interface Dynamics in 2D Neural Field Models

Helmut Schmidt
University of Nottingham, England
(Stephen Coombes)

Neural field models describe the coarse grained activity of populations of interacting neurons. Because of the laminar structure of real cortical tissue they are often studied in 2D, where they are well known to generate rich patterns of spatio-temporal activity. Such patterns have been interpreted in a

variety of contexts ranging from the understanding of visual hallucinations to the generation of EEG signals. Typical patterns include localised solutions in the form of travelling spots as well as intricate labyrinthine structures. These patterns are naturally defined by the interface between low and high states of neural activity. Here we derive the equations of motion for such interfaces and show that for a single population, with Mexican-hat connectivity and Heaviside firing rate, that the normal velocity of an interface is given in terms of a Biot-Savart interaction over the boundaries of the high activity regions. This exact, but dimensionally reduced, system of equations is solved numerically and shown to be in excellent agreement with the full nonlinear integral equation defining the neural field. Importantly we develop a linear stability analysis for the interface dynamics that allows us to understand the mechanisms of pattern formation that arise from instabilities of spots, fronts, and stripes.

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Oscillations in Pulse-Coupled Adaptive Exponential Integrate-and-Fire Neurons

Liejune Shiau
University of Houston, USA
(John Rinzel and Klaus Obermayer)

Various neuron models and their coupling behaviors have been studied extensively over the years. In this particular study, we investigate the adaptive exponential integrate-and-fire model (recently proposed by Brette and Gerstner 2005), because its parameters have a physiological interpretation and are of biophysical relevance to real recordings of pyramidal cells. We show that the stability of phase locked in two identical pulse-coupled adaptive exponential integrate-and-fire neurons can vary, depending on the type of bifurcation which periodic spiking is produced. Further more, we also show that the speed of post-synaptic input or the reset character may also play an important role in altering the stability in phase locked of such two pulse-coupled neurons.

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Detailed and Population Models for Epilepsy

Stephan van Gils
University of Twente, Netherlands
(Sid Visser and Hil Meijer)

We present analysis of two models developed to study neuronal activity during normal and pathological state. A detailed model describes a microcolumn of the neocortex. It contains many parameters, whose values are based on experimental data. Depending on global parameters, i.e. the total network excitation and inhibition, the model exhibits different types of behavior like saturated desynchronized activity, irregular bursts, fast oscillations and burst suppression. These different states are compared with the stable states in a delay differential population model for the activity of neuronal populations.

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The Weakly Nonlocal Limit of a One-Population Wilson-Cowan Model

John Wyller
Norwegian University of Life Sciences, Norway
(Anna Oleynik and Igor Wertgeim)

We investigate the weakly nonlocal limit of a one-population neuronal field model of the Wilson-Cowan type in one spatial dimension. By transforming this equation to an equation in the firing rate variable, it is shown that stationary periodic solutions exist by appealing to a pseudo-potential analysis. The solutions of the full nonlocal equation obey a uniform bound, and the stationary periodic solutions in the weakly nonlocal limit satisfying the same uniform bound are characterized by finite ranges of pseudo energy constants. The time dependent version of the model is reformulated as a Ginzburg-Landau-Khalatnikov type of equation in the firing rate variable where the maximum (minimum) points correspond stable (unstable) homogeneous solutions of the weakly nonlocal limit. We develop a numerical method for the Ginzburg-Landau-Khalatnikov formulation based on the wavelets-Galerkin approach. The method is illustrated by means of some testing examples where we detail the nonlinear evolution numerically.

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How Do Neurobiological Mechanisms Influence the Speeds of Traveling Waves in Mathematical Neurosciences?

Linghai Zhang
Lehigh University, USA
(Ping-Shi Wu and Melissa Anne Stoner)

We study the speeds of the traveling wave fronts of the following integral differential equation

$$u_t + f(u) =$$
$$(\alpha - \beta u)\int_{\mathbb{R}} K(x-y)H\Big(u\big(y, t - \frac{1}{c}|x-y|\big) - \vartheta\Big)\mathrm{d}y.$$

This model equation arises from synaptically coupled neuronal networks. In this equation, f is a smooth function of u, usually representing sodium

current in the network, K represents synaptic coupling between neurons, H stands for the Heaviside step function. The parameters c, α, β and ϑ are positive, each representing some biological mechanism.

We investigate the influence of neurobiological mechanisms on the speeds of traveling wave fronts. We will derive new estimates for the speeds.

We also perform various numerical simulations to investigate how c, α, β and ϑ influence the wave speeds.

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Special Session 23: Nonlinear Elliptic PDEs and Geometric Analysis

Wenxiong Chen, Yeshiva University, USA
Guofang Wang, Freiburg University, Germany

Introduction: We will address the recent advances on the theories and applications of nonlinear elliptic partial differential equations, integral equations, and nonlinear geometric analysis. This includes, but not limited to, the existence, symmetry, asymptotic behavior, and other qualitative properties of solutions. The aim of this special session is to bring together both experts and young researchers to exchange their ideas and developments in these areas.

Symmetry of Solutions for Nonlinear PDEs and Integral Systems

Wenxiong Chen
Yeshiva University, USA
(Congming Li)

We consider fully nonlinear integral systems involving Wolff potentials in R^n, and use the method of moving planes in integral forms to obtain the radial symmetry and monotonicity of positive solutions. In a special case, this system is associated with Hardy-Littlewood-Sobolev inequality, and the classification of solutions would provide the best constant in the inequality. We also prove that it is equivalent to a system of semilinear elliptic PDEs:

$$(-\Delta)^{\alpha/2} u = v^q \text{ and } (-\Delta)^{\alpha/2} v = u^p.$$

In particular, when $\alpha = 2$, it reduces to the well-known Lane-Emden system.

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Willmore Surfaces of Revolution Satisfying Natural Boundary Conditions

Anna Dall'Acqua
University Magdeburg, Germany
(Matthias Bergner, Steffen Fröhlich)

The Willmore functional is the integral of the square of the mean curvature over the unknown surface. We consider the minimisation problem among all surfaces which obey suitable boundary conditions. The Willmore equation as the corresponding Euler-Lagrange equation.

We study the Willmore boundary value problem for surfaces of revolution which satisfy the following boundary conditions: the height at the boundary is prescribed, and the second boundary condition is the natural one. Using direct methods of the calculus of variations, we prove existence and regularity of minimising solutions.

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Finite Morse Index Solutions of an Elliptic Equation with Supercritical Exponent

Yihong Du
University of New England, Australia
(E. N. Dancer and Zongming Guo)

We study the behavior of finite Morse index solutions of the equation

$$-\Delta u = |x|^\alpha |u|^{p-1} u \text{ in } \Omega \subset \mathbb{R}^N,$$

where $p > 1$, $\alpha > -2$, and Ω is a bounded or unbounded domain. We show that there is a critical power $p = \overline{p}(\alpha)$ larger than the usual critical exponent $\frac{N+2}{N-2}$ which plays a crucial role for the behavior of such solutions.

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Normal Coulomb Frames for Surfaces in Euclidean Spaces

Steffen Froehlich
Free University of Berlin, Germany
(Frank Mueller)

We construct so-called Coulomb frames in the normal bundle of surfaces in Euclidean spaces, discuss their regularity properties and possible applications. The method of Coulomb frames provides a useful tool in differential geometry and harmonic analysis.

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Density of Smooth Maps in $W^{k,p}(M,N)$ for a Close to Critical Domain Dimension

Andreas Gastel
Universität Erlangen-Nürnberg, Germany
(Andreas J. Nerf)

Assuming $m-1 < kp < m$, we prove that the space $C^\infty(M,N)$ of smooth mappings between compact Riemannian manifolds M,N is dense in the Sobolev space $W^{k,p}(M,N)$ if and only if $\pi_{m-1}(N)=\{0\}$. If $\pi_{m-1}(N)\neq\{0\}$, then every mapping in $W^{k,p}(M,N)$ can still be approximated by mappings $M\to N$ which are smooth except in finitely many points.

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Dirac Harmonic Maps and Boundaries

Jürgen Jost
Max Planck Inst. for Math. in the Sci., Germany

Reporting about joint work with Qun Chen, Guofang Wang, and Miaomiao Zhu, I'll discuss the boundary value problem (called D-branes in the physics literature) for the mathematical version of the nonlinear supersymmetric sigma model of quantum field theory, the so-called Dirac harmonic maps. Such maps couple a map from a Riemann surface into a Riemannian manifold N with a spinor field with values in the pullback of the tangent bundle of N. I'll describe the underlying rich geometric structure and the analytic difficulties and how to overcome them.

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Null Quadrature Domains and a Free Boundary Problem for the Laplacian

Lavi Karp
ORT Braude College, Israel
(Avmir S. Margulis)

Null quadrature domains are unbounded domains in $\mathbb{R}^n$ $(n\geq 2)$ with external gravitational force zero in some generalized sense. The known examples of null quadrature domains are half-spaces, exterior of ellipsoids, of elliptic paraboloids and of strips, and cylinders over domains of these types. The characterization of these domains in $\mathbb{R}^n$ for $n\geq 3$ has been an open problem for almost three decades.

An essential difficulty in the classification is the establishment of a quadratic growth of a global solution to a free boundary problem. The quadratic growth of solutions to free boundaries problems without sign restriction is the means to obtain the limitation of the second order derivatives. Therefore it has an essential role in the studies of the regularity of the free boundaries. A similar problem occurs in the local version of these types of problems; in that case the quadratic growth in this case was proved in [1]. Recently we proved the quadratic growth in the global case and by using theorem of Caffarelli, Karp and Shahgolian we conclude that any null quadrature domain is the complement of a convex set with analytic boundary. The talk will also discuss the relations to classical problems in potential theory.

[1] L. A. Caffarelli, L. Karp and H. Shahgolian, *Regularity of a free boundary problem with application to the Pompeiu problem*, Ann. Math. 151, (2000), 269-292.

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Nodal Properties of Radial Solutions of Polyharmonic Boundary-Value Problems

Monica Lazzo
University of Bari, Italy
(Paul G. Schmidt)

We present some general results regarding the nature and dstribution of the zeros of regular radial solutions of semilinear polyharmonic equations under Dirichlet or Navier boundary conditions and discuss consequences regarding the existence or nonexistence of such solutions.

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On a Nonlocal Elliptic Problem Arising in the Confinement of a Plasma in a Current Carrying Stellarator

Fengquan Li
Dalian University of Technology, PR China
(Weilin Zou)

This paper deals with a nonlocal free boundary problem arising in the study of the dynamics of the confinement of a plasma in a Stellarator device. The free boundary represents the separation between the plasma and vacuum regions, and the nonlocal term involves the notions of relative rearrangement and monotone rearrangement. We will be the first to

establish some new properties on the decreasing rearrangement which can be used to prove the convergence of the approximate problem, and then we prove the existence of solutions by Galerkin method.

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Constant Q-Curvature Conformal Metrics: The Critical Case

Mohameden Ould Ahmedou
Giessen University, Germany
(Cheikh B. Ndiaye)

We study the problem of existence of conformal metrics with constant Q-curvature on a given compact connected four-dimensional Riemannian manifold (M, g) without boundary. Using a new topological argument, we solve the problem in some cases left by the remarkable works of Chang-Yang and Djadli-Malchiodi and which were still open.

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Spectral Radius, Index Estimates for Schrödinger Operators and Geometric Applications

Marco Rigoli
Università degli Studi di Milano, Italy

We provide a sharp estimate for the first eigenvalue of the Laplace-Beltrami operator on the exterior of geodesic balls on a complete manifold M. This estimate extends to the case of a superexponential volume growth for the the geodesic balls of M and is based on a carefull analysis of the distance between two consecutive zeros of an oscillatory solution of a linear equation related to the geometry of M. Further geometric applications are also given.

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Blow-Up Solutions for Critical Trudinger-Moser Equations in R^2

Bernhard Ruf
Università di Milano, Italy
(Manuel del Pino, Monica Musso)

We consider elliptic equations in R^2 with exponential nonlinearities which have critical Trudinger-Moser growth. We give conditions for the existence of solutions which blow up in exactly k points when a parameter tends to zero. We show that this conditions holds always for $k = 1$, and so there exists always a sequence of solutions with a single point blow-up. If the domain is not simply connected, then the condition is verified for $k = 2$, and thus for such domains there exists always a sequence of solutions blowing up in exactly two points. Furthermore, information on the location of the blow-up points is given.

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Is the Trudinger-Moser Nonlinearity a True Critical Nonlinearity?

Kyril Tintarev
Uppsala University, Sweden
(Adimurthi)

The counterpart of the Sobolev inequality in the two-dimensional case is the Trudinger-Moser inequality

$$\sup_{u\in H_0^1(\Omega),\|\nabla u\|_2=1} \int e^{4\pi u^2} dx < \infty.$$

When Ω is a unit disk, a stronger inequality

$$\sup_{u\in H_0^1(\Omega),\|\nabla u\|_2=1} \int \frac{e^{4\pi u^2}-1}{(1-|x|^2)^2} dx < \infty$$

can be derived by requiring “translation invariance. The relevant translations are Möbius transformations, which, if Ω is regarded as the Poincaré model of the hyperbolic space $\mathbb{H}^2$, are isometries on $\mathbb{H}^2$. Relevant non-compact mininization problems are discussed.

A similar requirement of dilation invariance, implemented for the subspace of radially symmetric functions, yields yet another improved version of Trudinger-Moser inequality. The relevant blowups are nonlinear, $u(r) \mapsto s^{-1/2}u(r^s)$, $s > 0$. As a consequence we prove existence of minimizers for the two-dimensional dilation-invariant counterpart of the Hardy-Sobolev inequality.

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A Nonlocal Model for Image Processing

Guofang Wang
Freiburg University, Germany
(Y. Jin, J. Jost)

In this talk, we will talk about a nonlocal model for denosing in image processing.

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A Classification of the Wulff Shape by an Overdetermined Anisotropic PDE

Chao Xia
Albert-Ludwigs-Universität Freiburg, Germany
(Guofang Wang)

We consider an anisotropic elliptic PDE in a

bounded domain with overdetermined boundary value. We prove that the Wulff shape is the only domain on which the overdetermined problem admits a C^2 solution. It's an anisotropic version of a classic result of Serrin. We use a geometric approach to show that the anisotropic mean curvature of the level sets are all constant. The proof combines a maximum principle of so-called P-function and a Pohozaev-type inequality.

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Regularity of Harmonic Maps into Spheres and Applications to Bernstein Problems

Ling Yang
Max Planck Inst. for Math. in the Sci., Germany

We show the regularity of weakly harmonic maps from a Riemannian manifold into a Euclidean sphere under the assumption that the image avoids a neighborhood of a half-equator. The proofs combine constructions of strictly convex functions and the regularity theory of quasilinear elliptic systems. Applying these results to the spherical and Euclidean Bernstein problems for minimal hypersurfaces, we obtain new conditions under which complete minimal hypersurfaces are trivial.

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Compactness for Dirac-Harmonic Maps from Degenerating Spin Surfaces

Miaomiao Zhu
ETH Zürich, Switzerland

We shall consider sequences of Dirac-harmonic maps from degenerating spin surfaces and show some compactness results.

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Special Session 24: Nonlinear Systems of Mixed Type and Applications

Zhaosheng Feng, University of Texas-Pan American, USA
Claire David, University Piere and Marie Curie, Paris, France
Tingwen Huang, Texas A&M University, Qatar

Introduction: This session will be devoted to the study of mixed type of nonlinear dynamical systems arising in mechanics and biological phenomena. Equally emphasized are real-world applications in terms of modelling and computations.

Finite Time Extinction Property in Some PDE's with Dynamic Boundary Conditions

Alfonso Casal
Univ. Politecnica de Madrid, Spain
(Jesus I. Diaz, Jose M. Vegas)

We study the "finite extinction phenomenon" (there exists $t_0 \geq 0$ such that $u(t,x) = 0$ for all $t \geq t_0$, and a.e. $x \in \Omega$) in some delayed boundary conditions, and the implications of the behavior of these for the solutions of some partial differential equations (defined by operators ranging from very simple to very abstract ones), to which they are associated. If Ω is an open and bounded set in R^N, $b \geq 0$, f is a continuous increasing function, the $u(t,x)$ satisfies some PDE, a Neumann boundary condition $\frac{\partial u}{\partial t} - \frac{\partial u}{\partial n} + b(t)f(u(t-t,x)) = 0$ on $(0,+\infty) \times \Omega$ and some functional initial condition $u(s,x) = u_0(s,x)$ on $(-\tau,0) \times \Omega$ (for some given function $u_0 \in C([-\tau,0] : L^p(\Omega))$ and for some $p \in [1,+\infty])$. The term $b(t)f(u(t-t,x))$ can be understood as a delayed feedback control defined on the boundary.

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Pinning Control of Synchronization Over Complex Dynamical Networks

Guanrong Chen
City University, Hong Kong

Pinning control of synchronization over complex dynamical networks is addressed. Notion of synchronization and some criteria are firstly introduced for various dynamical networks, such as fully-connected, ring-shaped and star-shaped regular networks, as well as small-world and scale-free complex networks. Notion of pinning-controlled synchronization is then discussed, for networks that do not satisfy self-synchronized conditions. Some commonly concerned questions are also discussed, such as what

kind of controllers to deploy, how many to use, and where to apply them on a given network, so as to achieve effective network synchronization. A simple state-feedback pinning control method is finally introduced, supported by numerical simulation examples.

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Delay-Derivative-Dependent Stability Criteria for Systems with Both Time-Varying and Distributed Delay

Shumin Fei
Southeast University, Peoples Rep. of China
(ShuminFei, Tao Li and Aiguo Song)

In recent years, many important results on the linear systems with time-delay have appeared, but there still exist some points on stability for delayed system waiting for the further improvements, for instance: though the delay-partitioning ideas have been employed to tackle dynamics on delayed systems, but they cannot efficiently deal with time-variant delay, especially the variable interval one. Secondly, based on most present results, they have not taken the lower bound of delay's derivative into consideration. In fact, if the delay's derivative belongs to one interval, considering both the upper bound and the lower one will do great beneficence in reducing the conservatism of derived results. Thirdly, though the distributed delay has received much attention, the interval distributed delay has not been fully considered in present literature especially as that its lower bound is not necessarily restricted to be zero. In this talk, we have fully considered the points mentioned above and tackle them in an efficient way. Though choosing an improved Lyapunov-Krasovskii functional and utilizing generalized convex combination technique, the less conservative condition is presented in terms of LMIs and the feasibility can be easily checked by resorting to LMI in Matlab Toolbox. Finally, some comparing results are given to illustrate the superiorities of the derived results based on two numerical examples.

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Traveling Waves and Their Stability for a Public Goods Game Model

Wei Feng
UNC-Wilmington, USA
(Xiaojie Hou)

We study the traveling wave solutions to a reaction diffusion system modeling the public goods game with altruistic behaviors. The existence of the waves is derived through monotone iteration of a pair of classical upper- and lower solutions. The waves are shown to be unique and strictly monotonic. A similar KPP wave like asymptotic behaviors are obtained by comparison principle and exponential dichotomy. The stability of the traveling waves with non-critical speed is investigate by spectral analysis in the weighted Banach spaces.

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Dynamics of Switching Systems

Tingwen Huang
Texas A&M University at Qatar, Qatar
(Zhigang Zeng)

The motivation for studying switching systems comes from the fact that many practical systems are inherently multimodal in the sense that several dynamic systems are required to describe their behaviors, depending on various environmental factors, and from the fact that the methods of intelligent control design are based on the idea of switching between different controllers. Some stability properties of switching systems composed of a family of linear time-invariant configurations are studied. In addition, the obtained results are applied to the perturbed switching systems where nonlinear norm-bounded perturbations exist in the linear time-invariant configurations.

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Non-Linear Flows in the Porous Media: Structural Stability and Long Term Dynamics

Akif Ibragimov
Texas Tech. University, Lubbock, USA
(E. Aulisa, L. Bloshanskaya, L. Hoang)

We study the generalized Forchheimer equations for slightly compressible fluids in porous media. The structural stability is established with respect to either the boundary data or the coefficients of the Forchheimer polynomials. A weighted Poincare-Sobolev inequality with the weight related to the non-linearity of the equation is used to study the asymptotic behavior of the solutions. Also, we prove a perturbed monotonicity property with the linear correction with respect to the coefficients of the Forchheimer polynomials.

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Equations of the Camassa–Holm Hierarchy

Rossen Ivanov
Imperial College London, England

The squared eigenfunctions of the spectral problem associated with the Camassa–Holm (CH) equation represent a complete basis of functions, which

helps to describe the inverse scattering transform for the CH hierarchy as a generalized Fourier transform (GFT). All the fundamental properties of the CH equation, such as the integrals of motion, the description of the equations of the whole hierarchy, and their Hamiltonian structures, can be naturally expressed using the completeness relation and the recursion operator, whose eigenfunctions are the squared solutions. Using the GFT, we explicitly describe some members of the CH hierarchy, including integrable deformations for the CH equation. We also show that solutions of some (1+2) -dimensional members of the CH hierarchy can be constructed using results for the inverse scattering transform for the CH equation. We give an example of the peakon solution of one such equation.

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Data Mining Technique Based on Fuzzy Clustering

Samia Jones
Texas A&M University, Qatar

Data mining tools predict future trends and behaviors, allowing researchers and organizations to make proactive, knowledge-driven decisions. It predicts the future and the hidden information in the database. The automated, prospective analyses offered by data mining move beyond the analyses of past events provided by retrospective tools typical of decision support systems and statistics where you get analysis of what you have now. Data mining tools can answer business questions that traditionally were very time consuming to resolve and experts may miss because it lies outside their expectations. The most commonly techniques used in data mining are artificial neural networks, decision trees, and clustering which used in this project.

In this talk, the author presents a data mining method based on fuzzy clustering. The method provides a strategic decision base to be used in academia for learning purposes. The discovery of hidden knowledge is achieved through unsupervised learning case studies in Qatar to provide scientific answers using data mining tools.

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Approximated Solution of Ginzburg-Landau Equation Induced from Nearly Bichromatic Wave

Shuya Kanagawa
Tokyo City University, Japan
(Kiyoyuki Tchizawa, Takashi Nitta)

We consider an envelope function for the multi-dimensional nealy bichromatic wave $u_b(x,t)$ defined by a Fourier transformation and show that it satisfies a kind of Ginzburg-Landau equation under some conditions for the spectrum function $S(k)$ and the angular function $\omega(k)$. Furthermore we extend our result and estimate the distance between the envelope function and the solution of the Ginzburg-Landau equation under some mild conditions for $S(k)$ and $\omega(k)$.

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A Particle Method and Numerical Study of a Quasilinear Partial Differential Equation

Long Lee
University of Wyoming, USA
(R. Camassa, P.-H. Chiu, and T. W. H. Sheu)

We present a particle method for studying a quasilinear partial differential equation (PDE) proposed for the regularization of the Hopf (inviscid Burger) equation via nonlinear dispersion-like terms. These are obtained in an advection equation by coupling the advecting field to the advected one through a Helmholtz operator. Solutions of this PDE are "regularized" in the sense that the additional terms generated by the coupling prevent solution multivaluedness from occurring. We propose a particle algorithm to solve the quasilinear PDE. "Particles" in this algorithm travel along characteristic curves of the equation, and their positions and momenta determine the solution of the PDE. The algorithm follows the particle trajectories by integrating a pair of integro-differential equations that govern the evolution of particle positions and momenta. We illustrate the relation between dynamics of the momentum-like characteristic variable and the solution behavior of the PDE.

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Spatiotemporal Multiple Coherence Resonances and Calcium Waves in a Coupled Hepatocyte System

Qishao Lu
Beihang Univ., Peoples Rep. of China
(Baohua Wang)

In this paper Gaussian white noise is applied to investigate the spatiotemporal dynamical characters of coupled excitable hepatocytes. It is obtained that bi-resonances in hepatocytes system are induced by the competition of coupling between cells and noise. Furthermore, the intercellular annular calcium waves induced by noise are shown and the wave length is decreased with augmenting the noise intensity but increased monotonously with the coupling strength. For a fixed noise level, there is an optimal coupling strength that makes the coherence

resonance maximal.

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Duffing-van der Pol-Type Oscillator

Qingguo Meng
Tianjin University, Peoples Rep. of China
(Zhaosheng Feng)

In this paper, under certain parametric conditions, we are concerned with the first integrals of the Duffing-van der Pol-type oscillator equations which include the van der Pol and the Duffing oscillators etc. as particular cases. After making a series of variable transformations, we apply the Preller-Singer method as well as the Lie symmetry reduction method to find the first integrals of the simplified equations without complicated calculations. A class of nontrivial bounded solutions and their properties are discussed.

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Borel Divergent-Series Resummation Technique and Application to Numerical Simulation of Evolution Problems

Dina Razafindralandy
LEPTIAB, Universite de La Rochelle, France
(Aziz Hamdouni)

The perturbation method has proved to be an efficient tool for the numerical resolution of non-linear problems. However, it is not suitable for singular problems, of which the series solution is divergent. We propose to combine the perturbation method to the Borel series resummation technique for such a problem. The resulting algorithm is adapted to a numerical use. Applications to reduced models of the Navier-Stokes equations and to the heat equation are presented.

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The Best Constant in an Anisotropic Sobolev Inequality and Related Results

Eugenio Rocha
University of Aveiro, Portugal
(Jianqing Chen)

In this talk, we present a method for finding the best (sharpest) constant in some classes of integral inequalities with applications to PDEs. In particular, we study the best constant α in the anisotropic Sobolev inequality

$$\|u\|_p^p \leq \alpha \, \|u\|_2^{\frac{2(2N-1)+(3-2N)p}{2}} \, \|u_x\|_2^{\frac{N(p-2)}{2}} \prod_{k=1}^{N-1} \|D_x^{-1}\partial_{y_k} u\|_2^{\frac{p-2}{2}}$$

and the best constant β in the inequality

$$\|u\|_{p_*}^{p_*} \leq \beta \|u_x\|_2^{\frac{2N}{2N-3}} \prod_{k=1}^{N-1} \|D_x^{-1}\partial_{y_k} u\|_2^{\frac{2}{2N-3}},$$

where $V := (x, y_1, \cdots, y_{N-1}) \in \mathbb{R}^N$ with $N \geq 3$ and $2 < p < p_* = \frac{2(2N-1)}{2N-3}$. The method introduced seems to have independent interest. In fact, we use it to find the best constant of the Gagliardo-Nirenberg interpolation inequality involving the $m-$Laplacian and we briefly mention the best constant of other kinds of integral inequalities.

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Using Mathematical Modeling to Assess the Efficacy of Oxygen for Problem Wounds: Use of Hyperbaric or Topical Oxygen Therapies

Richard Schugart
Western Kentucky University, USA
(Jennifer Flegg, D. L. S. McElwain)

We extend a previously developed mathematical model (Schugart, R. C., Friedman, a., Zhao, R., Sen, C. K., Wound angiogenesis as a function of tissue oxygen tension: a mathematical model, PNAS USA 105: 2628 - 33, 2008) for acute wound healing to investigate the application of hyperbaric and topical oxygen therapies to treat acute, delayed, and chronic wounds. In this talk, I will present the model, a sensitivity analysis of the model, and simulation results for treating the wound with hyperbaric and topical oxygen therapies.

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Generalization of Superradiance Integral Equation and Applications

Indranil Sen Gupta
Texas A&M University, USA

Collective radiation phenomena has been an interesting subject since the pioneering work of R. H. Dicke in 1954. In that classic paper, Dicke considered two types of collective radiation phenomena: superradiance and subradiance in a collection of two-level atoms when all atoms are confined inside a volume much smaller than radiation wavelength. This problem is reduced to finding all the eigenfunctions of an integral equation. In our work we will consider a class of problems which are generalized versions of the three-dimensional superradiance integral equation. A commuting differential operator will be found for this generalized problem. For the three dimensional superradiance problem an alternative set of complete eigenfunctions will also be provided. The kernel for the superradiance problem when restricted to one-dimension is the same

as appeared in the works of Slepian, Landau and Pollak. The uniqueness of the differential operator commuting with that kernel is indicated.

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Stabilization of Uniform Solutions by Distributed Time-Delay Feedback in a Reaction-Diffusion Equation

Michael Stich
Centro de Astrobiologia (CSIC-INTA), Spain
(Carsten Beta)

We consider the complex Ginzburg-Landau equation complemented by a combined global and local time-delay feedback. Feedback terms are implemented as control scheme, i.e., they are proportional to the difference between the time-delayed state of the system and its current state. Besides the time delay, relevant parameters include the feedback strength, the phase shift between the feedback and the unperturbed dynamics, and the magnitudes of the the global and local feedback. Considering the complex Ginzburg-Landau equation in regime of spatiotemporal chaos, we perform a linear stability analysis of uniform oscillations with respect to space-dependent perturbations as function of the system parameters. Similarly, for the fixed point solution that corresponds to amplitude death in the spatially extended system, a linear stability analysis with respect to space-dependent perturbations is performed. We find a very good agreement between the stability analysis and the numerical simulations for the uniform oscillations and the steady state solution. Implications of the findings for the control of chaos in reaction-diffusion systems are discussed.

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A Mathematical Model for the Clt Effect on the Drug Resistance During Antiretroviral Treatment of HIV Infection

Nicoleta Tarfulea
Purdue University Calumet, USA

In treating HIV infection, strict adherence to drug therapy is crucial in maintaining a low viral load, but the high dosages required for this often have toxic side effects which make perfect adherence to Antiretroviral Therapy (ART) unsustainable. The imperfect patient adherence to ART and the development of resistant strains in the viral load has led to the development of alternative treatments that incorporate immunological response. In this talk we present the impact of immune effectors, such as the cytotoxic lymphocyte (CTL), in modeling HIV pathogenesis during primary infection. Additionally, by introducing drug therapy, we assess the effect of treatment consisting of a combination of several antiretroviral drugs. Nevertheless, even in the presence of drug therapy, ongoing viral replication can lead to the emergence of drug-resistant virus variances. Thus, by including two viral strains, wild-type and drug-resistant, we show that the inclusion of the CTL compartment produces a higher rebound for an individual's healthy helper T-cell compartment than does drug therapy alone and we characterize successful drugs or drug combination scenarios for both strains of virus.

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Mathematical Analysis of a System of PDE Arising from Epidermal Wound Healing

Haiyan Wang
Arizona State University, USA

Systems of reaction-diffusion equations have been proposed to study cell migration across the surface of an epidermal wound. Approximation of traveling waves which describe the cell migration into the wound has been studied by a number of researchers. We will investigate asymptotic behavior and speed of the travel wave solutions. Analysis of the model system reveals biological interactions in the wound healing process and the role of the model parameters in determining the speed.

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On Solving Nonlinear Eigen-Solution Problems

Jianxin Zhou
Texas A&M University, USA
(Changchun Wang)

Many problems in the study of solution pattern, (in)stability analysis and other properties lead to solve a nonlinear eigen-solution problem. Such problems are very difficult to solve due to their nonlinearity, multiplicity and non variational nature. Starting with a Schrödinger type nonlinear eigen-solution problem, the speaker will address several types of nonlinear eigen-solution problems, discuss their significant difference from linear eigen-solution problems and propose their solution concepts and methods. Numerical examples will be shown to illustrate the methods.

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Special Session 25: Analysis, Optimization and Control of Nonlinear Partial Differential Equation Evolution Systems

George Avalos, University of Nebraska-Lincoln, USA
Lorena Bociu, University of Nebraska-Lincoln and CNRS-INLN, USA and France

Introduction: The goal of the Session is to bring together specialists who, owing to their respective fields of study, might not otherwise have occasion to meet and exchange ideas and points of view. To this end, the discussions will include the development of high-minded PDE techniques and tools for the following (ostensibly disparate) classes of problems: (i) Qualitative and quantitative properties enjoyed by solutions to nonlinear wave equations; e.g., global existence, uniqueness, finite time blow-up, and asymptotic behavior of solutions; (ii) the study of PDE dynamics under the influence of feedback or open loop control, with a view towards obtaining stabilization, controllability or precise information of asymptotic behavior, for said dynamics.

On Some Recent Trends on Nonlinear Stabilization of PDE's and ODEs with Applications to Semi-Discretization PDE's

Fatiha Alabau-Boussouira
Metz Univ. and INRIA t.p. CORIDA, France

We shall present in this talk, a general approach for nonlinear stabilization of differential equations, in finite and infinite dimensions. We will give optimality results in the finite dimensional case with applications to semi-discretized PDEs and discuss optimality questions in the infinite dimensional framework.

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Solvability of Elliptic Problems with Immersed Interfaces

Giles Auchmuty
University of Houston, USA
(P. Kloucek)

This talk will describe spectral methods for obtaining solutions of self-adjoint elliptic problems where Dirichlet conditions are prescribed on a closed curve or surface in the interior of the domain. This is similar to the models used in fictitious domain, or domain decomposition approaches to solving boundary value problems. That is the unknown function must be found on both sides of an interface - so the usual trace theorems are not valid. The class of allowable interior data for finite energy H^1 solutions is described using a spectral approach similar to earlier spectral characterizations of trace spaces by the author. The analysis allows quite different equations to hold on either side of the interface.

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Rational Decay Rates for a Partial Differential Equation Model of Fluid-Structure Interaction

George Avalos
University of Nebraska-Lincoln, USA

In this talk we shall derive certain delicate decay rates for a partial differential equation (PDE) system which comprises (parabolic) Stokes fluid flow and a (hyperbolic) elastic structural equation. The appearance of such coupled PDE models in the literature is well-established, inasmuch as they mathematically govern many physical phenomena; e.g., the immersion of an elastic structure within a fluid. The coupling between the distinct hyperbolic and parabolic dynamics occurs at the boundary interface between the media. In previous work, we have established semigroup wellposedness for such dynamics, in part through a nonstandard elimination of the associated pressure variable. For this PDE model, we provide a uniform rational decay estimate for solutions corresponding to smooth initial data; viz., for initial data in the domain of the semigroup generator. The attainment of this result depends upon the appropriate use of a recently derived operator semigroup result of A. Borichev and Y. Tomilov.

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Linearization of a Coupled System of Nonlinear Elasticity and Viscous Fluid

Lorena Bociu
INLN-CNRS, Sophia-Antipolis, France
(J.-P. Zolesio)

We are concerned with the coupled system formed by an incompressible fluid and a nonlinear elastic body. We work with large displacement, small deformation elasticity (St. Venant elasticity). The elastic body is three-dimensional $\Omega \in \mathbb{R}^3$, and thus it can not be reduced to its boundary Γ (like in the case of a membrane or a shell). First we study the static problem. In view of the stability analysis we derive the linearization of the system which turns out to be different from the usual coupling of classical linear modelings. New extra terms (for example those involving the boundary curvature) play an important role in the final linearized system around some equilibrium.

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On the LQ-Problem for Composite Systems of Evolutionary PDEs with Boundary Control

Francesca Bucci
Università degli Studi di Firenze, Italy

We report on the progress made on the development of the theory of the quadratic optimal control problem for abstract systems arising from the modeling of evolutionary partial differential equations (PDEs) with boundary/point control. Focus is specifically on dynamics which may possess a parabolic component, yet do not yield an overall 'parabolic character'. Relevant composite systems of PDEs which constitute a prime motivation for pursuing this study will be discussed.

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Multiplicative Controllability for Reaction-Diffusion Equations with Target States Admitting Finitely Many Changes of Sign

Piermarco Cannarsa
University of Rome "Tor Vergata", Italy
(A. Y. Khapalov)

We study the global approximate controllability properties of a one dimensional reaction-diffusion equation governed via the coefficient of the reaction term. The traditional (linear operator) controllability methods based on the duality pairing do not apply to such a problem. Instead, we focus on the qualitative study of the diffusion and reaction parts of the evolution process at hand. We consider the case when both the initial and target states admit no more than finitely many changes of sign.

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Uniform Decay Rate Estimates for the Wave Equation on Compact Surfaces and Locally Distributed Damping

Marcelo Cavalcanti
State University of Maringá, Brazil
(Valeria Domingos Cavalcanti, Ryuichi Fukuoka, Daniel Toudykov)

In this talk we present new contributions concerning uniform decay rates of the energy associated with the wave equation on compact surfaces subject to a dissipation locally distributed. We present a method that gives us a sharp result in what concerns of reducing arbitrarily the area where the dissipative effect lies.

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On Qualitative Aspects for the Damped Korteweg-de Vries and Airy Type Equations

Valéria Domingos Cavalcanti
State University of Maringá, Brazil
(Marcelo Moreira Cavalcanti, Andrei Faminkii, Fabio Matheus Amorin Natali)

The exponential decay rate of L^2-norm related to the Korteweg-de Vries equation with localized damping posed on the whole real line is established. In addition, we determine that the solutions associated to the fully damped Korteweg-de Vries equation decay in H^1-level for any arbitrary initial data.

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Partial Continuity for Parabolic Systems

Mikil Foss
University of Nebraska-Lincoln, USA
(Verena Bogelein; Giuseppe Mingione)

Consider the parabolic system

$$u_t - \operatorname{div}[a(x,t,u,Du)] = 0 \quad \text{in } \Omega_T := \Omega \times (-T,0),$$

where $\Omega \subset \mathbb{R}^n$ is a bounded domain and $T > 0$. The vector field $a : \Omega_T \times \mathbb{R}^N \times \mathbb{R}^{N\times n} \to \mathbb{R}^N$ satisfies natural p-growth and ellipticity assumptions with $p \geq 2$. I will present a partial continuity result for weak solutions to the above problem. More precisely, the result establishes that there is an open set of full measure in Ω_T in which the solution is Hölder continuous. The key assumption for the problem being considered is that the vector field a is continuous with respect to the arguments x, t and u. This distinguishes the result from others which provide regularity for the gradient of the solution while requiring at least Hölder continuity with respect to x, t and u. The work to be presented was done in collaboration with V. Bögelein (Friedrich-Alexander-Universität Erlangen-Nürnberg, Germany)and G. Mingione (University of Parma, Italy).

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Lefloch Solutions to Initial/Boundary Value Problems for Scalar Conservation Laws

Helene Frankowska
University Pierre and Marie Curie, France

We consider an initial/boundary value problem for scalar conservation laws (CL) with strictly convex flux f on the strip $(0,\infty) \times [0,1]$. It is well known that it does not have classical solutions even for smooth initial value problem and has a unique entropy weak solution. When additional boundary conditions are present, then it may happen that weak solutions satisfying boundary conditions

pointwise do not exist. To apply a boundary control to solutions of (CL) it is necessary to give meaning to the boundary conditions. We show that an associated Hamilton-Jacobi equation with correctly defined initial and boundary conditions has a unique generalized solution V and that its gradient V_x is a solution to (CL). Furthermore traces of gradients V_x satisfy generalized boundary conditions introduced by LeFloch (Explicit formula for scalar nonlinear conservation laws with boundary condition, Math. Methods Appl. Sci., 10, (1988), 265–287) pointwise when the initial and boundary data are continuous and in a weak sense when they are discontinuous. It is also shown that V_x is continuous a.e. and a result concerning traces of sign of $f'(V_x(t,\cdot))$ is derived.

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Cahn-Hilliard Equations with Inertial Term

Maurizio Grasselli
Politecnico di Milano, Italy
(Giulio Schimperna, Sergey Zelik)

In order to account for separation processes in certain binary solutions (e.g., glasses), some physicists have proposed to modify the classical Cahn-Hilliard equation. This new nonlinear evolution equation is characterized by the presence of a second-order time derivative of the order parameter multiplied by a (small) inertial coefficient. The additional term changes the nature of the equation and instantaneous regularization effects are lost in the nonviscous case. In one spatial dimension, this modified Cahn-Hilliard equation has been analyzed by several authors. However, in two or three dimensions, the situation looks much more complicated. For instance, in the latter case, uniqueness of solutions is still an open issue, unless the inertial parameter is small enough. I intend to present an overview of some recent results on the longtime behavior of solutions in dimensions two and three. This research has been carried out in collaboration with Sergey Zelik (University of Surrey, UK) and Giulio Schimperna (Universitá di Pavia, Italy).

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Stabilization of a Structural Acoustic Model with Interface a Reissner-Mindlin Plate with Second Sound

Marié Grobbelaar
University of the Witwatersrand, So Africa

We consider the question of strong stabilization of a three-dimensional structural acoustic model which incorporates displacement, rotational inertia, shear and thermal effects in the flat flexible structural component of the model, a two-dimensional Reissner-Mindlin plate. The thermal disturbances in the plate are described by Cattaneo's law instead of Fourier's law so that the model is deprived of the usual coupled parabolic heat equation and instead contains a hyperbolic system in ϑ and q, which variables describe respectively the termperature variations in the plate and the flux vector. Moreover the model exhibits weaker coupling between the thermal and elastic variables than in models using the Kirchoff or Bernoulli plate equations, in the sense that there is direct coupling between the thermal and the shear angle variables, but only indirect coupling between the thermal variable and the displacement variable.

We show strong stabilization of the model without incorporating viscous or boundary damping in the equations for the gas dyamics and without imposing geometric conditions. It turns out that damping in the interior of the plate which may be as weak as viscous damping, is required. Our main tool is an abstract resolvent criterion due to Y. Tomilov.

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Fiber Spinning of the Upper-Convected Maxwell Fluid

Thomas Hagen
The University of Memphis, USA

Fiber spinning of a viscoelastic liquid modeled by the constitutive theory of the upper-convected Maxwell fluid is analyzed. The governing equations are given by one-dimensional mass, momentum and constitutive equations which arise in the slender body approximation by cross-sectional averaging of the two-dimensional axisymmetric Stokes equations with free boundary. Existence, uniqueness and regularity results are proved by means of fixed point arguments and weak convergence methods.

The difficulty in this problem lies with the constitutive model of the Maxwell fluid: when both the inflow velocity at the spinneret and the pulling velocity at take up are prescribed, a boundary condition can be imposed for only one of the two elastic stress components at the inlet. The absence of the second stress boundary condition makes the mathematical analysis of the problem hard and presents serious challenges for the numerical resolution of the flow. (Joint work with D. Kurmashev)

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Analysis and Control of Flows in Gas-Structurei Interactions

Irena Lasiecka
University of Virginia, USA

Dynamics for a class of nonlinear hyperbolic systems modeling gas -flow interactions will be considered.

These are Euler equations (flow of gas) coupled at the interface with nonlinear plate (structure) equations. The coupling leads to non-conservative and non-dissipative models with supercritical sources. Both subsonic and supersonic flows will be considered

Existence of weak and strong solutions will be presented. In addition control of long time behavior will also be discussed. The analysis is based on compensated compactness, harmonic analysis tools along with long time behavior methods presented in I. Chueshov and I. Lasiecka, *Long-time behavior of second order evolution equations with nonlinear damping*, Memoirs of AMS, vol.195, no. 912, AMS, 2008.

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Control of Crack Growth in Brittle Materials with Inclusions

Günter Leugering
University of Erlangen-Nuremberg, Germany
(A. Khludnev (Novosibirsk))

We consider materials with elastic or rigid inclusions and holes. The material is supposed to contain an incipient crack. The mathematical description leads to a nonlinear system of PDEs with complementarity conditions or, equivalently, to a variational inequaltiy. We discuss the influence of boundary controls and, in particular, the influence of shape variations for the inclusions or the holes with respect to the crack sensitivity. The goal is to extremize the energy release rate of the crack. This is important in crashworthiness and other applications in material sciences. Thus, we propose a new kind of crack control and optimization.

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On Ingham Type Inequalities

Paola Loreti
Sapienza Università di Roma, Italy
(Vilmos Komornik)

In this talk we discuss results related to Ingham type inequalities and to an inequality due to Bernstein. As well known, Ingham type inequalities are related to observability and controllability problems and they are proved by non harmonic analysis techniques.

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Strong Solutions for Semilinear Wave Equations with Damping and Source Terms

Petronela Radu
University of Nebraska-Lincoln, USA

I will present a local existence result concerning strong solutions for semilinear wave equations with power-like interior damping and source terms. A long standing restriction imposed on the range of exponents allowed for the two nonlinearities is removed through a natural argument that uses the physics of the problem. The methods apply to the Cauchy problem as well as for initial boundary problems with homogeneous Dirichlet boundary conditions.

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Wave Equations with Supercritical Interior Sources and p-Laplacian Damping

Mohammad Rammaha
University of Nebraska-Lincoln, USA
(Zahava Welstein)

In this talk we consider a wave equation with supercritical interior sources and a strong damping induced by the p-Laplacian. Under some restrictions on the parameters, we prove the existence and uniqueness of a global solution. We also working towards obtaining the uniform decay rates of energy.

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Invariant Manifolds and Stability for the Stefan Problem with Surface Tension

Roland Schnaubelt
Karlsruhe Institute of Technology, Germany

We study quasilinear parabolic systems with fully nonlinear dynamical boundary conditions, which arise e.g. after a transformation from problems with moving boundaries such as the Stefan problem with Gibbs-Thompson correction. We concentrate on the qualitative behavior near an equilibrium. Depending on the spectrum of the linearization, one obtains local stable, center and unstable invariant manifolds. We discuss their properties and the connection to the stability of the equilibrium. The proofs use maximal regularity results for inhomogeneous initial-boundary value problems, techniques from dynamical systems and semigroup theory.

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Observability and Reachability for Evolution Equations with Memory

Daniela Sforza
Sapienza Università di Roma, Italy
(Paola Loreti)

In this talk we speak about some control results for integro-differential equations arising, for example, in viscoelasticity theory. By way of nonharmonic analysis techniques we show observability estimates

for the solution of the adjoint problem written as a Fourier series. The so-called Hilbert Uniqueness Method allows us to prove our reachability results.

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Min-Max of a Fluid-Structure Interaction with Control and Disturbance at the Interface

Roberto Triggiani
University of Virginia, USA

(I. Lasiecka and J. Zhang)

We consider a fluid-structure interaction model: a structure (modeled by the system of dynamic elasticity) immersed in a fluid (modeled by the linearized Navier-Stokes equation). Both control and disturbance are acting at the interface between the two media. We set up a min-max game theory problem and we solve it in terms of a Riccati Equation. Such system fails to satisfy a singular estimate between control and state, due to a mismatch between the parabolic and the hyperbolic component. Accordingly a marginally smoothing observation operator is needed to obtain a singular estimate between control and observation.

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Eulerian Image Metrics Involving Monotone Rearrangement

Jean-Paul Zolesio
CNRS & INRIA, France

We extend former results concerning morphic metrics built through tube analysis using non smooth vector fields such that topological changes are possible during the evolution in a connecting tube [1] [2]. This Eulerian approach is an extension of the Courant metric introduced in '73 by A. M. Micheletti and generalized in [3]. We extend to images by considering the elements $\beta_{[u]}(x) = \text{meas}(\{y \in D \,|\, u(y) < u(x)\})$ for any non negative $u \in L^1(D)$, and minimizing $J(\beta, V) = (\int_0^1 (|V|^2 + |\text{div}\, V|^2 + \sigma\, |\beta(t)|^2_{H^\varepsilon(D)}) dt)^{(1+\varepsilon)/2}$, $\varepsilon < 1/2$ over all connecting tubes $(\beta(t), V(t))$, where $V(t))$ is divergence free. In fact, to derive a metric, as in previous works, we change the 2nd term for a "time capacity term" which vanishes when $V = 0$. Again the geodesics are described by some Euler flow for which we get existence of variational solutions.

[1] J. P. Z. *Complete Shape Metric and Geodesic*, IFIP proc. '07 and in Springer Verlag, IFIP series, March '09; pp.155-179;
[2] J. P. Z. *Variational formulation for incompressible Euler Flow /shape-morphing metric and geodesic.* Contr. and Cybernetic, 38 ('09) No. 4;
[3] M. Delfour and J. P. Z. Shapes and geometries, Advances in Design and Control, SIAM PA, '01, 2^{nd} ed. '10.

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Special Session 26: Nonsmooth Dynamical Systems Analysis, Modeling and Numerical Techniques

Bernard Brogliato, INRIA, France
Joachim Gwinner, Universität der Bundeswehr Muenchen, Germany
Lionel Thibault, Université de Montpellier 2, France

Numerical Solution for Contact Problem with Nonmonotone Friction

Krzysztof Bartosz
Jagiellonian University, Kraków, Poland
(Piotr Kalita, Michael Barboteu)

The presentation deals with an elastic frictional contact problem. The friction law considered in the process is assumed to be nonmonotone, and therefore the problem is modelled by means of hemivariational Inequality (HVI). We provide an error estimate which is the appropriate version of Cea lemma. We also present the results of numerical simulations obtained by the approach based on the Finite Element Method and Proximal Bundle Method.

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Well-Posedness, Stability and Invariance Results for a Class of Multivalued Lur'e Dynamical Systems

Bernard Brogliato
INRIA, France
(D. Goeleven)

This work analyzes the existence and uniqueness issues in a class of multivalued Lur'e systems, where the multivalued part is represented as the subdifferential of some convex, proper, lower semicontinuous function. Through suitable transformations the system is recast into the framework of dynamic variational inequalities and the well-posedness (existence and uniqueness of solutions) is proved, relying either on Kato's theorem or on maximal monotonicity. Stability and invariance results are also obtained for the case of dissipative operators. The problem is motivated by practical applications in electrical circuits containing electronic devices with nonsmooth multivalued voltage/current characteristics (e.g. diode bridge rectifiers), and by state observer design for multivalued systems.

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Dynamical Systems with Rate Independent Hysteresis

Martin Brokate
TU Munich, Germany

A common occurrence of rate independent hysteresis as part of a dynamical systems arises when balance equations from mechanics are complemented by a hysteretic consitutive law. We present some results concerning the latter, considered as an evolution in its own right, as well as some connections to the analysis of flows in porous media.

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On the Interconnection of Linear Passive Systems and Monotone Characteristics

Kanat Camlibel
University of Groningen, Netherlands

We investigate existence and uniqueness of solutions of a certain class of nonsmooth nonlinear input/state/output dynamical. The class of systems that are of interest in this paper are obtained by interconnecting a linear dynamical system and a static nonlinear characteristics. We show that the classical results on differential inclusions with maximal monotone set-valued mappings can be utilized in case the underlying linear system is passive and the nonlinear characteristics are given by maximal monotone set-valued mappings. Apart from the classical results on differential inclusions with maximal monotone set-valued mappings, a recent characterization of maximal monotonicity and the theory of time-varying differential inclusions are the two main ingredients of the paper.

$\longrightarrow \infty \diamond \infty \longleftarrow$

A Class on Non-Convex Sets and Their Applications in Control Theory

Giovanni Colombo
Universitá di Padova, Italy

$\rho(\cdot)$-prox-regular sets (other names are sets with positive reach or $\varphi(\cdot)$-convex sets) are now well studied as a good generalization of both convex and $\mathcal{C}^{1,1}$-sets. Functions having $\rho(\cdot)$-prox-regular epigraph (or, symmetrically, hypograph) are good generalizations of convex/concave, $\mathcal{C}^{1,1}$, and semiconvex/concave functions, from which they inherit several regularity properties, including a.e. twice differentiability. It turns out, and the talk will be mainly devoted to this fact, that under suitable but rather weak controllability assumptions the minimum time function belongs to this class.

[1] G. Colombo, Nguyen T. Kh., On the structure of the minimum time function, submitted, 43 pp., October 2009.
[2] G. Colombo, A. Marigonda, P. Wolenski, Some new regularity properties for the minimal time function, preprint *Siam J. Control* 44 (2006), 2285-2299.

⟶ ∞ ⋄ ∞ ⟵

Network Problems, Dynamic Games and Hybrid Dynamical Systems

Scott Greenhalgh
University of Guelph, Canada
(Monica G Cojocaru)

We present a hybrid systems method for describing the time evolution of a class of network equilibrium problems and dynamic Nash games. Our method is based on an approach from hybrid dynamical systems and blends in with previous approaches for studying equilibrium problems, coming from optimization and variational inequalities. In particular, we present applications of our method to computation of solutions for dynamic games in vaccinating behaviour.

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Towards Efficient Solution of Nonsmooth Dynamical Systems

Joachim Gwinner
Universität der Bundeswehr Muenchen, Germany

It is well-known that smooth dynamical systems can be efficiently solved by adaptive discretization methods based on a posteriori error analysis. Here we combine adaptive time discretization with special methods to cope with nonsmoothness. Jumps, kinks, and other issues of nonsmoothness are considered as events that have to be detected in the dynamic process and have to be incorporated in the step size control. Such an event detection can be accomplished by an additive zero search of a suitable event function or by solving additionally another differential equation. Both approaches have been numerically tested in relatively small sized examples that come form applications in mechanics (frictional contact of a nonlinear spring) and electronics (rectifier circuit).

This talk is based on joint work with B. Ewald.

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On Dynamics and Bifurcations of 3D Piecewise Systems

Hany Hosham Bakit
University of Cologne, Germany
(Tassilo Küpper (University of Cologne) and Daniel Weiß (University of Tübingen))

The talk will focus on some recent results on invariant cones for a class of 3D piecewise systems. Our results are motivated by results obtained in [1-5]. Necessary and Sufficient conditions for the existence of periodic orbits are given in two main cases, namely, transversal intersection as well as attracting sliding modes will be considered. The stability of the whole system will be investigated. Numerical illustrations of periodic orbits with and without sliding mode on the boundary are given. Finally, we present a representative example to illustrate the question: How it is possible to reduce piecewise non-linear systems to a lower dimensional invariant manifold?

[1] V. Carmona, E. Freire, E. Ponce, F. Torres, Bifurcation of invariant cones in piecwise linear homogeneous system. Int. J. Bifur. Chaos 15 (8) (2005) 2469-2484.
[2] V. Carmona, E. Freire, J. Ros, F. Torres, Limit cycle bifurcation in 3d continuous piecewise linear systems with two zones. Application to Chua's circuit, Int. J. Bifur. Chaos 15(10)(2005)3153-33164.
[3] H. A. Hosham, T. Küpper and D. Weiß, Invariant manifold in nonsmooth systems, in preparation.
[4] T. Küpper, Invariant cones for non-smooth systems, Mathematics and Computers in Simulation, 79 (2008) 1396-1409.
[5] T. Küpper and H. A. Hosham, Reduction to Invariant Cones for Non-smooth Systems, It will appear in Special Issue of Mathematics and Computers in Simulation (2010).

⟶ ∞ ⋄ ∞ ⟵

Nonlinear Waves in Granular Chains

Guillaume James
Grenoble University and CNRS, France

We study the dynamics of a chain of identical spherical beads interacting via Hertz's contact law. Hertz contact force is unilateral and depends nonlinearly on the beads compression, with a vanishing stiffness constant when beads have a single contact point. At this point the contact force is differentiable with respect to bead compression, but it is not twice differentiable. These different features introduce additional mathematical difficulties which are not present in classical regular models of interacting particles, such as the Fermi-Pasta-Ulam lattice. However, for Hertzian contacts one can re-

cover this classical setting when the chain is pre-constrained and bead displacements are small. In this talk we first present a short review of the mathematical theory of nonlinear waves (solitons, periodic travelling or standing waves, breathers, ...) in the pre-constrained situation, focusing on local tools from bifurcation theory. In a second part we discuss the existence of travelling and standing waves for chains of beads without precompression.

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Method of Rothe for Parabolic Hemivariational Inequalities

Piotr Kalita
Jagiellonian University, Kraków, Poland

Hemivariational Inequalities (HVIs) are partial differential inclusions with multivalued term in the form of Clarke subdifferential of a locally Lipschitz functional. This work is devoted to evolutionary HVIs of parabolic type. We present the method of Rothe (known also as the time approximation method) for such problems. The method can be used as a tool to prove existence, and in some cases uniqueness, of solutions. Moreover it generates a constructive numerical scheme that can be effectively used to approximate the numerical solution. We provide the error estimates for such scheme as well as we present the examples of numerical computations.

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Periodic Traveling Waves in the Burridge-Knopoff Model

Marion Lebellego
Université Paul Sabatier Toulouse, France

The Burridge-Knopoff model is a model of an earthquake fault, which consists in a linearly coupled chain of masses in contact with a rough moving surface, subject to a velocity weakening friction. Despite the one-dimensional version of this model remains simple, its dynamics is complex: it reproduces non linear phenomena like stick-slip motion. We are interested in the mathematical study of this phenomenon, which raises some new difficulties linked to the non regularity of the motion equations. The dynamics is described by an infinite dimensional system of coupled differential inclusions with a non linear feature due to the velocity-weakening friction of Coulomb type, presenting a singularity at zero. Looking for traveling periodic waves, we obtain a non linear differential inclusion of order 2 with an advance and delay term, and which depends on some physical parameters (especially a coupling parameter ℓ). In this presentation, we show how to prove existence of periodic traveling waves when ℓ is close to zero, both for a regularized friction law and non regularized friction law, based on local methods (Lyapounov-Schmidt method).

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The Bouncing Ball System: Lyapunov Stability and Symptotic Attractivity

Remco Leine
ETH Zurich, Switzerland

(Thomas Heimsch)

The stability of non-smooth dynamical systems is a novel research field which is receiving much attention in the mathematical as well as engineering community. Mechanical systems with impact phenomena and unilateral constraints form an important class of non-smooth systems as they arise in many engineering applications. This research project aims to investigate the Lyapunov stability of equilibria in non-smooth non-autonomous systems. The focus is on the bouncing ball system, which consists of a rigid ball bouncing on a harmonically oscillating table. Global attractive stability conditions for the equilibrium of the bouncing ball system are proven using an extension of Lyapunov's direct method to non-autonomous non-smooth systems. Instead of requiring that the Lyapunov function decreases monotonically, it is merely required that the net change of the Lyapunov function is negative over every interval between two consecutive impacts. Furthermore, it is proven that the attractivity of the equilibrium is symptotic in the sense that there exists a finite time for which the solution has converged exactly to the equilibrium.

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Non-Convex Sweeping Processes

Manuel Marques
University of Lisbon (CMAF, FCUL), Portugal

Sweeping processes are differential inclusions where derivatives (or expressions involving derivatives) are required to belong to normal cones to sets which, in most mechanical applications, are assumed to be convex. The case of non-convex sets, which has also been the object of mathematical investigations, seems to be gaining momentum in the applications. An overview of some of my contributions (with several co-authors, such as C. Castaing, G. Colombo and N. Chemetov) towards existence results for non-convex sweeping processes will be presented.

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Quasistatic Viscoelastic and Thermoviscoelastic Contact Models

Stanisław Migórski
University of Krakow, Poland

We consider hemivariational inequalities modeling quasistatic viscoelastic and thermoviscoelastic problems which describe nonsmooth frictional contact between a body and a foundation. We study the asymptotic behavior of sequences of solutions to evolution problems when a small parameter in the inertial term tends to zero. We provide results on the existence of solutions.

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Linear-Quadratic Optimal Control with Lipschitz State and Costate Trajectories: Existence and a Unified Numerical Scheme

Jong-Shi Pang
University of Illinois, USA

(Lanshan Han, Kanat Camlibel, and W. P. Maurice H. Hemmels)

We provide a comprehensive treatment of the convex linear-quadratic optimal control problem with mixed polyhedral state and control constraints, via a constructive numerical scheme that unifies a time-stepping method based on the differential variational inequality reformulation of the problem and model predictive control. The scheme solves a sequence of finite-dimensional convex quadratic programs whose optimal solutions are employed to construct a sequence of discrete-time trajectories dependent on the time step. Under certain technical primal-dual assumptions primarily to deal with the algebraic constraints involving the state variable, we prove that such a numerical trajectory converges to an optimal trajectory of the continuous-time control problem as the time step goes to zero, with both the limiting optimal state and costate trajectories being Lipschitz continuous. This provides a constructive proof of the existence of a solution to the optimal control problem with such regularity properties, and as a by-product, the necessity of the well-known Pontryagin optimality conditions. Additional properties of the optimal solutions to the linear quadratic problem are also established that are analogous to those of the finite-dimensional convex quadratic program.

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A Proximal-Like Algorithm for Vibro-Impact Problems with a Non-Smooth Set of Constraints

Laetitia Paoli
University of Saint-Etienne, France

We consider a discrete mechanical system sujected to perfect unilateral contraints characterized by some geometrical inequalities $f_\alpha(q) \geq 0$, $\alpha \in \{1, \ldots, \nu\}$, with $\nu \geq 1$. We assume that the transmission of the velocities at impacts is governed by a Newton's impact law with a restitution coefficient $e \in [0, 1]$, allowing for conservation of kinetic energy if $e = 1$, or loss of kinetic energy if $e \in [0, 1)$, when the constraints are saturated.

Starting from a formulation of the dynamics as a first order measure-differential inclusion for the unknown velocities, time-stepping schemes inspired by the proximal methods can be proposed. Convergence results in the single-constraint case ($\nu = 1$) are recalled and extended to the multi-constraint case ($\nu > 1$), leading to new existence results for this kind of problems.

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The State Transfer Principle for Switching Port-Hamiltonian Systems

Arjan van der Schaft
University of Groningen, Netherlands

(M. K. Camlibel)

Instantaneous charge/flux transfers may occur in switched electrical circuits when the switch configuration changes. Characterization of such state discontinuities is a classical issue in circuit theory which, typically, is based on the so-called charge and flux conservation principle. In this talk we discuss a general state transfer principle for arbitrary switching port-Hamiltonian systems. The direction of the impulsive motion of the state at switching times is dictated by a Dirac structure which is derived from the Dirac structure of the overall system. This state transfer principle coincides with the charge and flux conservation principle in the special case of linear RLC circuits, but also covers circuits with nonlinear capacitors and inductors, and of arbitrary topology. Moreover, the principle is applied to switching mechanical systems. Next we discuss the extension of these results to switching port-Hamiltonian systems containing elements like ideal diodes. This will require the extension to cones.

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Special Session 27: Computational Dynamics

Michael Dellnitz, University of Paderborn, Germany
Oliver Junge, TU München, Germany
Kathrin Padberg-Gehle, TU Dresden, Germany

Introduction: Computational techniques for dynamical and control systems have grown to powerful tools beyond mere simulation. This session presents state of the art computational methods for the analysis and synthesis of systems, and showcases their applications. The focus is on detecting new links between dynamical systems resp. systems theory and such diverse fields as graph theory, numerics, mechanics, optimization, statistics, and topology.

Computation of Two-Dimensional Invariant Manifolds at a Shilnikov Bifurcation

Pablo Aguirre
University of Bristol, England
(Bernd Krauskopf, Hinke M. Osinga)

Invariant manifolds of saddle equilibria are a key ingredient for understanding the global dynamics in model vector fields.

The qualitative behavior of the phase space is organized by (un)stable manifolds that act as boundaries for the different kinds of trajectories. Moreover, their topological and geometrical properties may change at bifurcations. Thus, the interactions of invariant manifolds of equilibria and periodic orbits are essential for determining the overall dynamics near bifurcations.

While it is fairly easy to study the related one-dimensional invariant manifolds, the same is not true for (un)stable manifolds of higher dimension. Nevertheless, today these manifolds can be computed with high accuracy with state-of-the-art numerical methods based on the continuation of orbit segments, defined as solutions of suitable boundary value problems.

We consider, as an example, the well-known Shilnikov homoclinic bifurcation where a homoclinic orbit connects a saddle-focus back to itself. Under certain conditions, this bifurcation may yield infinitely many saddle periodic orbits giving rise to chaotic dynamics. In a model of an optically injected laser we show which topological and geometrical changes the two-dimensional stable manifold of a saddle-focus undergoes in the Shilnikov bifurcation. In this way, we are able to understand not only how the phase space is organized in this process, but also how it changes from simple to chaotic dynamics.

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Rigorous Verification of Global Properties of Dynamical Systems

Zin Arai
Hokkaido University, Japan

We propose rigorous computational algorithms for proving global properties of dynamical systems such as the structural stability of the system and the occurrence of homoclinic bifurcations. The algorithms involve subdivision algorithm, rigorous interval arithmetic, computational homology theory and the Conley index. Using these algorithms, we study the dynamics of the Hénon map and some dynamical systems arising from chemical reaction theory. In particular, we prove the uniform hyperbolicity of the system for large parameter regions, and show the occurrence of homoclinic tangencies for specified parameter values. We also apply our algorithms to polynomial dynamical systems with complex variables and prove that the topology of the two-dimensional generalization of the Mandelbrot set is totally different from that of the original Mandelbrot set. Furthermore, we prove that the monodromy of the complex Hénon map determines the dynamics of the real Hénon map, provided some hyperbolicity conditions. Combining this observation with our computational algorithms, we rigorously obtain the complete description of the dynamics of real Hénon map for various parameter regions.

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Scalable Methods for Uncertainty Quantification

Andrzej Banaszuk
United Technologies Research Center, USA
(Amit Surana, Tuhin Sahai, Sorin Costiner)

The objective of Robust Uncertainty Management (RUM) project was to develop methodology and tools for quantifying uncertainty in ways that are orders of magnitude faster than Monte Carlo with near-linear scaling in the system size, and demonstrate them in molecular dynamics and UAV search challenge problems. This research was conducted by team including UTRC, UCSB, Caltech, Stanford, Yale, Georgia Tech, Princeton, Aimdyn Inc., and PlainSight Inc. Several methods including Polynomial Chaos, and new Stochastic Response Surface, and Dynamic Sampling methods allowed the team to calculate the mean and variance of the phase transition temperature in molecular dynamics calculations with 10K atoms 2000 times faster than

using Monte Carlo sampling. We also proposed an iterative UQ approach that exploits the weak subsystem interactions to overcome the dimensionality curse and radically accelerate uncertainty quantification in large systems.

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The Computation of Invariant Sets Via Newton's Method

Mirko Hessel-von Molo
University of Paderborn, Germany

(R. Baier (University of Bayreuth), M. Dellnitz (University of Paderborn), I. Kevrekidis (Princeton University), St. Sertl (University of Paderborn))

We will introduce a set oriented framework for the computation of invariant sets of dynamical systems via Newton's method. As in the case of fixed (or periodic) points this technique is not just more efficient numerically but it also allows, in principle, to approximate unstable invariant sets with a more complicated dynamical structure.

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On Hybrid Stochastic-Deterministic Models for Biochemical Reaction Kinetics

Tobias Jahnke
Karlsruhe Institute of Technology, Germany

Biochemical reaction systems are traditionally modelled by ordinary differential equations representing the concentrations of the species. The reaction-rate approach is computationally cheap, but becomes insufficient if some of the species contain only a very low number of particles, and if small-scale stochastic fluctuations can cause large-scale effects. This is the case, e.g., in gene regulatory networks where gene expression is regulated by a few activators or repressors, or in viral kinetics where the fate of very few infectious individuals decides whether the infection spreads over large parts of the population.

For such applications, the appropriate description is provided by stochastic reaction kinetics. Here, the system is represented by a time-dependent probability dstribution $p(t, x_1, \ldots, x_d)$ which indicates the probability that at time t exactly x_i particles of the i-th species exist. It is well-known that p is the solution of the chemical master equation, but solving this equation numerically is only possible for rather small systems because the number of degrees of freedom grows exponentially with the number of species.

This dilemma has motivated a number of attempts to construct hybrid models which combine the simple but coarse reaction-rate approach with the precise but expensive stochastic kinetics. A particularly appealing hybrid model has recently been proposed by A. Hellander and P. Lötstedt. In this talk, we present an extension of their model and show that, under certain conditions, this extension yields a more accurate approximation of the true dynamics.

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Computation of Invariant Measures with Model Reduction Methods

Jens Kemper
Bielefeld University, Germany

We present algorithms to compute invariant measures in high-dimensional discrete dynamical systems. The algorithms are based on subdivision techniques developed by Dellnitz, Junge and co-workers combined with the Proper Orthogonal Decomposition (POD) method as a model reduction approach to avoid the 'curse of dimension'. It turns out that the Prohorov metric is a suitable distance in the space of measures that can be computed analytically. Important steps towards a broad convergence theory for the POD-based algorithms are worked out including the long-term behavior of POD systems and approximation properties of discrete measures. Finally, we present numerical approximations of invariant measures computed by our algorithms for discretized parabolic equations. We visualize our results by a histogram-like representation.

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Universality of Algebraic Decays in Hamiltonian Systems?

Roland Ketzmerick
TU Dresden, Germany

(G. Cristadoro)

Hamiltonian systems with a mixed phase space typically exhibit an algebraic decay of correlations and of Poincaré recurrences, as it was first observed by Chirikov and Shepelyansky in 1981. While numerical experiments over finite times show system-dependent power-law exponents, we conjecture the existence of a universal asymptotic decay. This conjecture is based on (i) analytical and numerical results for a Markov tree model with random scaling factors for the transition probabilities (extending a model by Meiss and Ott) and (ii) a statistical hypothesis on the phase-space structure of 2D Hamiltonian systems at the boundary of regular and chaotic motion on sufficiently small scales. Numerical simulations for different Hamiltonian systems support this conjecture and permit the determination of the universal exponent.

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Fast Approximation of Long Term Dynamical Behaviour for Continuous-Time Systems

Péter Koltai
Technische Universität München, Germany
(Gary Froyland (UNSW) and Oliver Junge (TUM))

Transfer operator methods are widely used in applications to determine long term dynamical behaviour. They are based on simulations of the dynamical system. However, for systems arising from ODEs, simulation is computationally very expensive. Instead of the associated transfer operator we propose to analyze the infinitesimal generator of the system, to avoid trajectory computation. For illustration we present several numerical examples. A special attention is payed to the computation of domains of attraction.

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Towards Density-Based Methods for Nonlinear Conservation Laws

Michael Kratzer
TU München, Germany
(Oliver Junge, Utz Wever)

The evolution of the density of an ensemble of particles in a dynamical system is governed by a linear conservation law. We attempt to extend the numerical methods used to study this problem to the nonlinear case. In this talk, we show how the multivalued approach of Brenier can be interpreted in this way and present preliminary results towards density-valued approximations for the solution of the Euler equations.

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Computation of Optimally Controlled Trajectories Using Invariant Manifolds and Symmetries

Sina Ober-Blöbaum
University of Paderborn, Germany
(Kathrin Flaßkamp, Marin Kobilarov)

The computation of control policies to steer dynamical systems optimally w.r.t. a predefined objective is of great interest in many different research areas, such as astrodynamics, robotics or vehicle dynamics. However, most optimal control techniques provide only locally optimal solutions and in general a good initial guess is required. On the other hand, the use of global methods leads to high computational cost since a search over the whole phase space has to be performed.

In this talk, we will demonstrate how to exploit inherent properties of the dynamical system to approximate a global optimal solution that also provides a good first initial guess for a local optimizer. To this aim, the space of trajectories is quantized by representative small pieces of energy efficient solution trajectories which can be combined in various ways. Candidates for these so called motion primitives can be obtained by the inherent dynamical properties of the system under consideration, such as motion curves induced by symmetries and invariant (un)stable manifolds of hyperbolic fixed points. Additionally, short controlled maneuvers are required to arrange these trajectories in sequences. The approach will be demonstrated by means of the optimal control of a spherical pendulum.

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An Implicit Numerical Method for Differential Inclusions

Janosch Rieger
Goethe-Universität Frankfurt, Germany
(W.-J. Beyn)

We consider the differential inclusion

$$\dot{x}(t) \in F(t, x(t)) \text{ a.e. in } [0,T],\ x(0) = x_0 \tag{1}$$

in the Euclidean space $\mathbb{R}^m$, where F is continuous, has convex and compact values, and satisfies the relaxed one-sided Lipschitz condition (ROSL), which generalizes both, the single-valued one-sided Lipschitz and the set-valued Lipschitz condition.

In most sources which deal with numerical analysis of differential inclusions, the right-hand side is required to be Lipschitz continuous or ROSL, so that the only suitable numerical methods are those of order one. As it was more or less believed that implicit set-valued methods make no sense, almost all efforts have been focussed on the explicit Euler scheme

$$\Phi_h^{\exp}(t, x) := \{y \in \mathbb{R}^m : y \in x + hF(t, x)\}. \tag{2}$$

Our aim is to establish an implicit set-valued Euler scheme for continuous and ROSL right-hand sides. We present a solvability theorem for ROSL maps which guarantees that the implicit Euler steps

$$\Phi_h^{\mathrm{imp}}(t, x) := \{y \in \mathbb{R}^m : y \in x + hF(t + h, y)\} \tag{3}$$

are well-defined. Furthermore, we prove that the numerical method (3) converges and has favourable stability properties, and we demonstrate that the implicit scheme is indeed implementable.

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The Treatment of Uncertainties with Polynomial Chaos in an Industrial Environment

Utz Wever
Siemens AG, Germany
(Albert Gilg, Meinhard Paffrath)

The mathematical description of industrial processes generally leads to differential equations. These processes could be electric circuits, multibody systems, chemical reactions or transport-diffusion processes. In real industrial applications often uncertainties enter the model in term of scattering temperature, scattering geometry due to manufacturing tolerances of scattering densities of a material. The efficient mathematical treatment of these uncertainties is a topic of increasing interest. Monte Carlo evaluations, which require a large number of samples, are very costly for complex differential equations. During the last years the Polynomial Chaos expansion introduced by Wiener, which can be interpreted as a serial expansion of L_2 random variables in a random space, has become a powerful tool for treating uncertainties. The stochastic differential equations solved by the calcul of Wiener are called Random Ordinary Differential Equation (RandomODE).

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Special Session 28: Stochastic Partial Differential Equations

Wilfried Grecksch, University Halle (Saale), Germany
Björn Schmalfuss, University of Paderborn, Germany

Introduction: Stochastic partial differential equations is a modern part of stochastic analysis. This special session should give a general overview about new tendencies in the field of stochastic partial differential equations. The invited speakers will discuss issues related to numerics/computations, large deviations, and existence and uniqueness of spde. In addition another main goal will be to discuss spde driven by fractional Brownian and Levy noise, dynamics of spde and modeling and applications of spde.

On Some Stochastic Shell Models of Turbulence

Hakima Bessaih
University of Wyoming, USA
(Franco Flandoli and Edriss S. Titi)

Recently, it has been proposed that the Navier-Stokes equations and a relevant linear advection model have the same long-time statistical properties, in particular they have the same scaling exponents of their structure functions. In this paper, we investigate the validity of this assertion for certain stochastic phenomenological shell models of turbulence driven by an additive noise.

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Hyperbolic Equations with Dynamical Boundary Conditions

Peter Brune
University of Paderborn, Germany

We consider a hyperbolic equation with dynamical boundary conditions and with a noise which acts in the domain but also on the boundary. We show that this problem has a solution which generates a random dynamical system, in addition we investigate the dynamics. In particular we show the existence of random attractors.

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Stabilization of Differential Inclusions and PDEs without Uniqueness by Noise

Tomás Caraballo
Universidad de Sevilla, Spain

We prove that the asymptotic behaviour of partial differential inclusions and partial differential equations without uniqueness of solutions can be stabilized by adding some suitable Ito noise as an external perturbation. We show how the theory previously developed for the single-valued case can be successfully applied to handle these set-valued cases. The theory of random dynamical systems is used as an appropriate tool to solve the problem.

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Evolution Equations Driving by a Fractional Brownian Motion

Maria Garrido-Atienza
University of Sevilla, Spain

(Björn Schmalfuss)

In this talk we present some results on the existence and uniqueness of mild solutions to stochastic evolution equations driven by a fractional Brownian motion (fBm). To prove our results the main tool consists of combining suitable estimates of pathwise integrals with respect to the fBm with some properties of the semigroup.

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A Filtering Problem for a Fractional Stochastic Partial Differential Equation

Wilfried Grecksch
University Halle (Saale), Germany

We introduce a stochastic evolution equation

$$dX(t) = AX(t)dt + dB_{h,Q}(t),\ X(0) = 0, \qquad (1)$$

where A is an infinitesimal generator of a strong continuous semigroup in a Hilbert space H and $(B_{h,Q}(t))_{t\geq 0}$ is a Hilbert space valued fractional Brownian motion with Hurst parameter $h \in \left]\frac{3}{4}, 1\right[$ and a kernel operator Q as covariance. Finite dimensional observations are given by

$$dY(t) = G(t)X(t)dt + dB_{h,n}(t) + dW(t),\ Y(0) = 0 \qquad (2)$$

for the mild solution of (1). $(B_{h,n}(t))_{t\geq 0}$ is a finite dimensional part of the process $(B_{h,Q}(t))_{t\geq 0}$ and $(W(t))_{t\geq 0}$ is a finite dimensional Brownian motion.

Let $\mathcal{F}_t^Y$ be the σ-algebra generated by the observations $\{Y(s) : s \in [0,t]\}$.

The filtering problem consists in the estimation of the state $X(t)$ by $\hat{X}(t) = E\left[X(t)|\mathcal{F}_t^Y\right]$ for $t \geq 0$. We will show the existence of a family $(\Phi(t,s))_{s\in[0,t]}$ of deterministic linear operators with

$$\hat{X}(t) = \int_0^t \Phi(t,s)dY(s).$$

Equations are deduced for $\Phi(t,\cdot)$. The estimation error is also given.

The filtering problem is discussed in [1] without fractional noise in the observations by using of a so called innovation process. However here we can't use this technique since the fractional noise processes in (1) and (2) are dependent processes. We use an infinite dimensional theorem of conditional Gaussian correlation.

[1] W. Grecksch, C. Tudor: A filtering problem for a linear stochastic evolution equation driven by a fractional Brownian motion. Stochastics and Dynamics, Vol. 8 No 3, (2008), 397 - 412.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Pointwise Approximation of Stochastic Heat Equations with Additive Noise

Daniel Henkel
TU Darmstadt, Germany

We consider stochastic heat equations on the spatial domain $(0,1)^d$ with additive nuclear noise as well as additive space-time white noise, and we study approximation of the mild solution at a fixed time point. The error of an algorithm is defined by the average L_2-distance between the solution and its approximation. The cost of an algorithm is defined by the total number of evaluations of one-dimensional components of the driving Brownian motion at arbitrary time nodes. We want to construct algorithms with an (asymptotically) optimal relation between cost and error. Furthermore we wish to determine the asymptotic behaviour of the corresponding minimal errors. We show how the minimal errors depend on the spatial dimension d and, in the case of nuclear noise, on the decay of the eigenvalues of the associated covariance operator. Asymptotic optimality is achieved by a spectral Galerkin approximation together with a nonuniform time discretization. This optimality cannot be achieved by uniform time discretizations, which are frequently studied in the literature. This work was supported in part by the DFG.

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Shift of Stability in Spatially Extended Systems by Additive Noise and Application to Neural Populations

Axel Hutt
INRIA Nancy, France

The work studies the effect of additive noise on the stability of high-dimensional systems. The first system under study is two-dimensional, evolves close to the deterministic stability threshold. The work shows that the system exhibits an additive noise-induced shift of the control parameter when driving one variable by uncorrelated Gaussian noise. After a detailed analytical and numerical study of this effect, the work focusses on a spatially extended system subjected to global noise, i.e. noise constant in space and uncorrelated in time. This spatial system exhibits a phase transitions induced by additive global noise similar to the stability shift in the two-dimensional system. Numerical studies confirm this effect. Further closer investigations reveal that the occurence of the noise-induced shift is subjected

to the model nonlinearity and the direction of the shift depends on the sign of the prefactors of the nonlinearities involved.

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Non-Globally Lipschitz Counterexamples for the Stochastic Euler Scheme

Arnulf Jentzen
Bielefeld University, Germany
(Martin Hutzenthaler)

The stochastic Euler scheme is known to converge to the exact solution of a stochastic differential equation with globally Lipschitz coefficients and even with coefficients which grow at most linearly. For super-linearly growing coefficients convergence in the strong and numerically weak sense remained an open question. In this article we prove for many stochastic differential equations with super-linearly growing coefficients that Euler's approximation does not converge neither in the strong sense nor in the numerically weak sense to the exact solution. Even worse, the difference of the exact solution and of the numerical approximation diverges to infinity in the strong sense and in the numerically weak sense.

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Solutions for Stochastic Equations of Schrödinger Type

Hannelore Lisei
Babes-Bolyai University, Cluj-Napoca, Romania
(W. Grecksch)

We study nonlinear stochastic evolution equations of Schrödinger type perturbed by infinite dimensional Wiener processes. We use the Galerkin method to prove the existence of the solution.

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Generalized Polynomial Chaos Expansions

Antje Mugler
University of Applied Sciences Zwickau, Germany
(Hans-Joerg Starkloff)

We discuss properties of generalized polynomial chaos expansions and their use for the solution of random partial differential equations.

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Random Differential Equations with Random Delay

Arne Ogrowsky
Universität Paderborn, Germany
(Björn Schmalfuss)

We investigate a random differential equation with random delay and clarify under which assumptions the concept of a solution to such an equation is meaningful. Then we want to show that this solution generates a random dynamical system. Hence, due to the random delay, the measurability of that random dynamical system needs to be defined at first. Finally we study some further concepts with respect to the generated random dynamical system.

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Q-Fractional Brownian Motion in Infinite Dimensions. Itô's Formula and Isometry

Christian Roth
University Halle-Wittenberg, Germany
(Wilfried Grecksch)

We consider a white noise calculus for fractional Brownian motion with values in a separable Hilbert space, whereby the covariance operator Q is a kernel operator (Q-fractional Brownian motion). We prove a Q-fractional version of the Itô's formula. Furthermore we introduce Malliavin derivative for Q-fractional motion, prove a Q-fractional integration by parts formula and a Q-fractional version of the Itô isometry.

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Uniqueness of Invariant Measures for Stochastic Delay Equations

Michael Scheutzow
Technische Universität Berlin, Germany
(Martin Hairer, Jonathan Mattingly)

We show uniqueness of an invariant measure on an appropriate function space for solutions of a large class of stochastic functional differential equations with bounded memory. For an equation with a past-dependent diffusion coefficient, the solution Markov process on a suitable space of continuous functions does in general not enjoy the strong Feller property and therefore classical methods for proving uniqueness of an invariant measure do not apply. We show uniqueness by constructing an *asymptotic coupling.* This is joint work with Martin Hairer and Jonathan Mattingly.

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Stochastic Nonlinear Schrödinger Equation with Harmonic Potential

Ji Shu
Sichuan Normal University, Peoples Rep. of China

This paper discusses a class of stochastic nonlinear Schrödinger equations with harmonic potential and we derive a result on global existence of solutions in corresponding energy space, which is consistent with the case without harmonic potential.

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Stochastic Stability of the Ekman Spiral

Wilhelm Stannat
TU Darmstadt, Germany
(Matthias Hieber, TU Darmstadt)

We consider the Navier-Stokes equations with Coriolis term on a bounded layer perturbed by a cylindrical Wiener process. Weak and stationary martingale solutions to the associated stochastic evolution equation are constructed. The time-invariant dstribution of the stationary martingale solution can be interpreted as the long-time statistics of random fluctuations of the stochastic evolution around the Ekman spiral, which is an explicit stationary solution of the Navier-Stokes equations with Coriolis term. This is the stochastic analogue of the asymptotic stability of the Ekman spiral recently proven by Hess.

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The Existence and Asymptotic Behaviour of Energy Solutions to Stochastic 2D Functional Navier-Stokes Equations Driven by Levy Processes

Takeshi Taniguchi
Kurume University, Japan

Let D be a bounded or unbounded open domain of 2-dimensional Euclidian space. If the boundary exists, then we assume that the boundary is smooth. In this talk we discuss the existence and asymptotic behaviour of energy solutions to 2-dimensional stochastic functional Navier-Stokes equation perturbed by Levy processes.

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About the Navier-Stokes System with Infinite Delay

José Valero
Universidad Miguel Hernández, Spain
(Pedro Marín-Rubio, Joseé Real)

We prove existence of solutions for a Navier-Stokes model in dimension two with an external force containing infinite delay effects in a suitable weighted space. After that, under additional suitable assumptions we prove existence and uniqueness of a stationary solution and exponential decay of the solution of the evolutionary problem to this stationary solution. Finally, we study the asymptotic behaviour of solutions. In particular, we obtain the existence of pullback attractors for the dynamical system associated to the problem under more general assumptions. Moreover, we check that each component of the family of sets of the pullback attractor is connected. The attraction property is considered in two situations. In the first case the pullback attractor attracts fixed bounded sets of the phase space; in the second, it attracts time-dependent families of sets satisfying suitable assumptions.

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Special Session 29: Applied Analysis and Dynamics in Engineering and Sciences

Thomas C. Hagen, University of Memphis, USA
Janos Turi, UTD, USA

Introduction: This session will address theoretical, numerical and computational developments and their applications to fluid dynamics, solid mechanics and life sciences. Areas of analytical interest include the theory of linear/nonlinear partial differential equations, the qualitative behavior of solutions, stability and asymptotics, control theoretic issues, and related aspects. The areas of application range from viscoelasticity and flow simulation to industrial flows and applications in mathematical biology.

On Stability and Trace Regularity of Solutions to Reissner-Mindlin-Timoshenko Equations

George Avalos
University of Nebraska-Lincoln, USA

The uniform stability of Reissner-Mindlin-Timoshenko (RMT) plates is addressed. Similar

to what is seen for waves, Kirchhoff plates, and elastodynamics, boundary stabilization of the RMT model depends upon the derivation of a certain observability inequality; which in turns calls for the existence of certain boundary trace regularity estimates. In this talk, we will provide details on the securing of said observability and boundary trace regularity estimates. This is joint work with Daniel Toundykov.

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Power-Law Asymptotics and Applications to Polycrystal Plasticity

Marian Bocea
NDSU, USA

The asymptotic behavior of a general class of power-law functionals acting on fields subject to constant rank differential constraints is studied via Γ-convergence. The effective yield set of a polycrystal is characterized by means of variational principles associated to the limiting functionals in several model cases.

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A Study of Stability of Rogue Wave Solutions of the Nonlinear Schrödinger Equation

Annalisa Calini
College of Charleston, USA
(Constance Schober (University of Central Florida))

Homoclinic orbits of the integrable focusing Nonlinear Schrödinger (NLS) Equation have been widely studied as models of rogue waves in deep water. Coalescence of spatial modes in homoclinic orbits of NLS solutions with several unstable modes appears to be a key mechanism for formation as well as persistence of high amplitude rogue waves in NLS-based models of water wave dynamics. In this talk we discuss the linearization of the NLS equation about homoclinic orbits of unstable plane wave solutions with two unstable modes. Using Bäcklund transformations to construct a complete set of solutions of the linearized equation, we show how saturation of instabilities makes the largest dimensional homoclinic orbits the most stable in the sense of linear stability.

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The Dispersion Relations of Kevin-Helmholtz Instability Wave of Supersonic N Jets ($N > 3$)

Joshua Du
Kennesaw State University, USA

Because of the need for large thrust and flexibility, high performance military aircraft could be propelled by multiple jet engines housed close to each other. Since the complexity of the configuration of multiple jets and large tangential gradient of the velocity of the jets plumes, the Kelvin-Helmholtz instability wave and upstream propagated acoustic wave including the screech tone, would cause the instability of the jets and the damage of the engines vie metal fatigue.

In this work, Vortex sheet model has been used to investigate Kelvin-Helmoltz instability waves. The governing equations of virtual N-jet flow are derived by using three physics laws, the Conservation of Mass, Momentum, and Energy. The general solution is obtained.

The general solution is classified into $2N$ families by using a special Toeplitz matrix A with property $A = A^{-1} = A^T$. By applying Kinematical and Dynamical boundary conditions, we are able to derive $2N$ dispersion relations for Kelvin-Helmholtz Instability waves and acoustic waves, which provides the mathematical foundation to investigate the characteristics to those waves numerically.

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Numerical Determination of the Basin of Attraction for Asymptotically Autonomous Dynamical Systems

Peter Giesl
University of Sussex, UK, England
(Holger Wendland)

Nonautonomous ordinary differential equations arise in many applications from engineering to biology. Numerical methods to determine the basin of attraction for autonomous equations focus on a bounded subset of the phase space. For nonautonomous systems, any relevant subset of the phase space, which now includes the time as one coordinate, is unbounded in t-direction. Hence, a numerical method would have to use infinitely many points.

To overcome this problem, we introduce a transformation of the phase space. Restricting ourselves to asymptotically autonomous systems, we can map the infinite time interval to a finite, compact one. Now we are able to generalise numerical methods from the autonomous case. More precisely, we characterise a Lyapunov function as a solution of a suitable linear first-order partial differential equation and approximate it using Radial Basis Functions.

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Film Casting Processes: Simulation and Optimization

Thomas Goetz
TU Kaiserslautern, Germany

We consider a film casting process modelling the production of polymer films used for plastic bags or packing materials. In an isothermal environment, the uni-axial stretching of the film leads to a necking-in, i.e. the film gets thinner along its centerline than at the edges. This raises the question, how should one design the geometry of the nozzle, to obtain a uniform film thickness at the take-up point of the film. We propose an optimization problem to determine this shape. Numerical simulations show the results of the optimization.

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Global Existence Results for Stretching of Viscous Fibers

Thomas Hagen
The University of Memphis, USA

In this presentation we address some recent results about the Matovich-Pearson equations of fiber spinning of viscous liquids. The governing equations consist of a nonlinear mass conservation equation coupled to a momentum conservation equation in 1D. The underlying modeling hypothesis is that viscous forces dominate. Since standard a priori estimates are not applicable to prove global existence of solutions, we devise a contradiction argument to show that viscous fibers do not break in finite time. This result suffices to deduce global existence of solutions. (Joint work with Michael Renardy.)

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Asymptotic Behavior of Nonlinear Volterra Integral Equations

Ferenc Hartung
University of Pannonia, Veszprem, Hungary
(I. Győri)

In this talk we investigate the growth/decay rate of solutions of an abstract Volterra-type integral equation which frequently arises in quasilinear differential equations applying a variation-of-constants formula. We will show that under some conditions the exponential growth/decay rate of solutions of an associated linear equation is preserved for the nonlinear equation. Our results can be applied for the case when the characteristic equation of the associated linear equation has complex dominant eigenvalue with higher than one multiplicity. We also show analogous results for discrete Volterra difference equations. Examples are given to illustrate the sharpness of the results.

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Davydov-Scott Models of Wave Motion in α-Helix Protein and Exactly Energy-Momentum Conserving Discretizations for Hamiltonian Systems

Brenton Lemesurier
College of Charleston, South Carolina, USA

Long-range energetic pulse propagation in α-helix protein has been modeled by Davydov, Scott and others with Hamiltonian ODE systems, which in a continuum limit and a singular "infinite stiffness" limit, lead to the integrable NLS Equation. The soliton solutions of the latter suggest the possibility of coherent long-range propagation of energetic pulses in such proteins. These predictions need to be tested with more accurate models, as each limit can fundamentally change the nature of solutions, and exact solutions are unknown, so numerical solution of these large, stiff Hamiltonian systems is needed.

Simulation is done by introducing a time discretization method based on discretizing the Hamiltonian form using a finite difference calculus for gradients. This ensures exact conservation of all conserved quantities and allows a simple, highly stable iterative method for solving the resulting implicit system.

The results show that as the parameters get further from the continuum limit regime, substantial changes occur in the solution form, including pulse narrowing and slowing, and the development of additional pulses with speeds that tend to infinity in the NLS limit.

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Numerical Investigation of Particle-Laden Thermally Driven Turbulent Flows in Enclosure

Emmanuel Leriche
Université Jean-Monnet, Saint-Etienne, France
(R. Puragliesi and M. O. Deville and A. Soldati)

Nowadays, the global increase of energy demand and the necessity to satisfy high safety standards, have led engineers and scientists to focus their efforts in order to understand and describe fundamental phenomena that are crucial for a correct design of the new generation nuclear power plants. In this framework, the present work aims at providing a first insight of the mechanisms of deposition of aerosol particles inside a closed geometry where relatively strong currents are present due to turbulent natural

convective flows [1]. Direct Numerical Simulations were conducted coupling high-order pseudo-spectral code with a Lagrangian particle tracker. Laminar flows were computed in two and three dimensions in order to benchmark the code with published reference data. A parametric study was performed for three different aerosol microsize particle diameters and two super-critical Rayleigh numbers (as high as 10^{10}) in a square cavity. An extended analysis of the turbulent flows is provided in terms of first and second order statistics, time-averaged momentum and energy budgets, and moreover, important terms appearing in the transport equations of turbulent kinetic energy and temperature variance are also briefly discussed. Furthermore, the evolution in time of particle concentration for the three different diameters is considered. The text provides information about the deposition velocity, the deposition patterns on the cavity surfaces, the influence of lift and thermophoretic forces and the fractal dimension. The same size dependent parametric study for the three different sets of micro- size particles was carried out in a fully three-dimensional closed cubic cavity for one super-critical Rayleigh number (as high as 10^9). A detailed investigation of the turbulence was performed by means of statistical quantities, signal processing and conditional averaging, in order to get a general view of the complexity of the flow and its characteristics. Further on, the sedimentation process is studied in the same way as for the two dimensional case. Finally a simple theoretical deposition model is provided in order to interpret the numerical results for the aerosol phase.

[1] R. Puragliesi. *Numerical investigation of particle-laden thermally driven turbulent flows in enclosure.* PhD thesis, n^o:4600, Ecole Polytechnique Fédérale de Lausanne, Lausanne, Switzerland, 2010.

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Stokes Eigenmodes, Potential Vector – Vorticity Correlation and Corner Vortices in Trihedral Rectangular Corner

Emmanuel Leriche
Université Jean-Monnet, Saint-Etienne, France
(G. Labrosse and P. Lallemand, Université Paris-Sud, France)

The knowledge of Stokes eigenmodes in a square/cube (or in any bounded domain) could provide some insight into the understanding or analysis of turbulent instantaneous flow field in a geometry as simple as, for instance, the driven cavity. Stokes eigenmodes are not analytically known except when they are periodic in all, or in all but one, space directions. If they are indeed constrained to verify velocity no-slip conditions on a closed boundary they can only be determined by numerical approach. The aim of the present contribution is to provide a first deep insight into the Stokes eigenspace in the most simple confined geometries, namely a square or a cube, and then to provide some generalisation to any bounded domain.

Only a few attempts were made for computing Stokes eigenvalues and/or eigenmodes in fully 2D confined geometries. The present contribution has opted for using three different solvers, associated with each Stokes formulation: a spectral Chebyshev collocation method in primitive variable (2D/3D), the Galerkin-Reid-Harris (RH) decomposition in the stream function formulation (2D), and the Lattice Boltzmann approach (LBA) (2D/3D).

Computing the Stokes eigenmodes in cubical domain is a double challenge. First, owing to the reduced knowledge of the corresponding spectrum, from the theoretical predictions, the PrDi and LBA Stokes solvers are used for assessing the correctness of the numerically computed eigenspace. Secondly, in order to accurately compute a significant part of the spectrum for determining the coefficients of the afore mentioned theoretical predictions both chosen solvers lead to handle a huge eigenproblem to be solved. Regarding this last aspect, the authors believe that it is at the frontier of the nowadays eigensolver capabilities. The numerical results of these solvers are compared in the 2-D (square) and 3-D (cube) cases. The Chebyshev approach is by far the most accurate, even though the associated solver does not provide a divergence free velocity but asymptotically. The good agreement between the LBA and PrDi results clearly confirms that the LBA approach is an interesting alternative [5].

Owing to the lack of knowledge of the Stokes spectrum, except from the theoretical asymptotic predictions, $\lambda_k \simeq k^{2/d} + \mathcal{O}(k^{1/d})$ for a d-D problem, proposed by Constantin and Foias [1], an accurate computation of a significant part of the spectrum is needed to determining the coefficients of the afore mentioned theoretical predictions (see the analysis limited to the square case published in [2] and preliminary results for the cubical case spectrum in [5]).

The unsteady dynamics of any Stokes flows is settled leading to a general exact vectorial relationship between the vector potential and the vorticity. The velocity Vector Potential ($\vec{\Psi}$) and Vorticity ($\vec{\omega}$) are shown to verify, to a good approximation, and in any bounded domain, a co-linearity relation specifically characterizing the Stokes eigenmode core dynamics. This relation comes from an exact algebraic relation between the three pseudo-vectors required for describing the dynamics of any Stokes flow, namely $\vec{\Psi}$, $\vec{\omega}$ and $\vec{\Pi}$, the pressure gradient Vector Potential. This relation is applied to the Stokes eigenmodes confined in the disk, in the plane channel, in the

square and in the cube. It shows that, in the core of these domains, and therefore of any closed 2D or 3D domain, these modes verify a collinear relationship between the vector potential and the vorticity [3]. This is a definitive answer to the discussion opened by Batchelor in the sixties about the existence of such a relation for 2D inviscid (large Reynolds number) steady laminar flows or for 2D viscous flows (zero Reynolds number).

The confined Stokes eigenmode flows exhibit corners and edges behavior which, in 2D, meets the well known Moffatt's corner eddies occurring in an infinite sequence of decreasing intensities and enjoying the symmetry properties of the Stokes eigenmode parent. Over the last years, analytical description of 3D corner eddies have regained interest in the literature and have been reported like, for instance, in the cone where these 3D corner eddies are somehow the natural extension of the 2D Moffatt's ones [6]. The corner vortices in a fully confined trihedral rectangular corner bear some symmetry properties like in the square, but the 3D corner flow structure are now connected to 3D edge's one. Some illustrations will be given, and these structures are identified [4]. Regarding the 2D Moffatt's corner eddies, these 3D corner behaviors are of quite different nature. To the author's knowledge, such analytical or numerical description in a trihedral corner is still missing in the literature.

Finally, one can extend the present search of Stokes modes in a cube to other closed geometries with boundaries of arbitrary shape.

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[2] E. Leriche and G. Labrosse. Stokes eigenmodes in square domain and the stream function – vorticity correlation. *J. Comput. Phys.*, 200(2):489–511, 2004.
[3] E. Leriche and G. Labrosse. Vector potential - Vorticity relationship for the Stokes flows. Application to the Stokes eigenmodes in 2D/3D closed domain. *Theoretical and Computational Fluid Dynamics*, 21:1–13, 2007.
[4] E. Leriche, G. Labrosse, and P. Lallemand. Stokes eigenmodes in cubic domain and corner flows. in preparation, 2010.
[5] E. Leriche, P. Lallemand, and G. Labrosse. Stokes eigenmodes in cubic domain: primitive variable and Lattice Boltzmann formulations. *Applied Numerical Mathematics*, 58:935–945, 2008.
[6] P. N. Shankar. On Stokes flow in a semi-infinite wedge. *J. Fluid Mech.*, 422:69–90, 2000.

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An Ultra Weak Finite Element Method as an Alternative to a Monte Carlo for an Elasto-Plastic Problem with Noise

Laurent Mertz
Université Pierre et Marie Curie, Paris VI, France
(Alain Bensoussan, Olivier Pironneau, Janos Turi)

An efficient method for obtaining numerical solutions of an elasto-plastic oscillator with noise is considered. Since Monte Carlo simulations for the underlying stochastic process are too slow, as an alternative, approximate solutions of the partial differential equation defining the invariant measure of the process are studied. The regularity of the solution of that partial differential equation is not sufficient to employ a "standard" finite element method. To overcome the difficulty, an ultra weak finite element method has been developed and successfully implemented.

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Rogue Waves, Dissipation, and Downshifting

Constance Schober
University of Central Florida, USA

Damping plays an important role in stability and downshifting of waves. The physical and statistical properties of rogue waves in deep water are investigated using the focusing Nonlinear Schrödinger equation and the Dysthe equation with an additional damping term. The effects of both linear and nonlinear damping on the development of rogue waves and the interaction between rogue waves and downshifting are examined using numerical investigations and analytical arguments based on the inverse spectral theory of the underlying integrable model, perturbation analysis, and statistical methods.

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Nonlinear Stochastic Wave Equations in R^1 and R^2

Henri Schurz
SIU Carbondale (IL), USA

Semi-linear one-dimensional wave equations (nonlinear string and plates) with certain quasi-nonlinearities of power-law-type and perturbed by Q-regular space-time white noise are considered both analytically and numerically. These models as 2nd order SPDEs (stochastic partial differential equations) with non-random Dirichlet-type boundary conditions describe the displacement of vibrations of noisy nonlinear strings and plates as met in mechanical engineering, laser dynamics or chemical

processes. We discuss their mathematical analysis by the standard eigenfunction approach allowing us to truncate the corresponding infinite-dimensional stochastic systems (i.e. quasilinear SPDEs), to control its error, energy, existence, uniqueness and stability of solutions. The truncated system of obtained ordinary SDEs is numerically integrated by partial-implicit, midpoint based difference methods. These nonstandard methods guarantee an adequate control on the growth of related (nonlinear) mean energy functional with time t. To understand the qualitative behavior of both analytical solutions and numerical approximations, we investigate the existence and uniqueness of strong solutions using energy-type methods based on the construction of Lyapunov-functions. Moreover, probabilities of large fluctuations can be estimated while using consistent approximations. A more general applicable axiomatic approach to their numerical analysis is suggested on subspaces of certain Hilbert spaces of random processes with some uniformly bounded moments eventually.

If time permits, we shall talk on some quasi-linear stochastic heat equations as well.

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Optimal Control of Variational Inequalities

Janos Turi
UTD, USA

(A. Bensoussan and K. Chandrasekaran)

We consider control problems for the variational inequality describing a single degree of freedom elastoplastic oscillator. We are particularly interested in finding the "critical excitation", i.e., the lowest energy input excitation that drives the system between the prescribed initial and final states within a given time span. This is a control problem for a state evolution described by a variational inequality. We obtain Pontryagin's necessary condition of optimality. An essential difficulty lies with the non continuity of adjoint variables.

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Special Session 30: Lagrangian Coherent Structures and Invariant Manifolds: Analysis and Applications

George Haller, McGill University, Canada
Wenbo Tang, Arizona State University, USA

From Stratospheric Winds to Plankton Ecology: Geophysical Transport and Mixing from Lyapunov Exponents and Vectors

Francesco d'Ovidio
Inst. of Complex Systems (ISC-PIF), Paris, France
(E. Hernandez-Garcia, B. Legras, M. Levy, S. De Monte)

The calculation of the local Lyapunov exponents and vectors from a surface velocity is a well-known technique for the heuristic detection of transport barriers as unstable manifolds of embedded hyperbolic points. The application of this technique to geophysical datasets like stratospheric isentropic winds or satellite derived surface currents is an appealing technique for understanding the way in which passive and active tracers are segragated or mixed by their turbulent environment. Analyzing observational datasets, here I will discuss some atmospheric and oceanic questions where the Lyapunov technique can complement other traditional diagnostics. Using transport in the upper troposphere/lower stratosphere region as a case study, I will show how a diagnostic for parameterizing local mixing can be constructed by combining the Lyapunov exponents and vectors with the effective diffusivity. For the ocean, submesoscale information can be extracted with Lyapunov calculations from coarser grain datasets and this information can be used to understand the impact of the ocean turbulence on plankton dynamics. In particular, by quantifying the stirring of surface currents, Lyapunov calculations helps to identify a mechanism by which surface turbulence maintains complex planktonic community structures.

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Lagrangian Coherent Structures in Bio-Inspired Fluids

Melissa Green
Navel Research Laboratory, USA
(Clarence W. Rowley and Alexander J. Smits)

The method of identifying Lagrangian coherent structures (LCS) has proved useful in locating vortices and describing their evolution in three-dimensional aperiodic fluid flows. In the present work, we identify LCS in the wakes of rigid pitching panels with a trapezoidal planform geometry, chosen to model idealized fish caudal fins. The locomotion of fish and aquatic animals is achieved by the oscillation of these fins and flukes, which creates highly three-dimensional unsteady flow fields that are not yet well-understood. Digital particle image velocimetry experiments were performed to obtain the time-resolved velocity fields at two representative Strouhal numbers. A classic reverse von Kármán vortex street pattern was observed along the mid-span of the near-wake at low Strouhal number. However, at higher Strouhal number the complexity of the wake increased downstream of the trailing edge as the spanwise vortices spread transversely across the wake and lost coherence. This wake transition was shown to correspond to a qualitative change in the LCS at the same time and location. In this way, we are able to interrogate vortex formation and interactions within the wake and more fully understand the mechanisms of propulsion used by these animals.

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Lagrangian Coherent Structures, Finite-Time Hyperbolicity and Lyapunov Exponents

George Haller
McGill University, Canada

We review the fundamental physical motivation behind the definition of Lagrangian Coherent Structures (LCS) and show how it leads to the concept of finite-time hyperbolicity in non-autonomous dynamical systems. Using this concept of hyperbolicity, we obtain a self-consistent criterion for the existence of attracting and repelling material surfaces in unsteady fluid flows and in other non-autonomous dynamical systems. We also mention other proposed notions of finite-time hyperbolicity that lead to inconsistent results.

The existence of LCS is often postulated in terms of features of the Finite-Time Lyapunov Exponent (FTLE) field associated with the system. As simple examples show, however, the FTLE field does not necessarily highlight LCS, or may highlight structures that are not LCS. Under appropriate nondegeneracy conditions, we show that ridges of the FTLE field indeed coincide with LCS in volume-preserving flows. For general flows, we obtain a more general scalar field whose ridges correspond to LCS.

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Particle Transport in Unsteady Separated Flow

Gustaaf Jacobs
San Diego State University, USA
(Brian Gaston)

We discuss the dynamics and mixing of particles in separated flows. A numerical study is performed on the particle paths released in a separated cylinder and diffuser flows. Finite sized particles are shown to separate from a fixed location in an unsteady flow, even when the fluid particles' separation location is moving. Fluid particle transport barriers that attract fluid particles are visualized through contours of the Finite Time Lyapunov Exponent (FTLE) that measures the stretching of the fluid. Combinations of the transport barriers form Lagrangian Coherent Structures that are typically visualized through dye in experiments. Inertial particles closely follow these attracting transport barriers, as visualized by maxima in the FTLE field determined in backward time, for Stokes numbers smaller than unity. With increasing Stokes number the particles increasingly align with the transport barriers. At unity Stokes number when particles are well-known to exhibit particle focusing, large parts of a sharp particle streak are exactly aligned with the transport barrier in the FTLE field. The attracting transport barriers in the wake are hence excellent predictors of the location where particles focus. The inertial particles are shown to depart from the fluid particle transport barrier when criteria based on the rate of strain tensor are met.

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A Laboratory Study of Unsteady Flow Separation

Thomas Peacock
MIT, USA
(M. Weldon, G. Jacobs, M. Helu, G. Haller)

Unsteady separation is the phenomenon in which a spiked jet of fluid is ejected from a rigid boundary, resulting in significant alteration of the surrounding unsteady fluid motion. This is currently a limiting factor in the performance of most transport vehicles. Although separation is understood in steady flows, establishing a practical and widely accepted criterion for unsteady separation has proven elusive. This laboratory study further establishes the credibility of new criterion based on ideas concerning Lagrangian Coherent Structures. The study involves a dedicated series of laboratory experiments and coordinated comparisons with a numerical simulation. Perhaps most remarkably, the experiments clearly demonstrate that separation at a fixed location can occur even in a random experimental flow field that is kinematically equivalent to the conditions within a turbulent boundary layer.

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Lagrangian Coherent Structures in Scientific Visualization

Ronald Peikert
ETH Zurich, Switzerland

The goal of scientific visualization is to develop computer graphics based methods for understanding scientific data. In the case of (numerical) flow visualization, such data can arise either from simulation (computational fluid dynamics) or from measurement techniques such as particle image velocimetry. A challenge in visualization is to reveal those structures in time-dependent 3D velocity fields that are of importance in a particular application. For example, an engineer might want to study the creation of a particular vortex in a hydraulic machine. For the visualization scientist, this translates into extracting and tracking a feature from a stack of numerical simulation results. A popular approach has been to apply dynamical systems theory to the instantaneous velocity fields of each time step. A topological skeleton is thus computed per time step and its evolution over time is visualized, with the possibility to detect bifurcation events. However, in recent years, the visualization community became increasingly aware that this approach can lead to misleading results in the case of strongly unsteady flow. Consequently, interest has shifted from topological skeletons to Lagrangian coherent structures that more adequately describe the behavior of unsteady flow. When adapting this theory to scientific visualization, typical problems are how to extract geometric objects that can be rendered, the question about the most appropriate definition of a ridge or a ridge surface, or how to exploit temporal coherence to efficiently compute such structures. In my talk I will give an overview of the recent developments in this field and point out open problems.

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Lagrangian Coherent Structures in Cardiovascular Applications

Shawn Shadden
Illinois Institute of Technology, USA

This talk will overview work that has been done on applying the computation of Lagrangian Coherent Structures (LCS) to study transport related processes in the cardiovascular system. The motivation for this work is direct relationship between large and small scale blood transport processes and the regulation of cardiovascular health. Enabling a better

characterization of blood flow mechanics (hemodynamics) helps: to better understand biomechanical mechanisms of disease progression and health regulation, to diagnose adverse hemodynamic conditions and associated health risk, and planning surgical repair and device design. We will discuss applications related to each of these issues.

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Lagrangian Coherent Structure Extraction in Finite Domains

Wenbo Tang
Arizona State University, USA
(Pak Wai Chan, George Haller)

We develop a finite-domain finite-time Lyapunov exponent (FDFTLE) method to allow Lagrangian Coherent Structure (LCS) extraction from velocity data within limited domains. This removes spurious ridges as seen when trajectories are stopped at the domain boundaries. We find this extension useful in practical applications when LCS are extracted from LIDAR measurements at Hong Kong International Airport and used to determine airflow patterns around the airport. In addition to the FDFTLE method, we have developed a suite of mathematical tools to quantify different types of air motion near the LCS. This allows us to objectively describe the relative motion near LCS.

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Visual Analysis of Lagrangian Coherent Structures in Large-Scale Computational Fluid Dynamics Simulations

Xavier Tricoche
Computer Science, Purdue University, USA

The rapidly increasing size and complexity of computational simulations in science and industry put an enormous demand on post-processing tools to gain insight from the resulting deluge of data. A major research effort in Scientific Visualization strives to address this need through effective and interactive visual representations that convey the salient patterns present in the data. However, finding interesting features both reliably and efficiently remains a challenge and existing methods typically rely on ad-hoc criteria and heuristic algorithms to detect them.

In the context of computational fluid dynamics simulations, the concept of lagrangian coherent structures (LCS) and their differential-geometric characterization through the finite-time Lyapunov exponent (FTLE) are starting to gain popularity in the visualization community. This growing interest stems from the ability of LCS to objectively characterize remarkable structures as well as from the compelling visualizations that they create. This talk discusses the challenges and opportunities associated with the LCS approach in flow visualization. In particular, we will present our ongoing work on the efficient computation of FTLE and LCS in large-scale computational meshes and a novel approach that lets domain experts interactively explore the structural contents of their data across spatial and temporal scales.

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Special Session 31: Emergence and Dynamics of Patterns in Nonlinear Partial Differential Equations from Mathematical Science

Danielle Hilhorst, Universite Paris-Sud, France
Yoshihisa Morita, Ryukoku University, Japan
Kunimochi Sakamoto, Hiroshima University, Japan

Introduction: There are many nonlinear PDEs (partial differential equations) whose solution structures reveal the emergence of interesting patterns and the evolution of patterns. Such nonlinear-PDE models come from various fields of mathematical science, including material-science, life and biological sciences. In this session, we bring together recent studies on solution-structures of nonlinear PDEs related to pattern formation, dynamics, and bifurcations, presenting new aspects of solutions capturing the nonlinear phenomena together with underlying structures of solutions.

On Some Anisotropic Singular Perturbation Problems

Michel Chipot
University of Zurich, Switzerland

We consider the asymptotic behaviour of an anisotropic parabolic singular perturbations problem. The limit problem is an exotic nonlinear problem. (Joint work with S. Guesmia).

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On Interfacial Energy in a Phase-Transition Model

Elaine Crooks
Swansea University, Wales
(John Ball)

A simple way of incorporating interfacial energy into variational solid-solid phase transition models is via a perturbation of the elastic energy functional involving second gradients of the deformation. Such perturbations are well-known to yield the benefit over purely elastic energy minimization of predicting a length-scale for fine-scale mixtures of phases. This talk will discuss further consequences of such higher-gradient perturbations, for interfaces between gradients lying in two distinct energy wells, and for local minimizers.

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Equivariant Entire Solutions to Elliptic Systems with Variational Structure

Giorgio Fusco
University of L'Aquila, Italy
(N. D. Alikakos)

We consider the elliptic system

$$\Delta u = W_u(u), \quad x \in \mathcal{R}^n$$

for a class of potentials $W : \mathcal{R}^n \to \mathcal{R}$ that possess several global minima and are invariant under a finite reflection group G acting on $\mathcal{R}^n$. We establish existence of nontrivial G-equivariant entire solutions $u : \mathcal{R}^n \to \mathcal{R}^n$ connecting the global minima of W.

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Traveling Waves of Two-Component Reaction-Diffusion Systems Arising from Higher Order Autocatalytic Models

Jong-Shenq Guo
National Taiwan Normal University, Taiwan
(Je-Chiang Tsai)

We study the existence and uniqueness of traveling wave solutions for a class of two-component reaction diffusion systems with one species being immobile. Such a system has a variety of applications in epidemiology, bio-reactor model, and isothermal autocatalytic chemical reaction systems. We establish the existence and uniqueness of traveling wave solutions to the reaction-diffusion system for an isothermal autocatalytic chemical reaction of any order in which the autocatalyst is assumed to decay to the inert product at a rate of the same order.

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Spreading Speeds of Solutions of Reaction-Diffusion Equations

Francois Hamel
Aix-Marseille University, France

A fundamental aspect which accounted for the success of reaction-diffusion models is concerned with the descprition of spreading phenomena at large times in unbounded domains. Estimating the spreading speeds is one of the most important issues from a theoretical point of view as well as for the applications in biology and ecology. Usually, the solutions spread at a finite speed which is uniquely determined from the initial data. In this talk, I will report on some recent results obtained in collaboration with G. Nadin, L. Roques and Y. Sire which show that, even for the simplest reaction-diffusion models, propagation with a finite and uniquely determined speed may fail. Examples of propagation with an infinite speed and complex situations with non-trivial intervals of spreading speeds will be exhibited.

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Traveling Front Dynamics in Some Heterogeneous Diffusive Media

Hideo Ikeda
University of Toyama, Japan

We consider two component reaction-diffusion systems with a bistable and odd symmetric nonlinearity, which have the bifurcation structure of pitchfork-type traveling front solutions with opposite velocities. We introduce a spatial heterogeneity, for example, a Heaviside-like abrupt change at the origin in the space, into diffusion coefficients. Numerically, the responses of traveling fronts via the heterogeneity can be classified into four types of behaviors depending on the strength of the heterogeneity. The goal of this paper is to reduce the PDE dynamics to finite-dimensional ODE systems on a center manifold and show the mathematical mechanism to produce the four types of responses in the PDE systems using finite-dimensional ODE systems. This is a joint work with Shin-Ichro Ei.

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Approximation of Nonlinear Diffusion Systems by Reaction-Diffusion Systems

Hirofumi Izuhara
Université Paris-Sud 11, France

In this talk we deal with the relation between nonlinear diffusion systems and reaction-diffusion systems. It is known that a cross-diffusion system

can be approximated by an reaction-diffusion system with switching mechanism. We focus on the application of the reaction-diffusion systems with switching mechanism.

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On the Study of a Generalized Cahn-Hilliard Equation with Forcing and Stochastic Terms

Georgia Karali
University of Crete, Greece

We consider the Cahn-Hilliard equation on a bounded domain and introduce forcing terms of general type. We derive asymptotic results, the sharp interface limit in the multidimensional case and a linear diffusion equation in the one- dimensional case with noise-like non homogeneous term by making use of the Brownian scale. We also study existence for a generalized stochastic Cahn-Hilliard equation posed on bounded convex domains with additive space-time noise and a time noise term into the chemical potential.

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Singular Limit of a Haptotaxis Model with Bistable Growth

Elisabeth Logak
University of Cergy-Pontoise, France
(Chao Wang)

We consider a model for haptotaxis with bistable growth and study its singular limit. Haptotaxis refers to directed motion of cancer cells towards higher concentrations of some proteins located in the Extra-Cellular Matrix that do not diffuse. This mechanism plays a role in tumor invasion. We first give a short review of the literature concerning haptotaxis modelling. Next we establish that the singular limit of our model yields an interface motion where the normal velocity of the interface depends on the mean curvature and on some nonlocal haptotaxis term. We prove the result for general initial data after establishing a result about generation of interface in small time.

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Asymptotic Behaviors of Solutions to Evolution Equations in the Presence of Symmetries

Yasunori Maekawa
Kobe University, Japan
(Yoshiyuki Kagei, Kyushu University)

There are wide classes of linear or nonlinear evolution equations which possess invariant properties with respect to a scaling and translations. If a solution is invariant under the scaling then it is called a self-similar solution, which is a candidate for the asymptotic profile of general solutions at large time. In this talk we introduce an abstract method to improve the asymptotic profile by finding a suitable shift of self-similar solutions when the scaling and translations are suitably connected.

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Spike-Type Patterns in Degenerate Reaction-Diffusion Models of Proliferative Cell Populations

Anna Marciniak-Czochra
University of Heidelberg, Germany

In this talk we will explore a mechanism of pattern formation arising in the processes described by a system of a single reaction-diffusion equation couples with ordinary differential equations. Such models are very different from classical Turing-type models and the spatial structure of the pattern emerging from the destabilisation of the spatially homogeneous steady state cannot be concluded based on linear stability analysis. The models exhibit qualitatively new patterns of behaviour of solutions, including, in some cases, a strong dependence of the emerging pattern on initial conditions and quasi-stability followed by rapid growth of solutions. In numerical simulations, solutions having the form of periodic or irregular spikes are observed. Recently we have proposed models of spatially-distributed growth of clonal populations of pre-cancerous cells, which remained under control of endogenous or exogenous growth factors diffusing in the extracellular medium and binding to the cell surface. We found conditions for emergence of growth patterns, which took the form of spike-type spatially inhomogeneous steady states. This multifocality is as expected from the field theory of carcinogenesis. In this talk we approach the question of stability of spike solutions, which is essential for their observability in experiments. We show existence and stability of spatially inhomogeneous stationary solution of periodic type, the maxima and minima of which may be of the spike or plateau type.

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Refined Singular Limit of the Allen-Cahn Equation

Hiroshi Matano
University of Tokyo, Japan

It is known that the singular limit of the Allen-Cahn equation (with ε^{-2} being the coefficient of the nonlinearity) is the mean curvature flow. Intuitively,

this can be shown by a formal asymptotic expansion of the solution near the interface. And this formal asymptotic expansion has been successfully used to determine rigorously the position (hence the motion) of the limit interface. However, it was not known if the actual solution (for ε small) really possesses the nice profile predicted by the formal asymptotics. The only results known so far are for solutions with very special intial data. For more general solutions, the validity of even the first term of the asymptotic expansion has not been known. In this talk I will prove the validity of the first term for a rather general class of initial data. I will also present other recent results concerning the relation between the Allen-Cahn equation and the motion by mean curvature.

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Stability and Bifurcation of Solutions to a Mass-Conserved Reaction-Diffusion System

Yoshihisa Morita
Ryukoku University, Japan

We consider a reaction-diffusion system with conservation of a mass. In this system the stationary equations of the system are reduced to a scalar stationary equation with a nonlocal term. Namely, there is a one-to-one correspondence between equilibrium solutions of the system with a given mass and the scalar equation with the nonlocal term. As for the stability of equilibrium solutions, under some condition for the nonlinear term, we show that if the scalar equation allows a nondegenerate stable solution, then the system has a stable solution corresponding to the scalar one. On the other hand, we give a specific example having a spatially nonuniform periodic solution, which never arises in the scalar equation. We exhibit global bifurcation diagrams for this specific model by numerics.

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A Relation between Reaction-Diffusion Interaction and Nonlinear Diffusion

Hideki Murakawa
University of Toyama, Japan

This talk deals with nonlinear diffusion problems including degenerate parabolic problems, such as the Stefan problems and the porous medium equations, and cross-diffusion systems, such as the Shigesada-Kawasaki-Teramoto model in population ecology. We show that the solutions of the nonlinear diffusion problems can be approximated by those of semilinear reaction-diffusion systems. This indicates that the mechanism of nonlinear-diffusion might be captured by reaction-diffusion interaction.

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Upper Bounds for Coarsening for the Cahn-Hilliard Equation and Deep Quench Obstacle Problem

Amy Novick-Cohen
Technion-IIT, Haifa, Israel

(Andrey Shishkov)

In 2001, Kohn and Otto rigorously demonstrated the validity of coarsening upper bounds at large times of the form $t^{1/4}$ for the degenerate deep quench obstacle problem and of the form $t^{1/3}$ for the constant mobility Cahn-Hilliard equation. Similar results were demonstrated in 2009 by NC & Shishkov for to hold for all the degenerate Cahn-Hilliard equation, at arbitrary temperatures and concentrations. Moreover, the possibility of upper coarsening bounds was demonstrated also for all times $t \geq 0$. Recently, the possibility of transitions between the various coarsening predictions during the coarsening process has been investigated, and considerable insight has been gained with regard to the early time coarsening bounds for the deep quench obstacle problem.

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Mathematical Analysis to Liesegang Phenomena

Isamu Ohnishi
Hiroshima University, Japan

(Rein van der Hout, Danielle Hilhorst, Masayasu Mimura)

Our purpose is to start understanding from a mathematical viewpoint experiments in which regularized structures with spatially distinct bands and rings of precipitated material are exhibited, with clearly visible scaling properties. Such patterns are known as Liesegang bands or rings. In this paper, we study a one-dimensional version of the Keller and Rubinow model and present conditions insuring the existence of Liesegang bands. This is a joint work with Rein van der Hout, Danielle, Hilhorst, Masayasu Mimura.

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Symmetry Properties of Nonnegative Solutions of Elliptic Equations

Peter Polacik
University of Minnesota, USA

We consider nonnegative solutions of the Dirichlet problem for fully nonlinear elliptic equations on reflectionally symmetric domains. We show that such solutions do have certain symmetry properties although their symmetry pattern may be more complicated than that of positive solutions.

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Boundedness of Solutions of the Haptotaxis Model of Cancer Invasion

Mariya Ptashnyk
RWTH Aachen University, Germany
(Anna Marciniak-Czochra)

The two key factors governing migration of cancer cells during invasion are random motion and haptotaxis, i.e. the movement of cells follows the gradient of the nondiffusible molecules, degraded by cells. We prove that due to the structure of the kinetics system, the solutions of the haptotaxis model are uniformly bounded and exist globally for arbitrary non-negative initial conditions. The uniform boundedness is obtained by showing a priori estimates for the supremum norm and applying the method of bounded invariant rectangles to the reformulated system of reaction-diffusion equations in divergence form with a diagonal diffusion matrix. The analysis of the model shows also how the structure of kinetics of the model is related to the growth properties of the solutions and how this growth depends on the ratio of the sensitivity function (describing the size of haptotaxis) and the diffusion coefficient. One of the implications of our analysis is that in the haptotaxis model with a logistic growth term, cell density may exceed the carrying capacity, which is impossible in the classical logistic equation and its reaction-diffusion extension.

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A Comparison Principle for Hamilton-Jacobi Equations with Discontinuous Hamiltonian

Piotr Rybka
University of Warsaw, Poland
(Y. Giga, P. Górka)

In our problem the Hamiltonian depends upon the unknown function. More importantly, it is discontinuous in the time and space variables, to be precise, it has a jump across a Lipschitz curve. This kind of problem is rarely seen in the literature. The situation which we consider may arise when a driven singular weighted mean curvature flow governing evolution of a graph degenerates to a Hamilton-Jacobi equation.

We show a Comparison Principle when the supersolution and subsolution are bounded and uniformly continuous over the real line and in addition the supersolution is a piecewise C^1-function. The method of analysis is based upon a shift of the supersolution and the concept of a strict supersolution.

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Diffusion Systems with Interacting Flux on the Boundary

Kunimochi Sakamoto
Hiroshima University, Japan

Systems to be considered in this talk consist of diffusion equations for several species of reactants in a bounded domain and interactions among them on the domain boundary. The existence, stability and bifurcation of stationary solutions are studied as the interaction rates, as well as diffusion rates, are varied. Similarities and dissimilarities to the usual reaction-diffusion systems are emphasized.

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Emergence of Dissipation in the Dynamics of Elastic Phase Transitions – Travelling Waves in Lattice Models

Hartmut Schwetlick
University of Bath, England
(Johannes Zimmer)

We study the existence of travelling wave solution for a Fermi-Pasta-Ulam chain. Motivated by martensitic phase transitions the elastic interaction energy is assumed to be a multi-well potential. We focus on the special case where the potential is piecewise quadratic, with two wells representing two stable phases. In the physically interesting regime of subsonic speeds there exist phase transition solutions, that is, 'heteroclinic' travelling waves connecting the two distinguished stable wells. However, these waves do necessarily leave behind a non-decaying oscillating pattern. We deduce important information on the macroscopic dissipation of such transitions, namely, the kinetic relations governing the dependence of the configurational force on the speed of the moving interface. Furthermore, we discuss the non-existence of waves with constant tails.

This is joint work with Johannes Zimmer (Bath).

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Asymptotic Behaviors of Solutions in Multistable Reaction-Diffusion Equations

Masaharu Taniguchi
Tokyo Institute of Technology, Japan

In this paper I study asymptotic behaviors of solutions in multistable reaction-diffusion equations by assuming initial states are monotone. I study a condition on existence of a traveling front in multistable reaction-diffusion equations.

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Nonlocal Reaction-Diffusion Equations in Population Dynamics

Vitaly Volpert
CNRS, University Lyon 1, France

We will discuss recent result on integro-differential equations describing some models in population dynamics with nonlocal consumption of resources. Emergence of spatio-temporal patterns and propagation of waves will be presented from the point of view of related mathematical questions and biological applications.

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Global Bifurcation Structure on a Shadow System to the Gierer-Meinhart System with a Source Term

Shoji Yotsutani
Ryukoku University, Japan
(Hideaki Takaichi and Izumi Takagi)

As the first step to understand the Gierer-Meinhardt system with a source term, it is important to know the global bifurcation structure of a shadow system. There is a partial result due to I. Takagi (J. D. E.61, 1986). In this talk, we show the global bifurcation structure of a stationary problem.

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Special Session 32: Evolutionary Partial Differential Equations: Theory and Applications

Rainer Picard, TU Dresden, Germany
Michael Reissig, Technical University Bergakademie Freiberg, Germany
Karen Yagdjian, University of Texas-Pan American, USA

Introduction: The session is focused on new developments in the area of well-posedness, asymptotic profile, blow-up phenomena of systems described by evolutionary partial differential equations. These include: linear and non-linear wave models with mass or dissipation, hyperbolic systems, parabolic systems, Schrödinger equations, thermo-elastic and visco-elastic models, electromagnetic theory, Stokes and Navier-Stokes equations. These models have a wide range of applications that include wave propagation, fluid dynamics, theory of elasticity, engineering. The main goal of this session is to present state-of-the-art of results, to discuss further developments and open problems.

Energy Decay of Solutions to the Cauchy Problem for a Nondissipative Wave Equation

Benaissa Abbes
University of Sidi Bel Abbes, Algeria

In this paper we consider energy decay for nondissipative wave equation in unbounded domain (the usual energy is not decreasing). We use an approach introduced by Guesmia which leads to decay estimates (known in the dissipative case) when the integral inequalities method due to Komornik cannot be applied due to the lack of dissipativity. First we study the stability of a wave equation with a weak nonlinear dissipative term based on the equation

$$u'' - \Delta_x u + \lambda^2(x)u + \sigma(t)g(u') + \vartheta(t)h(\nabla_x u) = 0$$

in $\mathbb{R}^n$. We consider the general case with a function h satisfying a smallness condition, and we obtain decay of solutions under weak growth assumptions on the feedback function and without any control of the sign of the derivative of the energy related with the above equation.

In the second case we consider the case $h(\nabla u) = -\nabla\Phi\nabla u$. We prove some precise decay estimates of equivalent energy.

Key words and phrases: Stabilizability by a nonlinear feedback, Wave equation, Integral inequality. $\mathcal{AMS}$ Classification: 35L80, 35B40.

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Modulus of Continuity and Decay at Infinity in the Cauchy Problem for Evolution Equations with Real Characteristics

Massimo Cicognani
University of Bologna, Italy

It is well known that the modulus of continuity of the coefficients plays an essential role in the hyperbolic Cauchy problem. In particular, well-posedness in Sobolev spaces may fail with less than Lipschitz continuous coefficients in the time variable. We deal with non-kovalevskian equations of hyperbolic type, like the Euler-Bernoulli vibrating beam model, and show how a lack of regularity in time may be compensated by a decay condition as the space variable tends to infinity.

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Long Time Asymptotics for 2 by 2 Hyperbolic Systems

Marcello D'Abbicco
University of Bari, Italy
(M. Reissig)

The goal of this talk is to present some results about the behavior of the energy for 2 by 2 strictly hyperbolic systems. On the one hand we are interested in generalized energy conservation which excludes blow-up and decay of the energy for $t \to \infty$. On the other hand we present scattering results which take account of terms of order zero.

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Dissipation in Viscous and Inviscid Quantum Hydrodynamics

Michael Dreher
University of Konstanz, Germany

First we give a brief review on the theory of mixed-order parameter-elliptic systems as introduced by Agmon, Douglis, Nirenberg, and also Volevich, Agranovich, Grubb. Then we apply some results of this theory to a class of systems of quantum hydrodynamics. By a special diagonalization technique, we finally present a method of how to prove dissipativity for such systems.

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Similarity Solutions for the Semilinear Tricomi-Type Equations

Anahit Galstyan
University of Texas-Pan American, USA

In this talk we will discuss the existence of the similarity solutions for the semilinear weakly hyperbolic equations, appearing in the boundary value problems of gas dynamics. The necessary condition for the existence of the similarity solutions for the one-dimensional semilinear Tricomi-type equation will be presented.

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Hyperbolic Systems in $\mathbb{R}^n$ with Irregular Coefficients in t and Characteristics Admitting Superlinear Growth for $|x| \to \infty$

Todor Gramchev
Università di Cagliari, Italy

We study the well posedness of the Cauchy problem for diagonalizable hyperbolic systems of (pseudo)differential equations with characteristics which are not Lipschitz with respect to both the time variable t (locally) and the space variables $x \in \mathbb{R}^n$ for $|x| \to \infty$. We introduce sharp conditions guaranteeeing the well-posedness in scales of weighted Sobolev spaces in $\mathbb{R}^n$. The results are obtained in collaboration with M. Cappiello (Università di Torino) and D. Gourdin (Université de Paris 6).

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On the Well-Posedness of the CH Equation

Alex Himonas
University of Notre Dame, USA

We shall show that the solution map of the periodic CH equation is not uniformly continuous in Sobolev spaces with exponent greater than 3/2. This extends earlier results to the whole range of Sobolev exponents for which local well-posedness of CH is known. The crucial technical tools used in the proof of this result are a sharp commutator estimate and a multiplier estimate in Sobolev spaces of negative index. Also, we shall discuss the analogous result in the non-periodic case. This talk is based on work in collaboration with Carlos Kenig and Gerard Misiolek.

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On Bean's Critical State Model for Type-II Superconductors

Frank Jochmann
TU-Berlin, Germany

This talk is concerned with Bean's critical state model for the description of the electromagnetic field in type-II superconductors where the displacement current is not neglected at least in the surrounding insulating medium. The studied system consist of Maxwell's equations where the electric field and the current density satisfy a nonlinear current-voltage relation. For the purpose of generality, not only Maxwell's equations but a wider class of first-order systems is considered which includes the scalar wave equation with generally nonlinear multivalued damping. The large-time asymptotic behavior of the solutions is discussed.

Furthermore, the case where the displacement current is neglected on a certain subset of the spatial domain is considered. This mixed-type problem leads to an evolutionary variational inequality that has a unique solution under suitable initial-boundary conditions. It is also shown that the solution to this evolutionary variational inequality can be obtained from a singular limit in Maxwell's equations with a nonlinear voltage-current relation in which the dielectric permittivity tends to zero on the subset.

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Sharp Well-Posedness for the Hyperelastic Rod Equation

David Karapetyan
University of Notre Dame, USA

It is shown that the solution map for the hyperelastic rod equation is not uniformly continuous on bounded sets of Sobolev spaces with exponent greater than 3/2 in the periodic case and greater than 1 in the non-periodic case. The proof is based on the method of approximate solutions and well-posedness estimates for the solution and its lifespan.

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Einstein-Euler Systems with Polytropic Equations of State

Lavi Karp
ORT Braude College, Israel
(Uwe Brauer)

Einstein-Euler systems describe relativistic self-gravitating perfect fluids. Our current research deals with is the Cauchy problem for Einstein gravitational fields equations coupled with a perfect fluid. These systems are closed by an equation of state expressing the relations between the pressure, energy density and number density.

A common feature for polytropic equations of state is the fractional power of either the energy density, or the number density. Densities in isolated gravitational systems in general relativity vanish at certain regions or near infinity, therefore the fractional power causes serious technical difficulties. The talk will mainly focus on two issues: the reduction of the Euler equations to a first order symmetric

hyperbolic system; the compatibility problem of between the initial data of the matter variables of the gravitational fields and the data of the fluid. We will also discuss the affect of the fractional power on the regularity.

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Decay Estimates for Quasilinear Dissipative Hyperbolic Equations

Albert Milani
UWM, USA & TU Dresden, Germany
(Pascal Cherrier)

We establish decay estimates as $t \to \infty$ to strong solutions of the quasilinear hyperbolic dissipative evolution equation

$$u_{tt} + u_t - a_{ij}(\nabla u)\partial_i\partial_j u = 0.$$

These estimates are similar to those established by Matsumura for the linear case, and may help to study the related diffusion phenomenon of hyperbolic waves.

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Analytic Smoothing Effect for Nonlinear Schrödinger Equations

Tohru Ozawa
Waseda University, Japan

Analytic smoothing effect of solutions to the Cauchy problem for nonlinear Schrödinger equations in two space dimensions is discussed, where the nonlinearity vanishes of order three at the origin and the Cauchy data decay exponentially at infinity.

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Hodge-Helmholtz Decompositions of Weighted Sobolev Spaces

Dirk Pauly
University of Duisburg-Essen, Germany

We study Hodge-Helmholtz decompositions in nonsmooth exterior domains filled with inhomogeneous and anisotropic media. We show decompositions of alternating differential forms belonging to weighted Sobolev-spaces into irrotational and solenoidal forms. These decompositions are essential tools, for example, in electro-magnetic theory for exterior domains, in particular, to describe the low frequency asymptotic of time-harmonic electromagnetic fields properly.

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Flow Invariance for Nonlinear Partial Differential Delay Equations

Wolfgang Ruess
Universität Duisburg-Essen, Germany

A survey of results on existence, flow-invariance, regularity, and linearized stability of solutions to nonlinear partial differential delay equations of the form $\dot{u}(t) + Bu(t) \ni F(u_t)$, $t \geq 0$, $u_0 = \varphi$, with $B \subset X \times X$ ω-accretive, X a Banach space, is presented. The existence result solves the open problem of a subtangential condition for flow-invariance in the fully nonlinear case, including those known for the cases of no delay, and of ordinary delay equations. It is applied to place several classes of population models in their natural L^1-setting.

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A Hyperbolic Fluid Model Based on Cattaneo's Law

Jürgen Saal
University of Konstanz, Germany
(Reinhard Racke)

In various applications a delay of the propagation speed of certain quantities (temperature, fluid velocity, ...) has been observed. Such phenomena cannot be described by standard parabolic models, whose derivation relies on Fourier's law (Paradoxon of infinite propagation speed). One way to give account to these observations and which was successfully applied to several models, is to replace Fourier's law by the law of Cattaneo. In the case of a fluid, this leads to a hyperbolicly perturbed quasilinear Navier-Stokes system for which local and global existence and uniqueness results will be presented. This is a joint project with Reinhard Racke at the University of Konstanz.

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Propagation of Analyticity for Hyperbolic Systems in Several Space Dimensions

Sergio Spagnolo
University of Pisa, Italy

Since the pioneering paper of Lax of 1953, the propagation of analyticity (PA) for solutions to nonlinear equations with (possibly multiple) real characteristics, has been deeply investigated by many authors. Recently, in collaboration with G. Taglialatela, we stated the validity of the property (PA) for those solutions which are a priori bounded in a Gevrey class with order smaller than the Bronshtein bound, limited to systems in one space variables. Here we want to show how a suitable employment of the pseudodifferential techniques allows us to extend such a

result to the multidimensional case.

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On the Cauchy Problem for Noneffectively Hyperbolic Operators in the Gevrey Classes

Nishitani Tatsuo
Osaka University, Japan

We consider the Cauchy problem for noneffectively hyperbolic operators in the Gevrey classes. We discuss about the maximal Gevrey index s^* such that the Cauchy problem for noneffectively hyperbolic operators is well-posed in the Gevrey class $1 \leq s < s^*$.

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On the Cauchy Problem for Second-Order Hyperbolic Operators with the Coefficients of Their Principal Parts Depending Only on the Time Variable

Seiichiro Wakabayashi
University of Tsukuba, Japan

We consider the Cauchy problem for second-order hyperbolic operators with the coefficients of their principal parts depending only on the time variable in the C^∞ category. In the case where the coefficients are real analytic, we give a sufficient condition for C^∞ well-posedness, which is also a necessary one when the space dimension is less than 3 or the coefficients are semi-algebraic functions (e.g. polynomials) of the time variable. To prove “Sufficiency” we derive energy estimates, and to prove “Necessity” we use Ivrii-Petkov’s arguments.

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Dispersive Estimates for Hyperbolic Systems with Time-Dependent Coefficients

Jens Wirth
Imperial College London, England
(M. Ruzhansky)

The long-time behaviour of hyperbolic systems is studied based on diagonalisation of the full symbol of the operator within adapted symbol classes and resulting representations of solutions in terms of Fourier integrals. Dispersive estimates for solutions are derived from these Fourier integrals using a multi-dimensional van der Corput lemma assuming that the slowness surfaces associated to the phase functions are asymptotically convex for large time.

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On the Global Solutions of the Semilinear Hyperbolic Equations in the Friedmann-Robertson-Walker Spacetimes

Karen Yagdjian
University of Texas-Pan American, USA

We shall describe some qualitative behavior of the global solutions of the second order semilinear hyperbolic equations. In particular, we shall present some results for the global in time solutions of the equation for the Higgs boson field in the Friedmann-Robertson-Walker spacetimes in the case of sign-preserving total self-interaction function. For the global solutions of the last equation we shall formulate sufficient conditions for the creation of bubbles.

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Special Session 33: Operator Semigroups and Applications

Horst Thieme, Arizona State University, USA
Jürgen Voigt, Technische Universität Dresden, Germany

Introduction: Operator Semigroups are an effective concept to formulate and analyze a large variety of linear infinite-dimensional problems in partial, functional, and integral differential equations (and their combinations) and in their ubiquitous application in the natural sciences and engineering. They are also a stepping stone for handling semi- and quasilinear equations. It is one of the aims of this section to foster a fruitful exchange between theory and applications.

A Nonlinear Evolutionary Game Replicator-Mutator Model on the Space of Measures

Azmy Ackleh
University of Louisiana at Lafayette, USA
(John Cleveland)

A general nonlinear replicator-mutator model is formulated as a dynamical system on the state space of finite signed measure. Existence-uniqueness of solutions are first established using a fixed point argument. It is also proved that the total population is permanent and that for pure replicator dynamics the solution converges to a Dirac measure centered at the fittest strategy; thus this Dirac measure is a globally attractive equilibrium point which is termed a continuously stable strategy (CSS). Finally, it is shown that in the discrete case for pure replicator dynamics and even for small perturbation of pure replicator dynamics a globally asymptotically stable equilibrium exists.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Explicit Transport Semigroups Associated to Boundary Operators of Norm One

Luisa Arlotti
Universitá degli Studi di Udine, Italy

Consider, in the L^1 setting, transport equations with general external fields associated to a positive boundary operator H of norm one. It has been shown in a previous paper that an extension $\mathcal{H}$ of operator H exists such that the transport operator $\mathcal{T}_{\mathcal{H}}$ is the generator of a contraction semigroup $(V(t); t \geq 0)$. In this paper $(V(t); t \geq 0)$ is explicitly written and its expression is used in order to study the honesty of $(V(t); t \geq 0)$ itself. The developed theory is fully similar to the honesty theory of perturbed substochastic semigroups.

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Long Time Behaviour of Discrete Fragmentation – from Exponential Stability to Chaos

Jacek Banasiak
University of KwaZulu-Natal, So Africa

The fragmentation equation describes splitting of large clusters into smaller ones and, in the discrete case, is given by a Kolmogorov system with an infinite unbounded upper triangular matrix of coefficients. Intuitively, as time increases one expect all clusters to disintegrate with all mass of the system concentrated in particles of the smallest size. However, due to the fact that the system is reducible, the first proof of this fact is surprisingly involved, [1]. In this talk we give a simpler proof based on the theory of substochastic semigroups. Interestingly, the convergence to equilibrium is not always exponential. We shall show a large class of models for which the fragmentation semigroup is analytic and compact, yielding an exponential decay but also present an example of a natural fragmentation semigroup which is neither compact nor analytic and which converges no the equilibrium at a nonexponential rate. Moreover, we shall shaw that, upon occurrence of some growth of clusters due to e.g. attracting particles from the surrounding environment or internal division, the semigroup becomes chaotic.

This work is partly based on joined work with W. Lamb and M. Langer.

[1] J. Carr and F. da Costa, Asymptotic Behavior of Solutions to the Coagulation-Fragmentation Equations. II. Weak Fragmentation, *Journal of Statistical Physics*, Vol. 77, Nos. 1/2, 1994.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Operator Splitting and Product Formulas

András Bátkai
Universität Siegen, Germany

Operator splitting is a widely used method of numerical analysis based on the Lie-Trotter product formula

$$\left(e^{\frac{t}{n}A}e^{\frac{t}{n}B}\right)^n \to e^{t(A+B)}.$$

The idea is to decompose complicated operators into simpler ones and then use this to approximate the solutions of the corresponding Cauchy problem. In the talk we present a general abstract framework and present a list of examples (e.g., nonautonomous equations, abstract boundary value problems, inhomogeneous problems, semigroups associated to admissible control and observation systems, etc.) where the abstract theory can be applied.

Joint work with P. Csomós, K.-J. Engel, B. Farkas and G. Nickel. Supported by the Alexander von Humboldt-Stiftung.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Lord Kelvin's Method of Images in the Semigroup Theory

Adam Bobrowski
Lublin University of Technology, Poland

We show that the Lord Kelvin's method of images is a way to prove generation theorems for semigroups of operators. To this end we exhibit three examples: a more direct semigroup-theoretic treatment of abstract delay differential equations, a new derivation of the form of the McKendrick semigroup, and a generation theorem for a semigroup describing kinase activity in the recent model of Kaźmierczak and Lipniacki.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Douglis-Nirenberg Systems and Analytic Semigroups

Michael Dreher
University of Konstanz, Germany
(Robert Denk)

We study mixed order parameter-elliptic boundary value problems with boundary conditions of a certain structure. For such operators, we prove the analyticity of the semigroup via resolvent estimates in L^p based Sobolev spaces of suitable order; and we present an application of this theory to studies of the particle transport in a semi-conductor.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Semigroup Analysis of a Model for Structured Populations

Jozsef Farkas
University of Stirling, Scotland
(D. Green and P. Hinow)

We consider a class of physiologically structured population models, where individuals may be recruited into the population with distributed states at birth. The mathematical model which describes the evolution of such a population is a first-order nonlinear partial integro-differential equation of hyperbolic type. We use positive perturbation arguments and utilise results from the spectral theory of semigroups to establish conditions for the existence of a positive equilibrium solution of the model. We also investigate the stability of equilibrium solutions using positive and compact perturbations of the linearised semigroup generator.

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Singular Integrals for Operator Semigroups

Markus Haase
Delft University of Technology, Netherlands

Singular integrals involving strongly continuous groups are at the heart of many important questions in evolution equations. For instance, the maximal regularity property, important for the treatment of quasilinear parabolic problems, is connected to the singular integral

$$\mathrm{PV} - \int_{-1}^{1} \frac{U(s)x}{s}\, ds \qquad (x \in X),$$

where $(U(s))_{s\in\mathbb{R}}$ is a C_0-group on a Banach space X. By a so-called transference principle one can reduce the convergence of such a singular integral to a Fourier multiplier problem, solvable under appropriate conditions on the Banach space.

All known results so far were based on the presence of a group and no genuine semigroup approach was known. But recently we managed to establish a general *method to generate transference principles*, even for semigroups. These do not only cover the known instances of transference principles but also allow to treat singular integrals for C_0-semigroups $(T(s))_{s\geq 0}$, e.g.,

$$\mathrm{PV} - \int_{0}^{t+1} \frac{T(s)x}{s-t}\, ds \qquad (x \in X,\, t > 0).$$

In our talk we shall survey these new results; it is based on our paper [Math. Ann.345 (2) (2009), 245-265] and the recent preprint *Transference Principles for Semigroups and a Theorem of Peller.*

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On a Size-Structured Two-Phase Population Model with Infinite States-At-Birth

Peter Hinow
University of Wisconsin - Milwaukee, USA
(Jozsef Z. Farkas (University of Stirling, UK))

We introduce and analyze a linear size-structured population model with infinite states-at-birth. We model the dynamics of a population in which individuals have two distinct life-stages: an “active” phase when individuals grow, reproduce and die and a second “resting” phase when individuals only

grow. Transition between these two phases depends on individuals' size. First we show that the problem is governed by a positive quasicontractive semigroup on the biologically relevant state space. Then we investigate, in the framework of the spectral theory of linear operators, the asymptotic behavior of solutions of the model. We prove that the associated semigroup has, under biologically plausible assumptions, the property of asynchronous exponential growth.

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On the Abstract Cauchy Problem for Operators on Locally Convex Spaces

Michael Langenbruch
University of Oldenburg, Germany
(Pawel Domanski (Poznan))

The Abstract Cauchy Problem has been treated by several methods such as semigroup theory or Laplace transform. We will present a solution of this problem in the sense of Laplace hyperfunctions and Laplace dstributions for operators on complete ultrabornological locally convex spaces. This extends corresponding results of Komatsu for operators on Banach spaces. The crucial tools are a suitably general notion of a resolvent for operators on locally convex spaces and a general version of the Laplace transform for Laplace hyperfunctions with values in locally convex spaces. Basic examples are provided to explain the new definitions and their relation to the classical variants of these notions.
Report on joint work with P. Domanski (Poznan).

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Singular Limits in Nonlinear Dynamical Systems

Frieder Lippoth
Leibniz Universität Hannover, Germany

We prove the convergence of abstract dynamical systems to their associated quasistationary approximations and apply these results to a moving boundary problem describing the growth of avascular tumors. The abstract results rely on the theory of analytic semigroups and parabolic evolution operators. The setting is made to treat a quasilinear situation.

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Bifurcation Problems for Structured Population Dynamics Models

Pierre Magal
University of Bordeaux, France
(Shigui Ruan)

Several types of differential equations, such as delay differential equations, age-structure models in population dynamics, evolution equations with nonlinear boundary conditions, can be written as semilinear Cauchy problems with an operator which is not densely defined. The goal of this presentation is first to present several examples, and then to turn to normal form theory for semi-linear Cauchy problems with non-dense domain. Based on a previous work on the existence and the smoothness of the center manifold (i.e. by using Liapunov-Perron method and following the techniques of Vanderbauwhede and Iooss), we are now in position to discussion the normal form theory in such a context. In this talk we will mainly focus on Hopf bifurcation theory and provide several examples of applications in the context of structured population dynamics model.

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Strongly Degenerate Parabolic Equations and the Associated Evolution Operators in BV Spaces

Shinnosuke Oharu
Chuo University, Japan
(Hiroshi Watanabe)

A new notion of generalized solution to the initial boundary value problem for a strongly degenerate parabolic equation of the form $u_t + \nabla \cdot A(x,t,u) + B(x,t,u) = \Delta\beta(u)$ is introduced. Here this type of solution is called a BV-entropy solution. Since equations of this form are linear combinations of time-dependent conservation laws and porous medium type equations, it is interesting to investigate interactions between singularities of solutions associated with the two different kinds of nonlinearities. However, either of the part of conservation laws and that of porous medium type diffusion term may not be treated as a perturbation of the other. This observation leads us to the new notion of BV-entropy solution. Our objective of this talk is to discuss unique existence of such BV-entropy solutions under homogeneous Neumann boundary conditions in the framework of nonlinear evolution operator theory.

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Hopf Bifurcation for Non-Densely Defined Cauchy Problems

Shigui Ruan
University of Miami, USA
(Pierre Magal)

We establish a Hopf bifurcation theorem for abstract Cauchy problems in which the linear operator is not densely defined and is not a Hille-Yosida operator. The theorem is proved by using the center manifold

theory for non-densely defined Cauchy problems associated with the integrated semigroup theory. As applications, the main theorem is used to obtain a known Hopf bifurcation result for functional differential equations and a general Hopf bifurcation theorem for age structured models.

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Stability of Fronts

Stephen Schecter
North Carolina State University, USA
(A. Ghazaryan, Y. Latushkin)

We will discuss recent results on linear and nonlinear stability of fronts in reaction diffusion systems (possibly, degenerate) that arise in chemical applications and in combustion theory. In particular, an application of the Greiner spectral mapping theorem for strongly continuous semigroups will be given.

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Differentiability of Convolutions, Integrated Semigroups of Bounded Semi-Variation, and the Inhomogeneous Cauchy Problem

Horst Thieme
Arizona State University, USA

If $T = \{T(t); t \geq 0\}$ is a strongly continuous family of bounded linear operators between two Banach spaces X and Y and $f \in L^1(0, b, X)$, the convolution of T with f is defined by $(T * f)(t) = \int_0^t T(s)f(t-s)ds$. It is shown that $T * f$ is continuously differentiable for all $f \in C(0, b, X)$ if and only if T is of bounded semi-variation on $[0, b]$. Further $T * f$ is continuously differentiable for all $f \in L^p(0, b, X)$ $(1 \leq p < \infty)$ if and only if T is of bounded semi-p-variation on $[0, b]$ and $T(0) = 0$. If T is an integrated semigroup with generator A, these respective conditions are necessary and sufficient for the Cauchy problem $u' = Au + f$, $u(0) = 0$, to have integral (or mild) solutions for all f in the respective function vector spaces. A converse is proved to a well-known result by Da Prato and Sinestrari: the generator A of an integrated semigroup is a Hille-Yosida operator if, for some $b > 0$, the Cauchy problem has integral solutions for all $f \in L^1(0, b, X)$. Integrated semigroups of bounded semi-p-variation are preserved under bounded additive perturbations of their generators and under commutative sums of generators if one of them generates a C_0-semigroup.

Keywords: convolution, (C_0, integrated) semigroup, semi-variation, Stieltjes integral, inhomogeneous Cauchy problem, Hille-Yosida, resolvent positive, commutative sum, bounded additive perturbation
2000MSC: 45D05, 47D06, 47D62
J. Evolution Equations 8 (2008), 283-305.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Heat Kernel Estimates and Interpolation Inequalities

Hendrik Vogt
TU Dresden, Germany

We study stability of Gaussian type estimates for positive C_0-semigroups under perturbation by potentials. An example is the C_0-semigroup associated with the Cauchy problem

$$\partial_t u = \Delta u + Vu, \quad u(0) = f$$

(heat equation with absorption/excitation) in the half space $\Omega = (0, \infty) \times \mathbb{R}^{n-1}$, with Dirichlet boundary conditions, for a measurable potential $V \colon \Omega \to \mathbb{R}$. In this example, (stability of) the boundary behaviour of the kernel is of particular interest.

The main tool in the treatment of the above problem is an interpolation inequality for the kernels of perturbed semigroups. Versions of this inequality have long been known for semigroups associated with stochastic processes; in this case the Feynman-Kac formula can be used for the proof. We indicate a rather elementary proof that allows generalisation to the non-autonomous case, i.e., the interpolation inequality can also be used to study perturbation of strongly continuous propagators by (time-dependent) potentials.

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Nonlinear Evolution Inclusions with Nonlocal Initial Conditions

Ioan Vrabie
Department of Mathematics, Romania
(Angela Paicu)

We consider a nonlinear evolution differential inclusion, driven by a multivalued perturbation of an m-accretive operator, subjected to a nonlocal initial condition, and we prove a sufficient condition for the existence of at least one C^0-solution.

Acknowledgements. Partially supported by PN-II-ID-PCE-2007-1 Grant ID 397.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Positive Equilibrium Solutions in Some Nonlinear Age-Structured Population Models

Christoph Walker
Leibniz Universität Hannover, Germany

We consider equilibrium solutions to age-dependent population equations with nonlinear diffusion. Introducing a parameter measuring the intensity of the fertility, it is shown that a branch of positive

equilibria bifurcates from the trivial equilibrium. The approach is based on maximal regularity and evolution operators for quasilinear parabolic equations.

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Floquet Theory for Partial Differential Equations and Evolution Semigroups

Lutz Weis
KIT Karlsruhe, Germany
(Thomas Gauss)

We extend part of the classical Floquet theory for ordinary differential equations to non-autonomous evolution systems with periodic coefficients. In particular we give rather general conditions which insure that strong solutions can be represented as Floquet solutions. Our main tools are evolution semigroups, operator-valued Fourier multiplier theorems and the theory of analytic functions of Fredholmoperators. The general results are applied to systems of partial differential operators.

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Special Session 34: Dynamical Systems and Spectral Theory of Ergodic Operators

Daniel Lenz, Friedrich-Schiller-Universität Jena, Germany
Peter Stollmann, TU Chemnitz, Germany

Localization for an Anderson-Bernoulli Operator with Generic Interaction Potential

Hakim Boumaza
Université Paris 13 - LAGA, France

We present a result of Anderson localization in an interval of $\mathbb{R}$, for a matrix-valued random Schrödinger operator, acting on $L^2(\mathbb{R}) \otimes \mathbb{R}^N$ for an arbitrary $N \geq 1$, and whose interaction potential is generic in the real symmetric matrices. For this purpose, we prove the existence of an interval of energies on which we have separability and positivity of the N non-negative Lyapunov exponents of the operator. The method, based upon the formalism of Fürstenberg and a result of Lie group theory due to Breuillard and Gelander, allows an explicit contruction of the wanted interval of energies.

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On the Lyapunov Exponent and Hyperbolicity for $\mathrm{SL}(2, R)$-Valued Cocycles

Roberta Fabbri
Universitá di Firenze, Italy
(Russell Johnson, Luca Zampogni)

In the talk the study of the positivity of the Lyapunov exponent for a smooth SL(2, R)-valued cocycle defined over a flow, including the Kronecker one, is considered. In particular, the question of the density in the Hoelder class of the set of SL(2, R) cocycles exhibiting an exponential dichotomy when the base flow is of Kronecker type on a two torus is discussed.

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Lieb-Thirring Type Estimates for Non-Selfadjoint Operators

Marcel Hansmann
TU Clausthal, Germany

We present some Lieb-Thirring type estimates for non-selfadjoint operators. Applications to complex Jacobi and Schrödinger operators will be discussed.

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Non-Uniformly Hyperbolic Dynamics for Quasiperiodic Non-Linear Cocycles

Tobias Jaeger
TU Dresden, Germany

The concept of non-uniform hyperbolicity (NUH) plays a crucial role in the study of Schrödinger operators with quasiperiodic potential, where NUH of the associated Schrödinger cocycle is equivalent to Anderson localisation. The aim of the talk is to explore this concept further in a purely dynamical setting. We study quasiperiodic cocycles of non-linear circle diffeomorphisms, where NUH is often referred to as the existence of 'strange non-chaotic attractors'. Generalising results by Lai-Sang Young and Kristian Bjerklöv, we show that parameter families of qp non-linear cocycles exhibit NUH on a parameter set of positive measure, provided they satisfy certain (open) $\mathcal{C}^1$-conditions and the rotation number on the base is Diophantine. This further yields insights about the structure of the Arnold tongues, which correspond to the spectral gaps of the Schrödinger operator in the linear setting.

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Generalized Reflectionless Potentials and the Camassa-Holm Equation

Russell Johnson
Universitá di Firenze, Italy
(Luca Zampogni)

We consider some recent developments concerning the Camassa-Holm equation and its related spectral problem, the so-called acoustic equation. Emphasis will be placed on solutions which arise from initial data of “generalized reflectionless” type.

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Stability of Absolutely Continuous Spectrum under Random Perturbations on Trees

Matthias Keller
Friedrich Schiller University Jena, Germany
(Daniel Lenz, Simone Warzel)

We study a class of rooted trees which are not necessarily regular but still exhibit a lot of symmetries. The spectrum of the corresponding graph Laplace operator is purely absolutely continuous and consists of finitely many intervals. We show stability of absolutely continuous spectrum under small random perturbations.

⟶ ∞ ⋄ ∞ ⟵

Some Spectral Properties of Weighted Composition Operators Induced by Ergodic Transformations

Arkady Kitover
Community College of Philadelphia, USA

We prove that if a weighted rotation operator acts in a Banach space of functions analytic in the unit disk the under very mild conditions on the Banach space the spectrum of such an operator is either an annulus or a disk. Generalizations for some ergodic transformations more general than rotations are also considered.

⟶ ∞ ⋄ ∞ ⟵

Positive Lyapunov Exponent for Ergodic Schroedinger Operators

Helge Krueger
Rice / ESI, USA

I will discuss how to prove positive Lyapunov exponent for ergodic Schrödinger operators and why this is interesting.

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Ergodic Properties of Randomly Coloured Point Sets

Christoph Richard
Erlangen University, Germany
(Peter Mueller)

We provide a framework for studying randomly coloured point sets over a locally compact, second countable space, on which a locally compact, second countable group acts continuously and properly. We first construct and describe an appropriate dynamical system for uniformly discrete uncoloured point sets and characterise ergodicity geometrically. This framework allows to incorporate a random colouring of the point sets. We derive an ergodic theorem for randomly coloured point sets and characterise ergodic measures. Special attention is paid to the exclusion of exceptional instances for uniquely ergodic systems. The setup allows for a straightforward application to randomly coloured graphs.

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Preservation of Absolutely Continuous Spectrum of Periodic Jacobi Operators under Perturbations of Square–Summable Variation

Mira Shamis
Hebrew University, Israel
(Uri Kaluzhny)

We study self-adjoint bounded Jacobi operators of the form:

$$(J\psi)(n) = a_n\psi(n+1) + b_n\psi(n) + a_{n-1}\psi(n-1)$$

on $\ell^2(\mathbb{N})$. We assume that for some fixed q, the q-variation of $\{a_n\}$ and $\{b_n\}$ is square-summable and $\{a_n\}$ and $\{b_n\}$ converge to q-periodic sequences. Our main result is that under these assumptions the essential support of the absolutely continuous part of the spectrum of J is equal to that of the asymptotic periodic Jacobi operator. This work is an extension of a recent result of S. A. Denisov.

⟶ ∞ ⋄ ∞ ⟵

Asymptotics for the Toda Flow

Gerald Teschl
University of Vienna, Austria
(Helge Krueger)

The Toda flow induces a dynamical system on all Jacobi matrices. We are interested in its asymptotical behavior both for large t as well as for large n.

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Special Session 35: Geometric and Singular Analysis

Gerardo Mendoza, Temple University, USA
Michael Ruzhansky, Imperial College, UK
Bert-Wolfgang Schulze, University of Potsdam, Germany

Introduction: The Special Session "Geometric and Singular Analysis" focuses on a variety of topics in the analysis and geometry of differential and pseudo-differential operators on manifolds with singular geometry or on stratified spaces, and on pseudo-differential calculi on Lie groups. Topics include operator algebras, index theory, spectral theory, resolvents of singular elliptic operators, zeta functions, and asymptotic phenomena of solvabilitiy of equations with singular and degenerate behaviour.

On the Sub-Laplacian for Special Classes of Sub-Riemannian Manifolds

Wolfram Bauer
Universität Greifswald, Germany

We give examples of sub-Riemannian structures in a strong sense. Moreover, the spectral analysis of a corresponding sub-Laplacian Δ^{sub} is discussed. In particular, we analyze spectral functions of Δ^{sub} such as the spectral zeta-function. These are joint results together with C. Iwasaki and K. Furutani.

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Acoustic and Optical Black Holes

Gregory Eskin
UCLA, USA

Acoustic and optical black holes are the black holes for the wave equations describing the wave propagation in the moving medium. They include the black holes of the general relativity when the Lorentz metric corresponding to the wave equation satisfies the Einstein equations. We study the existence and the stability of black and white holes in the case of two space dimensions and in the axisymmetric case. The inverse problems in the presence of black holes will be discussed.

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On the Heat Trace Expansion for the Closure of an Elliptic Wedge Operator

Juan Gil
Penn State University, Altoona, USA

We will discuss the resolvent and heat trace expansion for the closure of an elliptic wedge operator. We consider the case when the domain of the minimal extension is a weighted Sobolev space. This is joint work with Thomas Krainer and Gerardo Mendoza.

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Blow-Up Analysis of Geodesics on Singular Surfaces

Daniel Grieser
Universität Oldenburg, Germany
(V. Grandjean)

Understanding the geodesics on a singular space is of interest both from the point of view of studying the inner geometry of such spaces and also from a PDE point of view, since the geodesics are the expected trajectories of singularities of solutions of the wave equation. In the case of conical singularities, it was proved by Melrose and Wunsch that the geodesics hitting the conical point foliate a neighborhood of that point smoothly. In other words, there is a smooth exponential map based at the singular point. We consider a different class of singularities and prove that while the exponential map based at the singularity is not smooth, its precise asymptotic behavior (to any order) near the singularity can be described completely in terms of certain blow-ups of the space and of its cotangent bundle.

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Explicit Green Operators for Quantum Mechanical Hamiltonians

Gohar Harutyunyan
University Oldenburg, Germany
(H.-J. Flad, R. Schneider, B.-W. Schulze)

We present a new approach to study the asymptotic behaviour of solutions of Schrödinger's equation for many-particle systems interacting via singular Coulomb potentials. The asymptotic behaviour of quantum systems near coalescence points of particles is of fundamental interest for electronic structure theory underlying most of chemistry and solid state physics.

Coalescence points of particles in quantum systems introduce a natural hierarchy of embedded geometric singularities. Depending on the number of particles at coalescence points these singularities range from conical to higher order corner singularities. The basic tasks are to establish ellipticity of

the Hamiltonians in the corresponding singular calculus and the explicit construction of the parametrices and Green operators which encode the asymptotic behaviour of the systems.

We discuss the conical, edge and corner degenerate structures of quantum mechanical Hamiltonians, the underlying principal symbolic hierarchies of operator-valued symbols on configurations with lower order singularities, and present first results concerning their ellipticity. These considerations lead to a recursive approach for the calculation of Green operators. An explicit asymptotic representation of the Green operator has been obtained for the hydrogen atom.

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Zeta Function on Surfaces of Revolution

Klaus Kirsten
Baylor University, USA

We consider the surface of revolution M generated by a positive, differentiable function f on $[0, L]$. Taking the metric induced by the standard Euclidean metric of R^3 we obtain a Riemannian manifold (M, g) with nonempty boundary. On (M, g) we consider the Laplacian and its associated zeta function. We analyze in detail how the zeta function determinant and the Casimir energy depend on the function f.

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Trace Expansions for Elliptic Cone Operators

Thomas Krainer
Penn State Altoona, USA
(Juan Gil and Gerardo Mendoza)

I will report on joint work with Juan Gil and Gerardo Mendoza on the expansion of the resolvent trace for closed extensions of general elliptic cone operators on compact manifolds with conical singularities as the spectral parameter tends to infinity along a ray (or a sector) in the complex plane. As was observed by other authors in special cases, the expansion in general exhibits unusual phenomena when compared to the classical (smooth) situation. Our work shows that these unusual phenomena can be fully explained utilizing a dynamical system that is naturally associated to the conic geometry and the closed extension of the operator under consideration. This viewpoint allows us to obtain a general result about the structure of the resolvent trace expansion for elliptic cone operators with minimal assumptions on the symbols of the operator.

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Characteristic Classes of the Boundary of a Complex b-Manifold

Gerardo Mendoza
Temple University, USA

A complex b-structure on a manifold with boundary is an involutive subbundle ${}^bT^{0,1}\mathcal{M}$ of the complexification of ${}^bT\mathcal{M}$ with the property that $\mathbb{C}{}^bT\mathcal{M} = {}^bT^{0,1}\mathcal{M} + \overline{{}^bT^{0,1}\mathcal{M}}$ as a direct sum. Such a complex b-structure induces a rich structure on the boundary of $\mathcal{M}$ similar to that of the circle bundle of a Hermitian holomorphic line bundle over a complex manifold. I'll discuss a classification theorem for boundary structures of complex b-manifolds structurally similar to the classification of complex line bundles over compact manifolds by their Chern classes, and one that resembles the classification of holomorphic line bundles over compact complex manifolds by the elements of the Picard group of the base.

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On the Hypoellipticity of Singular Differential Forms with an Isolated Singularity

Abdelhamid Meziani
Florida International University, USA

We will discuss the real-analytic hypoellipticity of a real-analytic, $\mathbb{C}$-valued, and formally integrable differential form Ω in a neighborhood of an isolated singularity in $\mathbb{R}^n$. The main result states that in the case $n = 2$, the form is not hypoelliptic, while in the case $n \geq 3$, the form is hypoelliptic at the singular point provided, that it is hypoelliptic outside the singularity and that the codimension-two foliation that it generates has no compact leaves.

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Variable Branching Asymptotics on Manifolds with Edge

Bert-Wolfgang Schulze
University of Potsdam, Germany
(A. Volpato)

Solutions to elliptic equations (degenerate in a typical way) on a manifold with edge are expected to have an asymptotic behaviour in the distance variable r to the edge, with complex exponents and logarithmic powers, depending on the variable y on the edge. Similarly as for conical singularities the coefficients of the asymptotics also depend on the variables x on the cross section of the local cones. Even for smooth solutions off the edge the dependence with respect to x, y can be very complicated. In general, after applying the Mellin transform in r we obtain families of meromorphic functions with poles

depending on y of variable multiplicity (i.e., of a branching behaviour). The position of poles is determined by the exponents in the asymptotics and the multiplicities by the exponents of the logarithmic terms (plus 1). If solutions are not smooth, the Sobolev smoothness s of coefficients of the asymptotics may also depend on y, with a corresponding variable and branching behaviour. We give a general answer in terms of elliptic regularity in edge spaces with such asymptotics by means of parametrices within the edge calculus, in fact, a refinement of the calculus with continuous asymptotics.

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Threshold Eigenfunctions and Threshold Resonances of Some Quantum Hamiltonians

Tomio Umeda
University of Hyogo, Japan

The talk will be devoted to investigations of asymptotic behaviors of the eigenfunctions of the Weyl-Dirac operator $\sigma\cdot(D-A(x))$ as well as the Pauli operator $(D-A(x))^2-\sigma\cdot B$ at the threshold eigenvalue 0, and asymptotic behavior of the eigenfunctions of the Dirac operator at the threshold eigenvalues $\pm m$. In the discussions, it turns out that zero modes (i.e., eigenfunctions corresponding to the eigenvalue 0) of the Weyl-Dirac operator play a basic and crucial role. Also, I shall discuss the threshold resonances of the Weyl-Dirac operator and the Pauli operator at the threshold energy 0.

This talk is based on joint works with Tsutomu Morita (University of Hyogo) and with Yoshimi Saitö (University of Alabama at Birmingham).

$\longrightarrow \infty \diamond \infty \longleftarrow$

Symbolic Parametrix Construction for the Strictly Hyperbolic Cauchy Problem

Ingo Witt
Göttingen, Germany

(Nhu Thang Nguyen)

The solution u to the strictly hyperbolic homogeneous Cauchy problem $Pu = f \equiv 0$ on X and $\gamma B_j u = g_j$ on X_0 for $1 \le j \le \mu$ can be written as $u = \sum_{j=1}^{\mu} E_j g_j$, where each E_j is a sum of Fourier integral operators. It is also known that real-principal type operators admit parametrices the kernels of which are one-sided paired Lagrangian dstributions. In this talk, by identifying the Lagrangian submanifolds of $T^*(X\times X)\setminus 0$, $T^*(X\times X_0)\setminus 0$, and so forth, that arise, e.g., in compositions, and by identifying the principal symbols on these Lagrangian submanifolds of the operators involved, we combine both constructions and develop a calculus in which the strictly hyperbolic, but now inhomogeneous Cauchy problem (i.e., with possibly $f \not\equiv 0$) appears as an operator with an invertible principal symbol that allows a parametrix in the calculus.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Special Session 36: Reaction Diffusion Systems

Julian Lopez-Gomez, Universidad Complutense de Madrid, Spain

Introduction: Reaction Diffusion Systems are imperative in modeling a great variety of natural phenomena in Biology, Chemistry, Physics and Engineering, as well as in Economics, where they are used to evaluate stock options in the context of the theory of Black-Scholes and Merton. Their significance not only relies on the huge number of their applications but also on the fact that they provide with a rather general class of linear and nonlinear differential operators whose mathematical analysis has shown to be a milestone for the development of applied, abstract and numerical Analysis, Algebra, Geometry and Topology. This session will cover a wide range of aspects around Reaction Diffusion Systems by some of the most relevant experts in this field.

Second-Order Boundary Estimates for Solutions to Singular Elliptic Equations

Claudia Anedda
University of Cagliari, Italy

(Giovanni Porru)

We investigate boundary blow-up solutions of the equation $\Delta u = f(u)$ in a bounded smooth domain $\Omega \subset R^N$. Under appropriate growth conditions on $f(t)$ as t goes to infinity we show how the mean curvature of the boundary $\partial\Omega$ appears in the asymp-

totic expansion of the solution $u(x)$ in terms of the distance of x from the boundary $\partial\Omega$.
We investigate the asymptotic behaviour of the solution to the homogeneous Dirichlet boundary value problem for the singular equation $\Delta u+f(u)=0$ in a bounded smooth domain $\Omega\subset R^N$. Under appropriate growth conditions on $f(t)$ as t approaches zero, we find an asymptotic expansion up to the second order of the solution in terms of the distance from x to the boundary $\partial\Omega$.

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Asymptotic Behavior at Infinity of the Positive Solutions of a Generalized Class of Thomas-Fermi Equations

Santiago Cano-Casanova
Universidad Pontificia Comillas, Spain

In this talk we will discuss about the exact decrease rate to zero at infinity of the unique positive solution of the generalized *Thomas-Fermi* problem

$$\begin{cases} u''=f(t)\varphi(u(t))\,, & t>0 \\ u(0)=\alpha>0\,, & u(\infty)=0 \end{cases}$$

where $f\in\mathcal{C}[0,\infty)$ is a positive and nondecreasing function, and $\varphi\in\mathcal{C}^1[0,\infty)$ is an increasing function satisfying when $u\downarrow 0$ that $\varphi(u)\sim\Upsilon u^q$, for some $\Upsilon>0$ and $q>1$.

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Approximating the Ideal Free Distribution Via Reaction-Diffusion-Advection Equations

Robert Stephen Cantrell
University of Miami, USA
(Chris Cosner and Yuan Lou)

We consider reaction-diffusion-advection models for spatially distributed populations that have a tendency to disperse up the gradient of fitness, where fitness is defined as a logistic local population growth rate. We show that in temporally constant but spatially varying environments such populations have equilibrium dstributions that can approximate those that would be predicted by a version of the ideal free dstribution incorporating population dynamics. This modeling approach shows that a dispersal mechanism based on local information about the environment and population density can approximate the ideal free dstribution. The analysis suggests that such a dispersal mechanism may sometimes be advantageous because it allows populations to approximately track resource availability. The models are quasilinear parabolic equations with nonlinear boundary conditions.

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Glaeser's Type Estimates for Viscosity Solutions of Some Nonlinear PDE's

Italo Capuzzo Dolcetta
Sapienza Universita di Roma, Italy
(A. Vitolo, Università di Salerno)

The talk will focus on some recent results on viscosity solutions of second order elliptic partial differential equations of the form

$$F(x,u(x),Du(x),D^2u(x))=f(x),\quad x\in B_R(0).$$

If u is a non-negative continuous viscosity solution of the above, then under some structure conditions on F

$$\sup_{\substack{x,y\in B_R(0)\\ x\neq y}}\frac{|u(x)-u(y)|}{|x-y|^\alpha}\leq C\left(\frac{u(0)}{R^\alpha}+R^{1-\alpha}\sup|f|\right)$$

for some $\alpha\in(0,1)$ and C depending on the space dimension and on the structure constants of F.
This Glaeser's type estimate extends previous results in this direction due to Li-Nirenberg.

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Evolution of Dispersal and the Ideal Free Distribution

Chris Cosner
University of Miami, USA
(R. S. Cantrell and Y. Lou)

The dispersal strategies of organisms can affect the outcome of interactions between species. It is natural to ask which dispersal strategies are most likely to evolve and persist. This question can be studied from a viewpoint suggested by adaptive dynamics, where strategies are compared by examining when a population using one strategy can invade another population using a different strategy. Strategies that resist invasion are considered to be evolutionarily stable. The stability of dispersal strategies can be studied by using reaction-diffusion-advection models for competing populations. In situations where resources are heterogeneous in space but constant in time, this approach has shown that for simple constant diffusion there is selection for lower diffusion rates. For models with spatially variable diffusion or advection the situation becomes more complicated, but in some cases it is possible to characterize which strategies are stable in terms of how well they allow populations to match resources. Resource matching is related to a verbal theory of the spatial dstribution of organisms called the ideal free dstribution. This talk will describe how the evolutionary stability problem is formulated and describe its solution in a particular case involving fixed diffusion rates but variable advection.

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Uniform Exponential Decay of the Free Energy for Voronoi Finite Volume Discretized Reaction-Diffusion Systems

Annegret Glitzky
WIAS, Berlin, Germany

Our focus are energy estimates for discretized reaction-diffusion systems. We introduce a discretization scheme (Voronoi finite volume in space and fully implicit in time) which preserves the main features of the continuous systems, namely positivity and dissipativity.

For a class of Voronoi finite volume meshes we investigate thermodynamic equilibria and obtain for solutions to the evolution system the monotone and exponential decay of the discrete free energy to its equilibrium value with a unified rate of decay for this class of discretizations. The fundamental idea is an estimate of the free energy by the dissipation rate which is proved indirectly by taking into account sequences of Voronoi finite volume meshes. Essential ingredient in that proof is a discrete Sobolev-Poincaré inequality.

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Nonlinear Diffusions with Nontrivial Dynamics

Patrick Guidotti
University of California, Irvine, USA

This talk will focus on a class of nonlinear diffusions which were motivated by applications to Image Pocessing and exhibit interesting mathematical features. They are degenerate diffusion equations which engender interesting non-trivial dynamical behavior.

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Small Spatial Variation of the Reproduction Rate in a Two Species Competition Diffusion System

Georg Hetzer
Auburn University, USA
(Tung Nguyen, Wenxian Shen)

The effect of a small spatially inhomogeneous perturbation of the reproduction rate of one species (mutant) is considered assuming that both species have otherwise identical features. The main result shows that the mutant can always invade, whereas the other species can only invade under certain conditions which yield uniform persistence of both species.

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On a Mathematical Model Arising from Competition of Phytoplankton Species for a Single Nutrient with Internal Storage:Steady State Analysis

Sze-Bi Hsu
National Tsing-Hua University, Taiwan
(Feng-Bin Wang)

In this talk we introduce a mathematical model of two microbial populations competing for a single limited nutrient with internal storage in an unstirred chemostat. First we establish the existence and uniqueness of positive steady state for a single population. The conditions for the coexistence of two populations are determined. Techniques include the maximum priciple, theory of bifurcation and degree theory in cone.

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Bifurcation Analysis for Indefinite Weight Boundary Value Problems with Nonlinear Boundary Conditions

Umezu Kenichiro
Ibaraki University, Japan

We consider bifurcation of positive solutions of nonlinear elliptic boundary value problems with indefinite weights and nonlinear boundary conditions, arising from populations dynamics. We discuss the global nature of bifurcation components emanating from two trivial lines of solutions. The result is based on the Lyapunov-Schmidt procedure, a variational argument, and the global bifurcation theory proposed by López-Gómez. An a priori bound for positive solutions is also discussed to obtain some result on the multiplicity of positive solutions.

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The Theorem of Characterization of the Maximum Principle

Julian Lopez-Gomez
Universidad Complutense de Madrid, Spain

In this talk we are going to discuss the genesis of the theorem of characterization of the maximum principle as well as some of its implications in the theory of nonlinear elliptic and parabolic differential equations.

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Non-Random Dispersal of Interacting Species in Heterogeneous Landscapes

Yuan Lou
Ohio State University, USA

From habitat degradation and climate change to spatial spread of invasive species, dispersals play a central role in determining how organisms cope with a changing environment. The dispersals of many organisms depend upon local biotic and abiotic factors, and as such are often non-random. In this talk we will discuss some recent progress on the effects of non-random dispersal on two competing species in heterogeneous environments via reaction-diffusion-advection models. This talk is based up on joint works with Andriy Bezugly, Steve Cantrell, Xinfu Chen, Chris Cosner and Richard Hambrock.

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Bounds for Blow-Up Time for Solutions to Reaction-Diffusion Systems

Monica Marras
University of Cagliari, Italy
(Piro Vernier Stella)

We consider the blow-up of the solutions to the following parabolic system

$$\begin{cases} div(p(u,v)\nabla u) + f_1(u,v) = u_t & \text{in } \Omega \times (0,t^*), \\ div(p(u,v)\nabla v) + f_2(u,v) = v_t & \text{in } \Omega \times (0,t^*), \\ \quad u(x,t) = 0 \quad \text{on } \partial\Omega \times (0,t^*), \\ \quad v(x,t) = 0 \quad \text{on } \partial\Omega \times (0,t^*), \\ \quad u(x,0) = u_0(x) \quad \text{on } \Omega, \\ \quad v(x,0) = v_0(x) \quad \text{on } \Omega, \end{cases} \tag{1}$$

where Ω is a bounded domain in $\mathbb{R}^3$, with smooth boundary $\partial\Omega$ and t^* is the blow up time. Under suitable hypotheses on nonlinearities, initial data and p, a differential inequality technique is used to determine a lower bound of the blow up time t^* for solutions of (1). In addition a sufficient condition which ensure that the blow up does not occur is determined.

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On the Multiplicity of Solutions of Singularly Perturbed Elliptic Problems on Riemaniann Manifolds

Anna Maria Micheletti
Dipartimento di Matematica Applicata, Italy
(Benci, Dancer, Micheletti, Pistoia)

Given (M,g) a smooth compact Riemaniann n-dimensional manifold $(n \geq 2)$ we look for functions $u \in H^1_g(M)$ satisfying

$$-\varepsilon^2 \Delta_g u + u = |u|^{p-2} u \text{ in } M.$$

Here $p > 2$ if $n = 2$, $2 < p < \frac{2n}{n-2}$ if $n \geq 3$. We show that the number of solutions of this problem depends on the topological properties of the manifold. Also the geometry of M has effect on the number of solutions and we prove the stable critical points of the scalar curvature of (M,g) generate solutions of this problem.

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A Parabolic-Elliptic System of Drift-Diffusion Type in Two Dimensions with Subcritical Initial Data

Toshitaka Nagai
Hiroshima University, Japan

In this talk we will focus on the Cauchy problem for a parabolic-elliptic system of drift-diffusion type in R^2, modeling chemotaxis and self-interacting particles. We discuss the global existence and asymptotic behavior of nonnegative solutions to the Cauchy problem for subcritical initial data.

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The Critical Case for the Drift-Diffusion System of Degenerate Type

Takayoshi Ogawa
Tohoku University, Japan

The drift-diffusion system is one of the simplest model that simulates the semiconductor device. It also appears in some other models such as the chemotaxis model or the mass concentration model in the gaseous star in astronomy. We consider the following simplest case:

$$\begin{cases} \partial_t \rho - \nabla \cdot (\nabla P(\rho) - \kappa \rho \nabla \psi) = 0, \ t > 0, x \in \mathbb{R}^n, \\ -\Delta \psi = \rho, \quad x \in \mathbb{R}^n, \\ \rho(0,x) = \rho_0(x) \geq 0, \quad P(\rho) = \rho^\alpha, \ \alpha \geq 1, \end{cases} \tag{1}$$

where $\rho = \rho(t,x) : \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}_+$ denotes the density of matter, $\psi = \psi(t,x) : \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}$ is the potential function and $P = P(\rho)$ denotes the pressure and we assume that it is only depending on the density ρ. While κ is the coupling constant and $\rho_0(x)$ is given initial data. Mathematically the sign of the constant κ is important since it separates the character of the system into attractive $(\kappa = 1)$ or repulsive (or defocusing).

In this talk, we formally comment the derivation for (1) and then we discuss various aspects of the solution of the drift-diffusion system (1) according to the sign of κ and power of $P(\rho) = \rho^\alpha$.

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Blow-Up Phenomena for Semilinear Heat Equations with Nonlinear Boundary Condition

Stella Piro Vernier
University of Cagliari, Italy
(L. Payne, G. Philippin)

We consider the blow-up of the solutions to semilinear heat equations with nonlinear boundary condition. We establish conditions on nonlinearities sufficient to garantee that $u(x,t)$ exists for all time $t > 0$ as well as conditions on data forcing the solution $u(x,t)$ to blow-up at some finite time t^*. Moreover an upper bound for t^* is derived. Under somewhat more restrictive conditions lower bounds for t^* are also derived. The results are based on a Sobolev-Talenti type inequality and on a differential inequality.

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Symmetry Breaking in Problems Involving Semilinear Equations

Giovanni Porru
Cagliari University, Italy
(Lucio Cadeddu)

This paper is concerned with three optimization problems where symmetry breaking arise. The first one consists in the maximization of the energy integral for a homogeneous Dirichlet problem governed by the elliptic equation $-\Delta u = \chi_F u^q$ in the annulus $B_{a,a+2}$ of the plane. Here $q \in [0,1)$ and F is a varying subset of $B_{a,a+2}$, with a fixed measure. We prove that a subset which maximizes the corresponding energy integral is not symmetric whenever a is large enough. The second problem we consider is governed by the same equation in a disc B_{a+2} when F varies in the annulus $B_{a,a+2}$ keeping a fixed measure. As in the previous case, we prove that a subset which maximizes the corresponding energy integral is not symmetric whenever a is large enough. The last case deals with the principal eigenvalue for the problem $-\Delta u = \lambda \chi_F u$ in the ball $B_{a+2} \subset \mathbb{R}^N$, where $N \geq 2$ and F varies in the annulus $B_{a+2} \setminus B_a$, keeping a fixed measure. We prove that, if a is large enough, the minimum of the corresponding principal eigenvalue is attained in a subset which is not symmetric.

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Nonlinear Diffusion and Geometry of Domain

Shigeru Sakaguchi
Hiroshima University, Japan

We consider the initial-boundary value problem for nonlinear diffusion equations in a domain in Euclidean space, where the boundary value equals a positive constant and the initial value equals zero. Then we study the relationship between the initial behavior of the solution and geometry of the domain. The Cauchy problem, where the initial data is given by the characteristic function of the complement of the domain, is also considered.

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On the Behavior of Solutions to a Parabolic-Elliptic System

Takasi Senba
Kyushu Institute of Technology, Japan

We consider radial solutions to a parabolic-elliptic system

$$M_t = M_{rr} + \frac{N+1}{r} M_r + M\,(rM_r + 2M)$$

in $[0,\infty] \times [0,\infty)$ with

$$M(0,t)_r = 0 \quad \text{for } t \in [0,\infty)$$

and

$$M(\cdot,0) = M_0 \quad \text{in } [0,\infty).$$

The equation is related to a simplified version of the Keller-Segel model. We consider solutions M such that the limit $\Lambda = \lim_{r\to\infty} r^2 M(r,t)$ exists and is independent of t. The limit Λ corresponds to the total mass of the solution to Keller-Segel system.

If $\Lambda > 4$, the solution blows up in finite time. If $\Lambda \leq 4$, the solution exists globally in time. Moreover, if $\Lambda < 4$, the solution is bounded globally in time t. If $\Lambda = 4$, the existence of infinite time blowup solutions was shown.

In this talk, we consider the asymptotic behavior of the solution.

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Asymptotics of Solutions and Inverse Bifurcation Problems for Nonlinear Sturm-Liouville Problems

Tetsutaro Shibata
Hiroshima University, Japan

We establish the local and global asymptotic behavior of the bifurcation branch of the nonlinear Sturm-Liouville problems. We also give some recent results for inverse bifurcation problems.

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Qualitative Behavior of Solutions for a Thermodynamically Consistent Stefan Problem with Surface Tension

Gieri Simonett
Vanderbilt University, USA
(Jan Prüss, Rico Zacher)

The qualitative behavior of solutions for a thermodynamically consistent two-phase Stefan problem with surface tension and with or without kinetic undercooling is studied. Existence and uniqueness of solutions is established. It is shown that the system conserves energy, and that the (negative) total entropy constitutes a Ljapunov functional. If a solution does not exhibit singularities, it exists globally in time and converges toward an equilibrium of the problem.

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On the Uniqueness of a Solution to the Boundary Blowup Problem for Curvature Equation

Kazuhiro Takimoto
Hiroshima University, Japan

The boundary blowup problem arises from physics, geometry and many branches of mathematics. In this talk, we discuss the uniqueness of the boudary blowup solution for the so-called k-curvature equation.

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A Reflection Priciple and the Regularity of the Nodal Set of Segregated Critical Configurations

Susanna Terracini
Universitá di Milano Bicocca, Italy
(Hugo Tavares)

We deal with a class of Lipschitz vector functions $U = (u_1, \ldots, u_h)$ whose components are non negative, disjointly supported and verify an elliptic equation on each support. Under a weak formulation of a reflection law, related to the Pohožaev identity, we prove that the nodal set is a collection of regular surfaces plus a low-dimensional residue.

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Exponential Attractors for Non-Autonomous Chemotaxis Systems

Atsushi Yagi
Osaka University, Japan
(Messoud Efendiev and Yoshitaka Yamamoto)

We will introduce a version of exponential attractors for non-autonomous diffusion systems as a time dependent set with uniformly bounded finite fractal dimension which is positively invariant and attracts every bounded set at an exponential rate. This is a natural generalization of the existent notion for autonomous systems. A generation theorem will be presented under the assumption that the evolution operator is a compact perturbation of a contraction. This theorem will be applied to some non-autonomous chemotaxis systems.

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On a Limit System for a Class of Lotka-Volterra Competition Systems with Cross-Diffusion

Yoshio Yamada
Waseda University, Japan

This talk is concerned with positive stationary solutions for a certain class of Lotka-Volterra competition systems with cross-diffusion. These systems were proposed by Shigesada-Kawasaki and Teramoto to describe the habitat segregation phenomena between two competing species. The purpose is to understand the structure of a set of positive stationary solutions. I will show that positive stattionary solutions of competition systems with cross-diffusion converge to a positive solution of a suitable limit system when cross-diffusion coefficients go to infinity. This fact gives us important information on the structure of positive solutions.

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Appearance of Anomalous Singularities in a Semilinear Parabolic Equation

Eiji Yanagida
Tokyo Institute of Technology, Japan
(Shota Sato)

We consider an initial value problem for a parabolic partial differential equation with a power nonlinearity. It is known that in some parameter range, there exists a time-local solution whose singularity is similar to that of a singular steady state for all existence time. In this talk, it is shown that there exists a solution whose singularity becomes anomalous in finite time.

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Multiplicity Results for Nonlinear Differential Equations with Indefinite Weight

Fabio Zanolin
University of Udine, Italy

We present some recent results about the existence

and multiplicity of positive solutions for boundary value problems associated to nonlinear differential equations where the main nonlinear terms are coupled with weight functions of non-constant sign. We use a dynamical systems approach. Our results apply to ordinary differential equations of the form

$$u'' + w(t)f(u) = 0$$

or to more general ODEs coming from the search of radially symmetric solutions of some PDEs.

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Special Session 37: Nonlinear Evolution Equations of Mathematical Physics

Yue Liu, University of Texas at Arlington, USA
Vladimir Varlamov, University of Texas-Pan American, USA
Zhijun (George) Qiao, University of Science and Technology, Hong Kong
Baofeng Feng, University of Texas-Pan American, USA
Andrew N. W. Hone, University of Kent, UK

Introduction: The topics of the session include the study of existence and uniqueness of solutions to nonlinear equations in various function spaces, blow-up and construction of solutions, modeling nonlinear dynamics in media with dispersion and absorption, long-time behavior, asymptotic expansions and qualitative behavior of solutions in question.

Stability of Standing Waves of a Class of Quasilinear Schrödinger Equation

Jianqing Chen
University of Aveiro, Portugal
(Boling Guo)

Variational methods are used to prove that the solution of the quasilinear Schrödinger equation

$$\begin{aligned} &iu_t + u_{xx} + |u|^{p-2}u + (|u|^2)_{xx}u = 0, \\ &u|_{t=0} = u_0(x), \quad x \in \mathbb{R} \end{aligned}$$

must blow up in a finite time for suitable initial data with positive initial energy and some restrictions on p. Then using this we prove that the standing wave is $H^1(\mathbb{R})$ strongly unstable with respect to the quasilinear Schrödinger equation. Similar results in the higher spatial dimensional are also given.

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An Explicit Formula for Exact Solutions to the Focusing NLS Equation

Francesco Demontis
Universitá di Cagliari, Italy
(T. Aktosun, T. Busse, C. van der Mee)

We present an explicit solution formula for the focusing nonlinear Schrödinger equation (NLS) equation by using a triplet of constant matrices (A, B, C) and using matrix exponentials. By exploiting certain symmetries in the solution formula, we show that such solutions are analytic in the entire xt-plane as long as the eigenvalues of the matrix A are not located on the imaginary axis and no two eigenvalues of A are located symmetrically with respect to the imaginary axis. Such solutions decay exponentially as $x \to \pm\infty$ at each fixed t, and they include all multisoliton solutions with any multiplicities of the poles. This is joint work with T. Aktosun and T. Busse of University of Texas at Arlington and C. van der Mee of University of Cagliari.

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Integrable Discretization and Self-Adaptive Moving Mesh Method for the Harry-Dym Equation

Baofeng Feng
The University of Texas-Pan American, USA
(Wei Yin)

In this talk, we will first present the semi- and full-discretizations for the Harry-Dym equation (HDE). The N-cuspon solutions of Casorati determinant form for the continuous, semi-discrete and fully discrete Harry-Dym equation will be provided. It is also shown that the resulting integrable discrete Harry-Dym equation can be served as an innovative numerical scheme, i.e., a self-adaptive moving mesh method for the numerical simulations of the HDE.

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Non-Uniform Dependence for the Euler Equations

Alex Himonas
University of Notre Dame, USA

We shall show that continuous dependence on initial data of solutions to the Euler equations of incompressible hydrodynamics is optimal. More precisely, we prove that the data-to-solution map is not uniformly continuous in Sobolev H^s for any s in the periodic case and for any $s > 0$ if the domain is the whole space R^n. This is work in collaboration with Gerard Misiolek.

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On the Well-Posedness of the Periodic Hunter-Saxton Equation

Curtis Holliman
University of Notre Dame, USA

It is proved that the flow map for the Hunter-Saxton (HS) equation from the periodic homogeneous Sobolev space $\dot{H}^s(\mathbb{T})$ into the space $C([0,T], \dot{H}^s(\mathbb{T}))$ is continuous but not uniformly continuous on

bounded subsets. To demonstrate this sharpness of continuity, two sequences of bounded solutions to the HS equation are constructed whose distance at the initial time converges to zero and whose distance at any later time is bounded from below by a positive constant. To achieve this result, appropriate approximate solutions are chosen and then the actual solutions are found by solving the Cauchy problem with initial data taken to be the value of approximate solutions at time zero. Then, using well-posedness estimates it is shown that the difference between solutions and approximate solutions is negligible.

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Peakon Equations with Spin

Andrew Hone
University of Kent, UK, England

Non-commutative analogues of partial differential equations (PDEs) that admit peakon solutions, such as the Camassa-Holm and Degasperis-Procesi equations, are considered. By allowing the fields to take values in an arbitrary associative algebra, both integrable and non-integrable non-commutative peakon equations are obtained. Upon restricting to the case of ordinary finite-dimensional matrix algebras, these PDEs are shown to have soliton solutions in the form of peakons with additional spin degrees of freedom. Poisson structures and explicit formulae for the solutions are given in some particular cases.

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Inverse Scattering Transform for the Degasperis-Procesi Equation

Rossen Ivanov
Imperial College London, England
(A. Constantin and J. Lenells)

We present the Inverse Scattering Transform method for the Degasperis-Procesi equation. The basic aspects of Inverse Scattering, such as fundamental analytic solutions, the Riemann-Hilbert problem formulation and the Zakharov-Shabat dressing method for the soliton solutions, are outlined.

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Nonlinear Evolution Equations with a Fractional Derivative on a Half-Line

Elena Kaikina
Istituto Tecnologico de Morelia, Mexico
(Martin P. Arciga Alejandre)

Consider the initial-boundary value problem on a half-line for the nonlinear evolution equations with a fractional derivative

$$\begin{cases} u_t + \lambda \left|u\right|^{\sigma} u + \left|\partial_x\right|^{\alpha} u = 0, \ t > 0, x > 0, \\ u(x,0) = u_0(x), \ x > 0, \end{cases}$$

where the $\left|\partial_x\right|^{\alpha}$ operator on a half-line is defined as follows for $\alpha \in (1,2)$

$$\left|\partial_x\right|^{\alpha} u = \vartheta(x) \int_{-i\infty}^{i\infty} e^{px} \left|p\right|^{\alpha} \left(\widehat{u}(p,t) - \frac{u(0,t)}{p}\right) dp.$$

Here the function ϑ is defined as

$$\vartheta(x) = \begin{cases} 1, x \geq 0 \\ 0, x < 0. \end{cases}$$

We study traditionally important problems of a theory of nonlinear partial differential equations, such as global in time existence of solutions to the initial-boundary value problem and the asymptotic behavior of solutions for large time.

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Bidifferential Calculus Approach to Solutions of Akns-Type Hierarchies

Folkert Mueller-Hoissen
MPI for Dynamics and Self-Organization, Germany
(Aristophanes Dimakis)

We express an AKNS-type hierarchy, admitting reductions to matrix NLS and matrix mKdV hierarchies, in terms of a bidifferential graded algebra. "Negative flows", extending the hierarchy, appear in a simple way in this framework. Application of a previously obtained general result quickly generates a large family of exact solutions, including e.g. the matrix solitons in the focusing NLS case.

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Short-Pulse Equation: Well-Posedness and Wave Breaking

Dmitry Pelinovsky
McMaster University, Canada

Sufficient conditions for global existence and for wave breaking are found for the short-pulse equation describing wave packets of few cycles on the ultra-short pulse scale. The analysis relies on the method of characteristics and conserved quantities of the short-pulse equation and holds both on an infinite line and in a periodic domain. Numerical illustrations of the finite-time wave breaking are given in a periodic domain.

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The Effect of the Linear Coupling on the Phase Shift of Interacting Solitons in Coupled Nonlinear Schrödinger Equations

Michail Todorov
Technical University of Sofia, Bulgaria

For a system of nonlinear Schrödinger equations (SCNLSE) coupled both through linear and nonlinear terms, we investigate numerically the taking-over interaction dynamics of elliptically polarized solitons. In the case of general elliptic polarization, analytical solution for the shapes of a steadily propagating solitons are not available, and we develop a numerical algorithm finding the shape. We use as superposition of generally elliptical polarized solitons as the initial condition for investigating the soliton dynamics.

In order to extract the pure effect of the linear coupling, we consider the case without cross-modulation: the Manakov system to which a linear coupling term with real coefficient is added, when the individual solitons are actually breathers whose breathing frequency is defined by the linear coupling parameter. The interactions properties of the breathers differ from the the stationary shapes, mostly in the fact that changes of the initial phase angle of the components of the vector soliton do not affect the values of polarization after the interaction. The masses of the individual quasi-particles (QPs) oscillate, but the sum of the masses for the two QPs is constant. Respectively, the total energy oscillates during one period of the breathing, but the average over the period is conserved. Involving, however, into the interaction a nontrivial cross-modulation combined with different initial phase angles causes velocity shifts of interacted solitons.

The results of this work outline the role of the initial phase, initial polarization and the interplay between the linear and nonlinear couplings on the interaction dynamics of soliton systems in SCNLSE.

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Existence and Stability of Solitary Water Waves with Weak Surface Tension

Erik Wahlén
Universität des Saarlandes, Germany
(M. D. Groves)

Two-dimensional solitary water waves with weak surface tension ($0 < \beta < 1/3$, where β is the Bond number) are constructed by minimising the energy subject to the constraint of fixed momentum. The stability of the set of minimisers follows by a standard principle since the energy and momentum are conserved quantities. 'Stability' must however be understood in a qualified sense because of the lack of a global well-posedness theory for the initial value problem. The variational method relies on the concentration-compactness principle and a penalisation argument. Solitary solutions in the form of periodic wave trains modulated by exponentially decaying envelopes, to leading order described by the focusing NLS equation, have previously been constructed by other methods. An important ingredient in the variational method is the construction of a minimising sequence which resembles these solutions.

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Formation of Singularities Near Morse Points

Ingo Witt
Göttingen, Germany
(Martin Lippl)

Given a Morse function f and a Riemannian metric h on a C^∞ manifold M, we study solutions u to the equation

$$\Box_g u = 0,$$

where $\Box_g$ is the wave operator associated with the Lorentzian metric

$$g = \|df\|_h^2\, h - \zeta\, df \otimes df,$$

and $\zeta > 1$ a constant. As turns out, at Morse points initially regular solutions u start to form singularities that are then propagated as usual. These singularities can be described in terms of certain classes of conormal dstributions.

Such models arise in quantum field theory on curved space-times with changing topology.

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Special Session 38: Navier-Stokes Equations and Related Problems

Reinhard Farwig, Technische Universität Darmstadt, Germany
Jiri Neustupa, Czech Academy of Sciences, Czech Republic
Werner Varnhorn, Universität Kassel, Germany

Introduction: The section is oriented to qualitative theory of mathematical models in fluid mechanics, based on the Navier-Stokes (or related) equations. The topics especially include questions of existence and uniqueness of solutions, regularity, stability, asymptotic behaviour, influence of boundary conditions and theoretical properties of associated operators or function spaces.

Sharp Convergence of Viscous to Inviscid Flows under Slip Boundary Conditions

Hugo Beirao da Veiga
Pisa University, Italy

In the first part of the talk we present some results of convergence of (local in time) solutions, in the half-plane, of the Navier-Stokes equations under a Navier slip-boundary condition, to the solution of the Euler equations under the zero-flux boundary condition. In the second part of the talk we consider the case of a non-flat boundary. In this case the problem is open. We are interested in proving convergence in $C(\,[0,T];\,X\,)$, where X is the initial-data space, and problems are in the three dimensional case.

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On Existence Analysis for Unsteady Flow of Incompressible Fluids with Implicit Constitutive Relation for Cauchy Stress in Orlicz Space Setting

Miroslav Bulíček
Charles University, Prague, Czech Republic
(P. Gwiazda, J. Málek, A.Świerczewska-Gwiazda)

There are many realistic models in which the Cauchy stress is not continuous function of the symmetric part of the velocity gradient and/or the symmetric part of velocity gradient is not continuous function of the Cauchy stress. Therefore, it seems to be quite natural to assume instead of some explicit relation some implicitly given constraint, that is in our setting some maximal monotone graph. Moreover, it also seems to be quite natural to replace the standard power-law coercivity and growth by corresponding but more general Orlicz space setting.

For such models of unsteady flows of incompressible fluids we establish the existence of a weak solution provided that a) the graph for the Cauchy stress and the velocity gradient is maximal monotone graph; b) a priori estimate in Orlicz space allows us to get compactness of the velocity in L^2; c) the N-function ψ describing the coercivity of the velocity gradient satisfies Δ_2 condition.

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Asymptotic Profiles for Stationary Linearized Incompressible Flow Around Rotating and Translating Bodies

Paul Deuring
Université Lille Nord de France
(S. Kračmar, Š. Nečasová)

We consider a stationary linearized incompressible flow around a body which is rotating and translating. In [1], a Green's formula and estimates of a fundamental solution for this type of flow were derived. We use and extend these results in order to discuss asymptotic profiles of the flow in question.

[1] P. Deuring, S. Kracmar, S. Necasova: On pointwise decay of linearized stationary incompressible viscous flow around rotating and translating bodies. Submitted.

$\longrightarrow \infty \diamond \infty \longleftarrow$

L^q Gradient Estimates for Non-Newtonian Fluids

Lars Diening
University of Freiburg, Germany
(Peter Kaplicky)

We study the nonlinear Stokes problem with p-growth. We show optimal integrability of the gradients in terms of the integrability of the force. The approach is based on the nonlinear Calderon-Zygmund theory introduced by Iwaniec.

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Stokes Resolvent System with Navier's Slip in Bounded and Unbounded Domains

Veronika Fišerová
TU Darmstadt, Germany

We consider the Stokes resolvent system with a partial slip boundary condition of Navier's type in a uniformly smooth bounded and unbounded domain. Since the Helmholtz projection and hence also the

Stokes operator are not well defined in various unbounded domains unless $q = 2$ we replace the space L^q, $1 < q < \infty$, by the sum and intersection spaces $\tilde{L}^q$, where $\tilde{L}^q = L^q + L^2$ for $1 < q < 2$ and $\tilde{L}^q = L^q \cap L^2$ for $2 \le q < \infty$, in order to define the Stokes operator. Investigating the corresponding resolvent equation it is our aim to show that the Stokes operator satisfies the resolvent estimate and generates an analytic semigroup in $\tilde{L}^q$ for both a uniformly smooth bounded and unbounded domain.

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2D Flow Around a Rotating Body

Toshiaki Hishida
Nagoya University, Japan

The exterior steady problem in 2D is quite difficult. I would like to report that the situation becomes less difficult when the obstacle is rotating. This is because the rotation in 2D implies better asymptotic behavior of the flow at infinity. If the obstacle is a rotating disk, it is easy to see that there is a Navier-Stokes flow, as an exact solution, which decays like $1/|x|$. In this talk I consider the plane Stokes flow around a rotating obstacle without any symmetry and show that it decays like $1/|x|$ provided the support of the external force is bounded. What is remarkable is that a rotating profile is found in the leading term of the flow (while it is not in the leading term but in the second term for 3D case).

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Global Compensated Compactness Theorem for General Differential Operators of First Order

Hideo Kozono
Tohoku University, Sendai, Japan
(Taku Yanagisawa)

Let $A_1(x, D)$ and $A_2(x, D)$ be differential operators of the first order acting on l-vector functions $u = (u_1, \cdots, u_l)$ in a bounded domain $\Omega \subset R^n$ with the smooth boundary $\partial\Omega$. We assume that the H^1-norm $\|u\|_{H^1(\Omega)}$ is equivalent to $\sum_{i=1}^2 \|A_i u\|_{L^2(\Omega)} + \|B_1 u\|_{H^{\frac{1}{2}}(\partial\Omega)}$ and $\sum_{i=1}^2 \|A_i u\|_{L^2(\Omega)} + \|B_2 u\|_{H^{\frac{1}{2}}(\partial\Omega)}$, where $B_i = B_i(x, \nu)$ is the trace operator onto $\partial\Omega$ associated with $A_i(x, D)$ for $i = 1, 2$ which is determined by the Stokes integral formula (ν: unit outer normal to $\partial\Omega$). Furthermore, we impose on A_1 and A_2 a cancellation property such as $A_1 A_2' = 0$ and $A_2 A_1' = 0$, where A_i' is the formal adjoint differential operator of $A_i (i = 1, 2)$. Suppose that $\{u_m\}_{m=1}^\infty$ and $\{v_m\}_{m=1}^\infty$ converge to u and v weakly in $L^2(\Omega)$, respectively. Assume also that $\{A_1 u_m\}_{m=1}^\infty$ and $\{A_2 v_m\}_{m=1}^\infty$ are bounded in $L^2(\Omega)$. If either $\{B_1 u_m\}_{m=1}^\infty$ or $\{B_2 v_m\}_{m=1}^\infty$ is bounded in $H^{\frac{1}{2}}(\partial\Omega)$, then it holds that $\int_\Omega u_m \cdot v_m dx \to \int_\Omega u \cdot v dx$. We also discuss a corresponding result on compact Riemannian manifolds with boundary.

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Very Weak Solutions in Weighted L_q-Spaces to Equations of Flow Around a Rotating Body

Stanislav Kracmar
Czech Technical University, Czech Republic

We study the time-periodic flow of viscous incompressible fluid around a rotating body in in an exterior domain. For the corresponding linear and nonlinear problems we study solvability in weighted L_q-spaces. Recalling the results for the related problems of existence of a strong solution and using the duality argument we derive the existence of a very weak solution of the problems.

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A Note on Navier-Stokes Equations with Mixed Boundary Conditions

Petr Kucera
Czech Technical University, Czech Republic
(Michal Benes)

We solve a system of the Navier-Stokes equations for incompressible heat conducting fluid with mixed boundary conditions (of the Dirichlet or non-Dirichlet type on different parts of the boundary). We suppose that the viscosity of the fluid depends on temperature.

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Global Existence of Solutions for the One-Dimensional Motions of a Compressible Viscous Gas with Radiation

Sarka Necasova
Czech Academy of Sciences, Czech Republic
(B. Ducomet)

We will consider an initial-boundary value problem for the equations of 1D motion of a compressible viscous heat-conducting gas coupled with radiation through a radiative transfer equations. Assuming suitable hypotheses on the transport coefficients, we prove that the problem admits a unique solution.

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Global Special Solutions to the Micropolar Fluid Equations

Bernard Nowakowski
Polish Academy of Sciences, Poland

We investigate the existence of solutions to the micropolar fluids equations in cylindrical domains. These equations were introduced by A. Eringen in 1966 to describe the motion of viscous and incompressible fluids which exhibits the micro-rotational effects and micro-rotational inertia. It is believed that when characteristic dimensions of the flow is small, this approach provides for a better model than the corresponding Navier-Stokes equations, as it was shown in several experiments on numerous real fluids (e.g. animal blood, liquid crystals). We prove the global existence of solutions under assumption that the derivatives of the initial and external data along the axis of the cylinder are small in L_2-norm. We do not impose any restrictions on the initial velocity and microrotation fields. The proof is divided into two steps: first the long time existence of regular solutions is proved, next the solutions are prolonged in time to the infinity. We prove the existence of regular solutions such that the velocity and microrotation fields belong to $W_2^{2,1}(\Omega\times\mathbb{R}_+)$ and the gradient of pressure to $L_2(\Omega\times\mathbb{R}_+)$.

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Evolutionary Navier-Stokes Equations and the Zero Viscosity Limit

Patrick Penel
USTV, France

Since the Euler equation for incompressible flows formally follows from the Navier-Stokes equations on the zero viscosity limit, a natural question arises to know whether the solution of the Euler equation (with the classical impermeability boundary condition for the velocity) is an inviscid limit of a possibly unique branch of solutions to an appropriate Navier-Stokes problem: How to find an appropriate additional boundary condition for the velocity, that enables us to produce the required extension? We deal with two positive answers, either (1) inhomogeneous generalized impermeability boundary conditions, or (2) inhomogeneous Navier-type boundary condition. Both of them lead to the following structure of velocities: $\mathbf{u}^\nu = \mathbf{u}^0 + \nu\mathbf{v}^0 + \nu\mathbf{w}^\nu$, in $L^\infty(0,T^0;\mathbf{W}^{1,2}_\sigma(\Omega)) \cap L^2(0,T^0;\mathbf{W}^{2,2}(\Omega))$, with $0 \le \nu < \nu^*$, where $\mathbf{u}^0$ is the local-in-time solution to the Euler problem and where $\nu\mathbf{v}^0$ solves a linear (hyperbolic) model in terms of $\nu\Delta\mathbf{u}^0$.

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Existence and Uniqueness of Time-Periodic Solutions with Finite Kinetic Energy for the Navier-Stokes Equations in $\mathbb{R}^3$

Ana Silvestre
Instituto Superior Técnico, Lisbon, Portugal

Consider a Navier-Stokes liquid filling the whole three-dimensional space and subject to a time-periodic external force. We construct a time-periodic strong solution with finite kinetic energy (at all times) for the corresponding time-periodic equations. For this, appropriate conditions on the external force have to be imposed, for instance, that, for all times, it is a function with null average and support contained in a fixed compact set, together with a smallness condition involving the viscosity of the fluid. First, a linearized version of the problem is analysed using the Fourier transform in the space variable, and then a strong solution to the full nonlinear problem is obtained by a fixed point procedure. We also show that such solution satisfies the energy equality and is unique within a larger class.

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Asymptotic Energy Concentration in Solutions to the Navier-Stokes Equations

Zdeněk Skalák
Czech Academy of Sciences, Czech Republic

We show that every nonzero global weak solution to the homogeneous Navier-Stokes equations satisfying the strong energy inequality exhibits the large-time energy concentration in a particular frequency. It is possible to see from some simple examples that the large-time energy concentration does not generally occur if the right hand side of the equations is a nonzero time-dependent function decreasing to zero with time approaching infinity. Nevertheless, we present a wide class of the initial conditions and the right hand sides, for which the associated solution still exhibits the above mentioned phenomenon.

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Parabolic Systems in Two Space Dimensions with Critical Growth Behaviour

Maria Specovius-Neugebauer
University of Kassel, Germany

(Jens Frehse)

Consider parabolic systems

$$u_t - \operatorname{div}\left(a(t,x,u,\nabla u)\right) + a_0(t,x,u,\nabla u) = 0$$

in two space dimensions with initial and Dirichlet boundary conditions. The elliptic part including a_0 is derived from a potential with quadratic growth in ∇u and is coercive and monotone. The term a_0 may grow quadratically in ∇u and satisfies a sign condition of the form $a_0 \cdot u \ge -K$. Existence of a regular long time solution verifying a regularity criterion of Arkhipova is shown without smallness assumptions on the data. The assumptions on the data may be weakened with the help of finite element techniques.

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Shape Stability of Incompressible Fluids Subject to Navier's Slip

Jan Stebel
Czech Academy of Sciences, Czech Republic

In a number of situations it is convenient to assume that an incompressible fluid can slip at the solid boundary. Mathematical analysis of resulting problems reveals some nice properties which are not available in presence of the widely used no-slip. On the other hand, the slip conditions are more sensitive to perturbations of the boundary, that is, a small imperfection or roughness can lead to big change in the solution. In this talk we show sufficient conditions for the smoothness of boundary perturbations under which the slip remains preserved.

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The Flow of a Liquid with Cavitation

Ivan Straskraba
Academy of Sciences, Prague, Czech Republic

In this contribution a mathematical model of a fluid flow in a tube with cavitation will be presented. The flow is described by a system of partial differential equations in one space dimension for the pressure, the velocity, and the concentration of possible gas bubbles. Adequate initial and boundary conditions are prescribed.

As a preliminary analysis, the special solutions are constructed under various assumptions on their structure. The cost for the explicit results is that initial and boundary data are satisfied in the fragmental way, that is this which is allowed by prescribed structure of the solutions.

Nevertheless, important physical situations are described thus being a base for particular experiments. Then the experimental data can be compared with those theoretically aquired.

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Global Regular Solutions to the Navier-Stokes Equations with Large Flux

Wojciech Zajaczkowski
PAN and Military Univ. of Warsaw, Poland
(Joanna Renclawowicz)

We consider the Navier-Stokes motion in a bounded cylinder with slip boundary conditions. We assume an inflow and an outflow of the fluid through the bottom and the top of the cylinder where the magnitude of the flux is not restricted. We require that the derivatives of initial velocity and the external force with respect to the variable along the axis of the cylinder are sufficiently small. Then our solutions are close to 2-dimensional solutions to NSE and we are able to prove the existence of long time regular solutions. Since the considered flux does not vanish in time we need to use the Hopf function to derive the global energy estimate. The proof is divided into three steps. First, we obtain a long time estimate for regular solutions. Next, we prove the existence by the Leray-Schauder fixed point theorem. Finally, we prolong the solution with respect to time up to infinity. Consequently, we are able to prove the existence of global regular solutions without restrictions on the magnitudes of the initial velocity and the external force.

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Special Session 39: Mathematical Cancer Modelling

Thomas Hillen, University of Alberta, Canada
Urszula Ledzewicz, Souther Illinois University, USA
Arnaud Chauviere, University of Texas - Houston, USA

Introduction: Cancer is made of complex biophysical and biochemical processes involved in many different phenomena at various temporal and spatial scales. A deep interest in cancer modelling among the research community has intensified in recent years. This intense activity has led to rapid improvements in the way we approach cancer modelling: From traditional models describing tumors as spheroids made up of only one cell type with constant density, to more recent multiscale models accounting for several types of cells (e.g., tumor and host cells moving, replicating and dying) that live in a complex environment filled with proteins and growth factors that affect single cell behavior. Resulting models offer many challenging problems to mathematicians, whose solutions may help the understanding of tumor growth and the development of treatment strategies. Our special session aims to gather scientists working in various fields of cancer modelling to discuss recent approaches and new results.

Open Discussion on Cancer Modelling

Arnaud Chauviere
University of Texas, Houston, USA
(Thomas Hillen and Urszula Ledzewicz)

We will close our special session “Mathematical cancer modelling” with a discussion session to summarize what we have learned during the talks and attempt to crosslink or contrast different approaches. One of our goals will be to summarize the advances that have been made from a modelling point of view, and how treatment strategy may be improved by means of mathematical approaches.

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Mathematical Modelling of Cell Proliferation, Circadian Rhythm and Pharmacokinetics-Pharmacodynamics to Optimise Cancer Treatments

Jean Clairambault
INRIA, France

Cell proliferation in health and disease, particularly cancer, is never a question involving a single isolated cell, but rather a problem to be considered at the level of cell populations. For this reason, physiologically structured partial differential equations modelling the cell division cycle in proliferating cell populations have been designed. The structure variables used are not space, but age in the cell cycle or in one of its phases, cell size, content in DNA or specific proteins (cyclins). Physiological cell proliferation control, that is disrupted in cancer, relies in particular on molecular circadian clocks (one is present in each nucleated cell), for which models, and models of their control on the cell division cycle, have also been developed. Another type of control on cell proliferation is exerted by anticancer drugs, that have effects on both healthy and cancer cells, and these effects on their targets at the cell level are again dependent on circadian clocks. More precisely, the pharmacokinetics-pharmacodynamics (PK-PD) of these drugs is dependent on circadian clocks, both at the whole body level (blood, liver, kidney) and at the intracellular level through their activation and detoxication mechanisms (enzymes, glutathione, ABC transporters). Hence an optimal control problem: destroying cancer cells with minimal damage to healthy cell populations, taking into account circadian influences. This can be tackled and partly solved by taking advantage of differences, known though not completely understood, between healthy and cancer cells with respect to the effects of drugs at different times of the 24-hour span. This is the object of cancer chronotherapeutics, in use in the clinic, for which modelling approaches will be presented.

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Analyzing Emergent Behaviour in Cellular Automaton Models of Cancer Invasion

Andreas Deutsch
Technical University Dresden, Germany

Deciphering the principles of tumour growth is crucial for the development of new therapy concepts. Besides increasingly complex molecular investigations, mathematical modelling and computer simulation of selected aspects of tumour growth have become attractive within the last few years. Here, we focus on the analysis of cancer invasion. Lattice-gas cellular automaton (LGCA) models allow for an adequate description of individual invasive cancer cell behaviour (microscopic level). We will show how analysis of invasive LGCA models allows for prediction of emerging macroscopic properties (in particular of the invasion speed). Furthermore, we will use our models for the interpretation of data from in vitro glioma cell invasion assays.

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Modelling the Evasion of Tumours from Immune Control

Alberto d'Onofrio
European institute of Oncology, Italy

The complex and nonlinear interplay of the immune system (IS) with non-self entities offers an ideal area of research for the mathematical physics of non-equilibrium systems and, indeed, has long been a source of fascination for theoretical researchers. In particular, the interaction of IS with tumours is classical challenge in the field of biomathematics. Indeed, Tumours are a wide family high-mortality diseases, all characterized by a remarkable lack of symptoms and by time courses that may be classified as non-linear since they macroscopically reflect a number of intra-cellular and inter-cellular phenomena which are strongly non-linear. This inherent non-linearity might be the most important key to explain why, despite the enormous efforts in medical research, "tumours" remain one of the leading causes of death worldwide. Besides non-linearity, another important point to stress is that the structure of the above mentioned interactions is also to some extent due to a series of phenomena whose nature may be classified as evolutionary.

Molecular biology has shown that tumour cells (TCs) are characterized by a vast number of genetic events leading to the appearance of specific antigens, which trigger actions by the IS. These experimental observations have provided a corpus of evidences to the old speculative hypothesis of immune surveillance, i.e. that the IS may act to control or eliminate tumours. An important point to stress is that the structure of the T-IS interactions is also time-varying at two different scales, fast and slow, which may be a cause of evasion of the tumour from immune control. The fast scale is related to the very initial phases of growth when immune system cells dynamically learn to recognize and target TCs, which might allow many neoplasms to escape from the immune control. At the slow scale, remarkably, the TCs are characterized by a considerable evolutionary ability to enhance their survival in a hostile environment through behavioural strategies interrelated with phenotype changes. Indeed, if the IS is not able to eliminate a neoplasm, a suboptimal control is possible by establishing a dynamic equilibrium, such that the tumour may only survive in a small steady state. However, over a long period of time the neoplasm may develop multiple strategies to circumvent the action of the IS, which may allow it to re-commence growing to its carrying capacity. Moreover, over the long temporal range not only those slow evolutionary processes but also the IS degradation due to natural senescence can explain long term evasions. However, evasions at temproal mesoscale are presumably representative of the vast majority of cases of immune surveillance failure. Here we face immunoevasion on these three scales by illustrating some "simple" nonlinear time-varying models, all derived by suitable modifications of a unique nonlinear model.

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A Mathematical Model for the Radiation Response of Tumour Spheroids

Alberto Gandolfi
IASI-CNR, Rome, Italy

(A. Bertuzzi, C. Bruni, A. Fasano, F. Papa, C. Sinisgalli)

The model is aimed at describing the radiation response observed in cultured tumour spheroids. The effect of an impulsive irradiation is assumed to instantly move the cells from a viable compartment to a compartment of lethally damaged cells. The dose dependence is described by the linear- quadratic model. Lethally damaged cells successively die and dead cell are degraded with volume loss. To take into account that cell death and cell degradation occur after a distributed delay, the lethally damaged cell compartment and the dead cell compartment are subdivided into a finite number of subcompartments having Poisson exit. The evolution of the spheroid is represented by a continuous model in spherical symmetry and the cell velocity is determined by assuming that the local volume fraction occupied by the cells is constant. Oxygen diffusion and consumption are included, and the radiosensitivity parameters depend on the local oxygen concentration. The identifiability of the parameters has been studied, and literature data have been used to test the model and estimate model parameters.

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Optimal Schedules of Cancer Screening Based on Models of Cancer Progression

Leonid Hanin
Idaho State University, USA

(Lyudmila Pavlova)

The problem of optimal cancer screening consists of finding optimal ages of medical exams for a specific cancer subject to certain constraints on their number and timing. As a functional to be optimized, we use the probability that by the time of primary tumor detection it has not yet metastasized. Such a selection of the efficiency functional leads to an improvement in the prevention of metastatic cancer. An explicit formula for the functional was found based on a comprehensive mechanistic model of cancer progression and detection that includes tumor latency, tumor growth, metastatic progression, and

a quantal response type model of cancer detection. Optimal screening schedules with a fixed number of exams (n = 10) were computed in the case of breast cancer with model parameters previously estimated from a large population data base of breast cancer incidence. These screening schedules are compared with those based on the efficiency functional defined as the expected reduction in tumor size at detection.

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Unraveling the Mechanisms of Glioma Tumor Invasion

Haralampos Hatzikirou
UT Houston, USA

Invasion of malignant glioma tumors is typically very aggressive and a highly complex phenomenon involving molecular and cellular processes at various spatio-temporal scales, whose precise interplay is still not fully understood. In order to identify the cellular mechanisms of glioma tumor invasion, we study an in vitro culture of glioma cells from the literature. By means of a computational approach based on a cellular automaton model, we compare theoretical results to the experimental data and deduce microscopic interactions (cellular mechanisms) from microscopic and macroscopic observables (experimental data). In particular, for the first time, it is theoretically shown that the migration/proliferation dichotomy plays a central role in the invasion of glioma cells. Additionally, we observe that radial persistence of glioma cells nearby dense areas accelerates the invasion process. We argue that persistence may result from a cell-cell repulsion mechanism. When glioma cell behavior is regulated through a migration/proliferation dichotomy and a self-repellent mechanism, our mathematical model reproduces faithfully the experimental observations.

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Tumor Control Probability Models for Radiation Treatment

Thomas Hillen
University of Alberta, Canada
(A. Dawson, J. Gong, G. de Vries, C. Finlay)

The tumor control probability (TCP) is a measure for the expected success of a given radiation treatment schedule. In my talk I will first review some simple TCP models which are based on Binomial and Poissonian statistics and the linear quadratic model. Then I will discuss the extensions from Zaider-Minerbo (ZM) and Dawson-Hillen (DH). The ZM- model allows to include arbitrary treatment schedules, while the DH- model includes cell cycle dynamics as well. We apply these theories to typical prostate cancer treatments. One result supports the use of hyperfractionation schemes as a method to reduce late effects on healthy tissue, while keeping the same effect on cancerous tissue. I will also discuss the use of this model related to cell-cycle synchronization methods.

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A Continuous Model for Microtubule Dynamics with Catastrophe, Rescue and Nucleation Processes

Peter Hinow
University of Wisconsin - Milwaukee, USA
(Vahid Rezania and Jack Tuszynski)

Microtubules are long, stiff polymers made of tubulin and form a major component of the cytoskeleton. Besides giving structural stability and rigidity to a cell, microtubules play key roles in many physiological processes such as intracellular vesicle transport and chromosome separation during mitosis. Due to their central role in the formation of the mitotic spindle, microtubules are a classical target of anti-cancer drugs (such as paclitaxel, vinblastine and others) that interfere with the polymerization and depolymerization processes of tubulin.

We propose a deterministic mathematical model at the population level that accounts for all known processes taking place during microtubule polymerization and depolymerization, namely growth, nucleation, catastrophic shrinkage and rescue events. It can be shown easily that the total amount of tubulin in both its free and bound forms is conserved. While striving for completeness in the mathematical description we have also attempted to introduce a small number of model parameters (precisely 7), most of which can be determined from experimental data or have been published in the literature. The resulting dynamical behavior of the microtubules agrees well with previously reported experimental data.

This is joint work with Vahid Rezania and Jack Tuszynski (Department of Physics, University of Alberta, Edmonton, Canada).

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Understanding the Role of Diacylglycerol (DAG) as a Second Messenger

Mary Ann Horn
National Science Found. & Vanderbilt Univ., USA
(Hannah L. Callender)

Diacylgylcerol (DAG) plays a key role in cellular signaling as a second messenger. In particular, it regulates a variety of cellular processes and the break-

down of the signaling pathway that involves DAG contributes to the development of a variety of diseases, including cancer. This talk will focus on a mathematical model of the G protein signaling pathway in RAW 264.7 macrophages downstream of P2Y6 receptors activated by the ubiquitous signaling nucleotide uridine 5'-diphosphate. The model, which is based on time-course measurements of inositol trisphosphate, cytosolic calcium, and diacylglycerol, focuses particularly on differential dynamics of multiple chemical species of diacylglycerol. Our primary goal is to better understand the role of diacylglycerol in the signaling pathway and the underlying biological dynamics that cannot always be easily measured experimentally. (Joint work with H. Alex Brown and the Brown Laboratory at Vanderbilt University.)

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A Mathematical Model of Bladder-Urothelium Carcinogen Penetration as an Initial Stage of Bladder Cancer Development

Eugene Kashdan
Tel Aviv University, Israel
(Svetlana Bunimovich)

Bladder Cancer is the seventh most common cancer worldwide. According to the existing statistics, 80% of its patients had occupational exposure to chemicals and/or were smoking regularly during long periods of time. The carcinogen from the bladder lumen affects umbrella cells of the urothelium (bladder surrounding epithelium) and then subsequently penetrates into the deeper layers of the tissue (intermediate and basal cells). The basic structure of our model is a two-dimensional grid, which represents a slice of the urothelium. To model tumor formation we introduce the carcinogen penetration process into the urothelial cell living cycle. The cell cycle is modeled using a Cellular Automata formulation, which consists of an array of automaton elements, identified with the real urothelial cells, and represents cell dynamics on the discrete level. We address carcinogen penetration as a continuous problem defined on the same domain and modeled by the two-dimensional nonlinear reaction-diffusion equation with the space-dependent diffusion coefficient and the singular source term representing carcinogen concentration on the bladder wall. The sequence of mutations is starting as soon as an average carcinogen concentration level inside the cell passes a predetermined threshold, and it is followed by the micro-tumors formation by the clones of mutated cells.

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Modeling the p53-Mdm2 Core Module in Neuroblastoma

Florian Lamprecht
DKFZ Heidelberg, Germany
(Daniel Dreidax, Frank Westermann, Thomas Höfer)

The p53-mdm2 regulatory unit plays a major role in the cell decision to undergo apoptosis or cell cycle arrest upon DNA damage. These two proteins are very frequently dysregulated in tumor cells. In neuroblastoma, a pediatric solid tumor of the sympathetic nervous system, the amplification of the *N-Myc* gene is closely related to the dysfunction of the p53-Mdm2 core module leading to impaired cell-cycle arrest and DNA-damage response. Unlike in other tumor entities, the aberrant function cannot be attributed to genetic mutations but appears to be due to an imbalance in the expression of the *p53* and *Mdm2* genes as well as other N-Myc targets. To rationalize how perturbed gene expression causes dysfunction of the p53-Mdm2 module, we developed a kinetic model that described the dynamics of these proteins as well as cell cycle-related (p21, p27) and apoptose-related (PUMA) p53 target proteins as readouts. First results indicate that the complex interplay of p53, Mdm2, E2F1 and N-Myc, can account for the experimentally observed dynamics under physiologic conditions and in *N-Myc* amplified tumor cells. Although in *N-Myc* amplified cells both pro-apoptotic p53 and anti-apoptotic Mdm2 are overexpressed, Mdm2 appears to exert a dominant inhibitory affect on p21 that impairs the DNA damage response.

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Chemotherapy Modeling Based on Clinical Data for Methotrexate Treatment of Non-Hodgkin Lymphoma and Optimal Control of Drug Schedules

Dirk Lebiedz
University of Freiburg, Germany
(Marcel Rehberg, Benjamin Kasenda, Melanie Franzem, Michael Engelhart, Sebastian Sager, Gerhard Illerhaus)

Although mathematical modeling of cancer and its treatment by chemotherapy are well established in bio-mathematical research, actual applications, for example optimal control of treatment schedules, in the clinical praxis are rare. One reason is that sufficient data, e.g. pharmacokinetic measurements for individual patients, is often not available. In the treatment of central nervous system lymphoma methotrexate based schedules are a standard procedure. Due to the high doses used the serum level of methotrexate is routinely measured which

allows individually fitted pharmacokinetics in modeling. In cooperation with the University Hospital Freiburg we developed a minimal model based on the data from patients treated with methotrexate. Besides the pharmacokinetic part and the pharmacodynamic effects on the tumor we also included the effect of the drug on the white blood cell count which is a major toxicity constraint in chemotherapy and might play an important role in determining optimal dosing and timing. The model was fitted to several groups of patients who received different amounts of methotrexate during their treatment. The resulting parameters were statistically analyzed and a sensitivity analysis was carried out to assess the predictive capabilities of the model. Further we used the model as foundation to investigate different optimal control scenarios. In order to obtain clinically feasible dosing schedules strategies from mixed-integer-optimal control have been employed. Overall, our approach shows prospects and difficulties of the application of mathematical modeling and optimal control in clinical cancer chemotherapy.

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Minimizing Tumor Volume for Mathematical Models of Combined Chemotherapy and Anti-Angiogenic Treatment with Pharmacokinetics: Optimal and Realizable Suboptimal Protocols

Urszula Ledzewicz
Southern Illinois University, USA
(Heinz Schaettler and Helmut Maurer)

Tumor anti-angiogenesis is an indirect cancer treatment approach that targets the vasculature of the tumor, but by itself does not kill the cancer cells. It therefore needs to be combined with other mechanisms, such as chemotherapy or radiotherapy, in order to simultaneously attack both the tumor cells and the vasculature that supports it. In this talk we use a mathematical model for tumor anti-angiogenesis originally formulated by Hahnfeldt et al. at Harvard School of Medicine as a base and expand it to include the action of a cytotoxic agent, both on the cancer cells and the vasculature. We consider the problem of minimizing the tumor volume for a priori given amounts of both agents. Mathematically generally this becomes a challenging multi-input optimal control problem. The problem gets even more complicated when the pharmacokinetics of the drugs is taken into account. Both analytical and numerical solutions indicate the presence of complex phenomena like singular controls (corresponding to time varying partial doses) or even chattering controls (infinitely many switchings in arbitrary time intervals), which are not practically realizable. On the other hand, it is being shown that from a practical point of view the solutions exhibit strong robustness properties and simple suboptimal protocols with a small number of switchings exist that achieve close to optimal behavior. Numerical results concerning the sequencing of the drugs in the model agree with experimental observations on combination therapy.

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Optimisation of Tumour Irradiation by Cell Cycle Synchronisation

Michael Meyer-Hermann
HZI Braunschweig, Germany
(Harald Kempf, Marcus Bleicher)

Heavy-ion cancer therapy has explored great success in the past years. The treatment of patients is very successful in precisely destroying the tumour while avoiding harm to the surrounding healthy tissue. Thus, only minor side-effects are encountered. Last November HIT, the Heavy-Ion-Therapy-Centre was opened in Heidelberg, a clinic devoted exclusively to cancer treatment with heavy ions. While the success of this method is obvious, there is still room for improving the treatment efficiency and for further reducing the side effects. We use a mathematical model based on the method Delaunay-Object-Dynamics (DOD), an agent-based modelling framework, to investigate the growth of the tumour between irradiation sessions and investigate different irradiation protocols. The model resolves the cell cycle phases of the tumour cells and the differences of irradiation efficiency in these phases. It is found that irradiation synchronises the cell cycle of the cells in the tumour and by this allows to induce a higher degree of tumour destruction in the next irradiation session. The established modelling framework will be explored to treat similar questions of clinical relevance and to support treatment planning in the future.

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The Bifurcation Analysis of Family of Angiogenesis Models with Discrete Delay(s)

Monika Piotrowska
University of Warsaw, Poland
(Urszula Foryś)

Angiogenesis is the process of new blood vessels formation from the pre-existing ones under the influence of angiogenic factors secreted by cells. It is a normal and vital process in growth and development of organisms. It is also required during the repair mechanism of damaged tissues such as wound healing processes. However, angiogenesis is also an essential step in the solid tumours transition from

the avascular forms (less harm for hosts) to cancers that are able to metastase and cause lethal outcome of the disease. On the other hand, understanding of the dynamics of angiogenesis might give a possibility for efficient treatment of cancer since the anticancer drugs can better penetrate the tumour structure when they are distributed with blood nourishing tumour mass. In our study we look more closely at a family of angiogenesis models which includes models earlier proposed by Hahnfeldt et al. and Ergun et al. Since we are interested in the influence of time lags in the processes of tumour growth and/or vessels formation on cancer development we have introduced discrete time delay(s) to the model(s). Our purpose is to investigate analytically and numerically the behaviour of the presented model(s) solutions and the stability of bifurcations occurring with increasing delay(s).

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Mechanical Aspects of Tumour Growth

Luigi Preziosi
Politecnico di Torino, Italy
(D. Ambrosi, A. Tosin, C. Verdier)

Tumour cells usually live in an environment formed by other host cells, extra-cellular matrix and extracellular liquid. Cells duplicate, reorganise and deform while binding each other due to adhesion molecules exerting forces of measurable strength. They also attach to the extracellular matrix and pull on it. In this talk a macroscopic mechanical model of solid tumour is investigated which takes both adhesion mechanisms into account. In order to define the relationship between stress and strain for the cellular constituents, the deformation gradient is decomposed in a multiplicative way distinguishing the contribution due to growth, to cell rearrangement and to elastic deformation. On the basis of experimental results at a cellular level, it is proposed that at a macroscopic level there exists a yield condition separating the elastic and dissipative regimes. A link is also drawn between microscopic measurement and macroscopic parameters to be used in the macroscopic model. Previously proposed models are obtained as limit cases, e.g. fluid-like models are obtained in the limit of fast cell reorganisation and negligible yield stress. The results obtained by the model are compared with several experimental observations.

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A Model of Dendritic Cell Treatment of Solid Tumors

Ami Radunskaya
Claremont Colleges, Pomona College, USA
(L. G. dePillis, A. Gallegos)

Immune therapies for cancer have garnered increasing interest over the last 10 years. One of the most promising immune therapies is dendritic cell treatment. Dendritic cells (DCs) are part of the adaptive immune response and function as antigen-presenting cells, triggering the production and activation of effector cells. DCs are naturally derived in the bone marrow and reside in peripheral tissues. Upon encountering pathogen, DCs travel to the lymphoid organs where they stimulate differentiation and maturation of cytotoxic T lymphocytes (CTLs). These activated CTLs then traffic to the infected tissue to form part of the adaptive immune response. Dendritic cell treatment consists of injecting primed cells into the patient to trigger an improved immune response to an existing tumor.

In this mathematical model, we describe the trafficking of DCs and immune cells between the peripheral blood, lymph, and tumor compartments, as well as the interaction between DC, activated and memory effector cells, and tumor cells. Delays in the model capture synaptic connection times, and data from murine studies of the effect of DC injections are used to calibrate the model. Sensitivity analyses and stability analysis suggests which model parameters are important in the prognosis of disease progression and response to treatment.

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Physical Limits in Tumor Cell Migration

Katarina Wolf
NCMLS, Nijmegen, The Netherlands
(Peter Friedl)

Tumor cell invasion into extracellular matrix is a key step towards tumor progression and metastasis that is controlled by multiple parameters including the physical spacing of ECM. Collagen type I as a main component of the ECM consists mainly of fibres and bundles, forming gaps and spaces that range from sub- to supracellular cell diameters. Invading mesenchymal tumor cells capable of pericellular proteolysis due to cell surface-expressed proteases, such as membrane-type matrix metalloproteinases (MT-MMPs), transmigrate subcellular pores by degrading constricting collagen fibres which is associated by spindle-shaped cell morphology and oval nucleus. We here define the mechanisms and physical limits of protease-independent migration, using spatially controlled 3D fibrillar collagen lattices and transwell chambers of different pore sizes. When treated with

MMP inhibitors, different tumor cells maintained amoeboid nuclear squeezing in subcellular-porous lattices, whereas microporous networks forced migration arrest, accompanied by non-moving round nuclei extenting thin, nose-like formations towards dynamic long cytoplasmic extensions. Thus, the physical dimensions of ECM gaps and pores control nuclear morphology, and therefore efficiency and protease requirements of cancer cell migration. Our observations on limits in tumor cell migration determined by different parameters such as collagen pore size, cell size, proteolysis, nuclear volume and deformability deliver parameters suitable for mathematical modelling.

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Special Session 40: Recent Developments in Optimal Control

Urszula Ledzewicz, Souther Illinois University, USA
Heinz Schaettler, Washington University, USA
Helmut Maurer, Universität Münster, Germany

Introduction: In this session recent developments in optimal control of dynamical systems described by ordinary differential equations will be presented. The topics cover a broad spectrum of theoretical, numerical and applied aspects of optimal control. They range from classical geometric techniques to modern non-smooth approaches and intertwine these theoretical and numerical developments with practical applications. These include quantum control, oil exploration, biological and medical systems. They provide a diversity of view-points of new developments in the control of nonlinear systems both from the type of methods used and areas from which the applications are drawn.

Optimal Control Models of Oil Exploration and Extraction

Matthias Bruns
Universität Münster, Germany
(Helmut Maurer, Willi Semmler)

Oil extraction and exploration is a prominent example of the optimal management of exhaustible resources; cf. Hotelling (1931), Pindyck (1978). In this talk, we discuss an optimal control model of oil extraction and exploration which extends the work by Liu, Sutinen (1982) and Greiner, Semmler (to appear). The model assumes monopolistic competition as market structure, and uses downward sloping demand curve with extraction rates, known stock of the resource and cumulated past extraction as determining factors.

The extraction control appears nonlinearly in the control system and is shown to be continuous. The control variable representing exploration appears linearly in the Hamiltonian which gives rise to bang-bang and singular arcs. The sequence of such arcs strongly depends on the form of the function modeling exploration costs. It is shown that in many cases the optimal extraction rate may follow an inverted U-shaped time path.

Moreover, we introduce a state delay $d > 0$ into the dynamics, which represents a few time periods to prepare explored deposits for extraction. We apply the necessary conditions and numerical discretization methods from Goellmann, Kern, Maurer (2009) and compute optimal solutions for some delays.

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Energy Minimization in Two-Level Quantum Control: The Integrable Case

Jean-Baptiste Caillau
Bourgogne University & CNRS, France
(Biari, M.; Bonnard, B.)

The aim of this contribution is to refine the computations of [2, 3]. The Lindblad equation modelling a two-level dissipative quantum system is investigated. The control can be interpretated as the action of a laser to rotate a molecule in gas phase, or as the effect of a magnetic field on a spin 1/2 particle (see [2] and references therein). For the energy cost, normal extremals of the maximum principle are solution to a three-dimensional Hamiltonian with parameters. Complete quadratures are given for an integrable submodel of the problem using rational or Weierstrass elliptic functions. Preliminary computations of spheres and conjugate loci are also provided for a two-dimensional subcase using [1].

[1] Bonnard, B.; Caillau, J.-B.; Trelat, E. Second order optimality conditions in the smooth case and applications in optimal control. ESAIM Control Optim. and Calc. Var.13 (2007), no. 2, 207–236. apo. enseeiht. fr/cotcot.

[2] Bonnard, B.; Cots, O.; Shcherbakova, N.; Sugny, D. The energy minimization problem for two-level dissipative quantum systems. Preprint (2009), 1-34.
[3] Bonnard, B.; Shcherbakova, N.; Sugny, D. The smooth continuation method in optimal control with an application to quantum systems. ESAIM Control Optim. and Calc. Var. (to appear).

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Discrete and Differential Homotopy in the Control of the Restricted Three-Body Problem

Bilel Daoud
Bourgogne University & CNRS, France
(Caillau, Jean-Baptiste; Gergaud, Joseph)

The planar circular restricted three-body problem [5] is considered. The control enters linearly in the equation of motion to model the thrust of the third body. The minimum time optimal control problem has two scalar parameters: The ratio of the primaries masses which embeds the two-body problem [2, 3] into the three-body one, and the upper bound on the control norm. Regular extremals of the maximum principle are computed by shooting thanks to continuations with respect to both parameters. Discrete and differential homotopy [1, 4] are compared. Second order sufficient conditions in optimal control are also tested.

[1] Allgower, E. L.; Georg, K. Introduction to Numerical Continuation Methods. SIAM Classics in Applied Mathematics, vol. 45, 2003.
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[4] Caillau, J.-B.; Cots, O.; Gergaud, J. apo. enseeiht. fr/hampath.
[5] Szebehely, V. Theory Of Orbits, The Restricted Problem of Three Bodies. Academic Press, 1967.

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A Nonsmooth Maximum Principle for Optimal Control Problems with State Constraints

Maria do Rosario de Pinho
University of Porto-FE, Portugal
(Haider Biawas)

A new version of the Nonsmooth Maximum Principle has been recently obtained by Clarke and de Pinho. A special feature of this result is the fact that it asserts the full Weierstrass condition together with the Euler form of the adjoint equation, thereby extending a result of de Pinho and Vinter. Applying well known techniques we show how this novel result can now be extended to cover some particular problems with state constraints.

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Numerical Approach of an Optimal Harvesting Problem for a Controlled Predator-Prey System with Age-Structure

Gabriel Dimitriu
University of Medicine and Pharmacy, Romania
(R. Stefanescu)

The control theory of age-dependent single species has been intensively researched and well developed. However, several papers concerning optimal control of age-dependent multi-species have come into view just very recently. In this talk, we investigate numerically an optimal harvesting problem (introduced by He Z. and Wang H. in *Appl. Math. J. Chinese Univ.*, 2009) for a system consisting of two populations with age-structure and interaction of predator-prey. The numerical implementation makes use of the optimality conditions derived by means of normal cone and adjoint system techniques.

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Notes on Solution Stability in Optimal Control Problems with Bang-Bang or Bang-Singular-Bang Control Behavior

Ursula Felgenhauer
Brandenburg University of Technology, Germany

In the paper, optimal control problems driven by ordinary differential equations are analyzed with respect to their parameter dependency. The focus is set on problems where the dynamics depend linearly on the control. Typical solution pattern then consist of bang-bang and/or singular control arcs.

In case of pure bang-bang optimal control structure, stability results for the switching points behavior have been found for vector-valued control functions. They include conditions for differentiability of the switching points w.r.t. parameters in case of so-called simple switches of at most one control component, and directional differentiability for the case when several control components switch simultaneously. Typically, the stability conditions closely correspond to sufficient optimality conditions.

Further, we consider the case of scalar controls with bang-singular-bang structure when the singularity is of first kind. Conditions are formulated which ensure Lipschitz stability of localization of

the bang-singular junction points and continuity of the singular feedback control law w.r.t. parameter changes.
The stability analysis of bang-singular-bang optimal controls is in progress yet. The results will be illustrated by examples.

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Application of Optimal Multiprocesses to Mixed Constrained Problems

M. Margarida Ferreira
FEUP-ISR, Portugal
(M. d. R. de Pinho)

Consider the following state and mixed constrained optimal control problem (P):

$$(\mathrm{P}) \quad \begin{cases} \text{Minimize } l(x(1)) \text{ subject to} & \\ \dot{x}(t) = f(t, x(t), u(t)) & \text{a.e. } t \in [0,1] \\ \gamma(t, x(t), u(t)) \leq 0 & \text{a.e. } t \in [0, t_1) \\ h(t, x(t)) \leq 0 & \text{a.e. } t \in [t_1, 1] \\ u(t) \in U(t) & \text{a.e. } t \in [0,1] \\ x(0) = x_0 & \\ t_1 \in [0,1]. & \end{cases}$$

Here $l : R^n \to R$, $f : [0,1] \times R^n \times R^k \to R^n$, $\gamma : [0,1] \times R^n \times R^k \to R^m$, $h : [0,1] \times R^n \to R$ are given functions and $U : [0,1] \to R^m$ is a given multifunction.

Using the theory of multiprocesses developed by R. Vinter and F. Clarke and techniques associated to mixed constrained optimal control problems, we derive necessary conditions for problem (P). Extensions to the case where the variable instant t_1 is replaced by a set of variable points distinguishing intervals with different constraints are also analyzed.

Problem (P) can be associated to a particular class of nonregular mixed constrained problems where the mixed constraint is reduced to a pure state constraint on certain subintervals of the original time interval.

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Optimal Control of Cancer Immune System Interactions under Treatment

Urszula Ledzewicz
Southern Illinois University, USA
(Heinz Schaettler and Mohammad Naghnaeian)

We are considering a model that describes the interactions between cancer and immune system as an optimal control problem with the action of a single cytotoxic agent as the control. In the uncontrolled system there exist both regions of benign and of malignant cancer growth that are separated by the stable manifold of a saddle point. The aim of treatment is to move an initial condition that lies in the malignant region into the region of benign growth. A Bolza formulation of the objective is given that includes a penalty term that approximates this separatrix by its tangent space and minimization of the objective is tantamount to moving the state of the system across this boundary.

In this talk, generalizing earlier results when the action of the cytotoxic agent on the immune system was assumed negligible, we now consider a killing action of the cytotoxic drug on both the cancer cells and the cells of the immune system. For various values of a parameter ε that describes the relative effectiveness of the drug onto these two classes of cells, we discuss the structure of optimal and near-optimal controls that move the system into the region of attraction of the benign stable equilibrium. In particular, the existence and optimality of a singular arc will be addressed using Lie bracket computations and the Legendre-Clebsch condition.

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On the Controllability of Second Order Systems with Constrained Controls

Marek Majewski
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(Dorota Bors, Bartłomiej Mysiński, Stanisław Walczak)

We consider the following second order equation of the form

$$\ddot{x}(t) = f(t, x(t), u(t)) \tag{1}$$

where $t \in [0, T]$,

$$u(\cdot) \in \mathcal{U} = \left\{ u(\cdot) \in L^2([0,T], \mathbb{R}^m) ; u(t) \in M \right\},$$

M denotes some compact and convex subset of $\mathbb{R}^m$, $x(\cdot) \in H^1([0,T], \mathbb{R}^n)$ and $T > 0$ is a fixed time limit. The system (1) is said to be controllable from the starting state (x_0, v_0) to the final state (x_T, v_T), if there exist an admissible control $u(\cdot) \in \mathcal{U}$ such that the system (1) possesses a global solution $x(\cdot) \in H^1([0,T], \mathbb{R}^n)$ satisfying the conditions $x(0) = x_0$, $\dot{x}(0) = v(0) = v_0$, $x(T) = x_T$ and $\dot{x}(T) = v(T) = v_T$. The system (1) is said to be approximately controllable, if the above mentioned trajectory satisfies the conditions $x(0) = x_0$, $x(T) = x_T$, $|\dot{x}(0) - v_0| \leq \varepsilon$, and $|\dot{x}(T) - v_T| \leq \varepsilon$, where ε is some nonnegative number. In the paper, some sufficient and necessary conditions for the exact and the approximate controllability of the system (1) with constrained controls are presented.

Linear systems (or linear systems with respect to x) were investigated in the following papers:

[1] Dauer J. P., "On controllability of systems of the form $\dot{x} = A(t)x + g(t, u)$," J. Optimiz. Theory Appl., vol. 11, pp.132-138, 1973.

[2] Pandolfi L., "Linear control systems: Controllability with constrained controls, " J. Optimiz. Theory Appl., vol. 19, pp.577-585, 1976.
[3] Schmitendorf W. and Barmish B. R., "Null controllability of linear systems with constrained controls, " SIAM J. Control and Opt., vol. 18, no. 4, pp.327-345, 1980.
[4] Schmitendorf W. and Barmish B. R., "New results on controllability of systems of the form $\dot{x}(t) = A(t)\,x(t) + f(t, u(t))$, " IEEE Trans. Automatic Control, vol. AC-25, no. 3, pp.540-547, 1980.

The controllability of nonlinear systems with the aid of analytical and geometrical methods were presented, among others, in:

[5] Sussmann H., "A sufficient conditions for local controllability, " SIAM J. Control and Opt., vol. 16, no. 5, pp.790-802, 1978.
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$\longrightarrow \infty \diamond \infty \longleftarrow$

Error Estimation and Construction of Piecewise Constant Suboptimal Controls

John Marriott
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Optimal controls often contain a singular arc or chattering, and in some systems this is not implementable due to physical constraints. A common solution is to develop a sub-optimal, piecewise constant control to approximate such an unrealizable optimal control. Due to practical constraints, these approximations typically have a small number of switchings. In practice, the construction of these suboptimal controls is done through heuristics and numerical simulation. Experimentation has shown that in many cases, a suboptimal control with only a small number of switchings can produce an excellent approximation. We are currently researching a mathematical framework for this process. In this talk we will discuss the development of a theory to (1) explicitly calculate the error bounds of a piecewise constant control with a fixed, small number of switchings and (2) provide a construction of such a control that satisfies these bounds.

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Dual Dynaming Programming in Solving Some Optimal Shape Control Problem

Andrzej Nowakowski
University of Lodz, Poland
(Jan Sokolowski)

Consider the following optimal control problem (P): minimize

$$J(u,v,\omega) = \frac{1}{2}\int_\Omega (u(x) - z_d(x))^2 dx + \frac{1}{2}\int_\Omega \chi_\omega(x)(v(x))^2 dx$$

subject to

$$-\Delta u(x) = f(x) + v(x)\chi_\omega(x) \ \text{ a.e. on } \ \Omega \qquad (1)$$

$$|v(x)| \geq \delta > 0 \ \text{ a.e. on } \ \Omega \qquad (2)$$

$$u(x) = 0 \ \text{ on } \ \partial\Omega \qquad (3)$$

where Ω is a given bounded with C^2 boundary domain of R^n and $\omega \subsetneq \Omega$ a finite number, at most N, of simply connected domains, such that $\varepsilon_2 \geq vol\ \omega \geq \varepsilon_1 > 0$, δ, ε_1, ε_2 are given numbers; $z_d : \Omega \to R$, $f : \Omega \to R$ and are given functions, χ_ω characteristic function of ω; $u : \Omega \to R$, $u \in W^{2,2}(\Omega)$ and $v : \Omega \to R$ is a Lebesgue measurable function. We assume that the functions $x \to z_d(x)$, $x \to f(x)$ belong to $L^2(\Omega)$. We call a trio $u(x)$, $v(x)$, ω to be admissible if it satisfies (1)-(3) and the conditions imposed on ω; then the corresponding trajectory $u(x)$ is said to be admissible and ω admissible set.

In this paper we provide a dual dynamic programming approach to control problems (1)-(3). This approach allows us to obtain sufficient conditions for optimality in problems considered and then calculate an optimal value.

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On Infinite Horizon Optimal Control of a Lotka-Volterra System

Sabine Pickenhain
BTU Cottbus, Germany

Still at the beginning of the previous century the optimal control problems with infinite horizon became very important with regards to applications in economics, where an infinite horizon seems to be a very natural phenomenon. Infinite horizon optimal control problems naturally arises not only in economics but also in natural sciences, like Biology. This can be explained by the fact that it is often unrealistic to assume the time is fixed or free. It is much more realistic to assume, that the termination time T of an admissible process is a random variable. The purpose of this contribution is to illustrate the use of optimal control theory for infinite horizon problems, to obtain an optimal strategy for the control of a prey-predator system.

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Controllability of Equations of Mathematical Physics with Euler Term

Andrey Sarychev
University of Florence, Italy

In recent work [1] one proceeded with study of approximate controllability by means of low-dimensional forcing for Euler/Navier-Stokes (NS) equation. In [2] controllability of Euler-Frahm equation for multidimensional rigid body has been studied. Both equations involve specific quadratic Euler term which determines their similarity in many aspects. In both studies geometric control approach has been employed to establish controllability.

According to the approach one starts with system controlled by low-dimensional input and searches for *Lie extensions* which enrich the system by new controlled vector fields. These vector fields are calculated via iterated Lie-Poisson brackets of the 'drift' (quadratic Euler term) and constant controlled vector fields, we call *direction.* The core of the method is in finding proper Lie extensions and in tracing their results.

The Lie extension, employed in [1, 2] in iterative way, is defined via double Lie bracket of the Euler term with two directions, one of which must be steady, i.e. must nullify the Euler term. This double Lie bracket corresponds to bilinear operator $\mathcal{B}$ defined on the space of directions. In order to prove controllability of our equations one has to verify *saturating property*, which means density of the space of extending controlled directions, calculated by repeated application of $\mathcal{B}$, in the space of all directions. This verification requires solution of some algebraic (combinatorial) problem for each particular domain and each particular choice of controlled directions.

To overcome the 'rigidity' of the approach, we suggested in [2] another method for studying controlled Euler-Frahm equation for multidimensional rigid body. This method goes over the algebro-combinatorial difficulties of the previous one and provides controllability criteria, which are structurally stable with respect to perturbation of inertia operator and of the controlled directions.

We describe the results achieved by these method for multidimensional rigid body and reestablish the method for NS/Euler equation formulating controllability criteria for this equation on various domains with generic choice of the controlled directions.

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On the Method of Characteristics in Optimal Control Theory

Heinz Schaettler
Washington University, USA

The method of characteristics, a classical construction of solutions of first order partial differential equations, can be adapted for the construction of solutions to the Hamilton-Jacobi-Bellman equation. Essentially, given a parameterized field $\mathcal{F}$ of extremals (consisting of controlled trajectories and an adjoint variable so that the conditions of the Maximum Principle are satisfied) the running cost along these controlled trajectories becomes a solution to the Hamilton-Jacobi-Bellman equation. The corresponding controls provide relative extrema when compared to other admissible controlled trajectories with the property that the corresponding trajectories lie in the region covered by the field $\mathcal{F}$. In this talk we show the applicability of these constructions to prove optimality of syntheses involving chattering controls (e.g., for the classical Fuller problem) without a need to successively integrate the cost along the infinitely many chattering control segments. We also use this method to discuss the behavior of the value function near singularities by relating these with singularities in the parametrization.

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Generalizations of Naismith's Problem: Minimal Transit Time between Two Points in a Heterogenous Terrain

Erik Verriest
Georgia Institute of Technology, USA

Naismith obtained a set of empirical rules for the time required to move through a terrain. In this paper we solve the problem of determining the path which minimizes the transit time between two points on a given terrain. We give an interpretation of Naismith's rule which leads to an elegant geometric construction of the optimal solution. This problem is a paradigm for the navigation of an autonomous vehicle in a heterogenous terrain. We generalize the solution for the motion of several vehicles with distance constraints, and discuss its limit: the flow of a two-dimensional incompressible medium.

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Special Session 41: Phase Field Methods in Material Sciences

Arnaud Rougirel, University of Poitiers, France
Appolinaire Abourou Ella, University of Poitiers, France

Introduction: This session is devoted to phase field methods as for instance, Cahn-Hilliard type equations, phase field systems or phase field crystal models. The issues of modelization, numerical simulations and theoretical analysis will be addressed.

A Phase Field Model for Electrowetting

Marco Fontelos
Consejo Sup. de Investigacines Cientificas, Spain

The term electrowetting is commonly used for phenomena where shape and wetting behavior of liquid droplets are changed by the application of electric fields. We develop and analyze a model for electrowetting that combines the Navier-Stokes system for fluid flow, a phase-field model of Cahn-Hilliard type for the movement of the interface, a charge transport equation, and the potential equation of electrostatics. The model is derived with the help of a variational principle due to Onsager and conservation laws. A modification of the model with the Stokes system instead of the Navier- Stokes system is also presented. The existence of weak solutions is proved for several cases in two and three space dimensions, either with non-degenerate or with degenerate electric conductivity vanishing in the droplet exterior. Some numerical examples in two space dimensions illustrate the applicability of the model. Joint work with C. Eck, G. Grün, F. Klingbeil and O. Vantzos. Reference: On a phase-field model for electrowetting, Interfaces and Free boundaries, Volume 11, Issue 2, 2009, pp. 259–290.

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Non-Locality and Fluctuations in Fast Spinodal Decomposition Controlled by Diffusion

Peter Galenko
German Aerospace Center (DLR), Germany
(David Jou)

Spinodal decomposition in a binary system deeply supercooled or superheated into the unstable region of miscibility gap in a phase diagram is analysed. A large driving force for phase separation in the spinodal region leads to the phenomenon of the fast spinodal decomposition, thermodynamic aspects of which are presented for locally nonequilibrium binary system with fluctuations. The role of deterministic and stochastic contributions is clarified for spinodally decomposing phases. Early time kinetics of phase separation is modelled and evaluated by the structure factor. Outcomes of the present model predictions are compared with the results obtained using the CHC (Cahn-Hilliard-Cook)-model and LBM (Langer-Baron-Miller)-model. The practically important cases of the fast spinodal decomposition in alloys are predicted and analyzed in comparison with accessible experimental data.

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Global Existence for a Class Higher-Order Parabolic Equations

Svetlin Georgiev
Sofia University, Bulgaria

In this talk we consider the Cauchy problem

$$\begin{cases} u_t = f(t,x)\Delta\Big(a_1(t,x)u^{k_1} + a_2(t,x)u^{k_2} \\ \qquad\qquad + a_3(t,x)\Delta u^{k_3}\Big), \quad x \in \mathbb{R}^n,\ t \in (0,\infty), \\ u(0,x) = u_0(x), \end{cases}$$

where $k_i \in \mathbb{N}$; $f, a_i \in \mathcal{C}([0,\infty) \times \mathbb{R}^n)$, $i = 1,2,3$, are given functions; $u : [0,\infty) \times \mathbb{R}^n \longrightarrow \mathbb{R}$ is unknown function.

We prove that the considered IVP has unique solution $u \in \mathcal{C}([0,\infty)H^m(\mathbb{R}^n))$, $m = 0,1,2,\ldots$. For investigation of this problem we propose new approach which gives new results. As an example of our results we consider Cahn - Hilliard equation.

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Electrowetting with Electrolyte Solutions: Analysis for a Two-Phase Model with Matched Densities and Some Perspectives

Günther Grün
University of Erlangen-Nürnberg, Germany

We prove existence of solutions to a thermodynamically consistent pde-system in three space dimensions. It couples Navier-Stokes and Cahn-Hilliard type-phase-field equations with Nernst-Planck equations for ion density evolution and with an elliptic transmission problem for the electrostatic potential. Crucial will be an iteration method to establish higher regularity for the ion densities. These are a priori only known to be bounded uniformly in time in the L log L Orlicz class (joint work with M. Fontelos and S. Jörres). Finally, we briefly mention a novel diffuse interface model for basic two-phase flow with different densities by Abels, Garcke, and

Grün which is thermodynamically consistent AND allows for divergence free velocity fields.

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Numerical Analysis for a Solidification Phase-Field Model

Juan Vicente Gutiérrez-Santacreu
University of Seville, Spain
(Francisco Guillén-González)

We analyze two numerical schemes of Euler type in time and C^0 finite-element type with $\mathbb{P}_1$-approximation in space for solving a phase-field model of a binary alloy with thermal properties. This model is written as a highly non-linear parabolic system with three unknowns: phase-field, solute concentration and temperature, where the diffusion for the temperature and solute concentration may degenerate.

The first scheme is nonlinear, unconditionally stable and convergent. The other scheme is linear but conditionally stable and convergent. A maximum principle is avoided in both schemes, using a truncation operator on the L^2 projection onto the $\mathbb{P}_0$ finite element for the discrete concentration. In addition, for the model when the heat conductivity and solute diffusion coefficients are constants, optimal error estimates for both schemes are shown based on stability estimates.

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Phase Transitions in Water

Pavel Krejčí
Institute of Mathematics ASCR, Czech Republic
(Elisabetta Rocca, Jürgen Sprekels)

We present a model for phase transitions in water, which includes both mechanical and thermal effects. Our objective is to explain the occurrence of high stresses during the solidification process, and their influence on the deformation of the container. Existence, uniqueness, and asymptotic convergence to equilibria are established for different material properties of the boundary. The results have been recently obtained in collaboration with Elisabetta Rocca and Jürgen Sprekels.

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Fast Spinodal Decomposition in a Binary System: Analysis of the Time-Evolution of the Structure Factor

Nicolas Lecoq
UMR 6634 CNRS, University of Rouen, France
(Helena Zapolsky, Peter Galenko)

Fast spinodal decomposition occurs at large gradients of concentration or at the initial stages of phase separation. To describe such strongly non-equilibrium phenomenon evolving with short periods of time, a hyperbolic model of spinodal decomposition described by the partial differential equation of a hyperbolic type is developped. Experimentally, a typical structure after spinodal decomposition is observed as interconnected worm-like patterns. Such structure has a characteristic lengthscale related to maximal amplification rate of decomposition. Experimental observations using scattering show a broad Bragg-like peak, from which information about the quenched structure during spinodal decomposition and the decomposition rate can be read off. A main quantitative and qualitative characteristic for the intensity of scattering is a structure factor. It can be taken as an important parameter also in modeling for characterization of analyzed phase decomposition and to verify model predictions in comparison with experimental data. The structure factor in fast spinodal decomposition is investigated numerically using the hyperbolic model of spinodal decomposition. A semi-implicit Fourier spectral scheme was used to numerically predict evolution of patterns and structure factor in spinodal decomposition. Important cases of the fast spinodal decomposition in a binary alloys are predicted and analyzed in comparison with experimental data.

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Phase Transitions and Nanoscale Phase Dynamics of Semiconductor Reconstructed Surfaces

Bang-Gui Liu
Inst. of Physics, Academy of Sciences, PR China

The interaction and time evolution of nanoscale phase structures is the key issue to the phase transition dynamics of solid materials. We propose a natural two-speed phase-field model for the phase dynamics of Si(111) 7×7 phase transition to high-temperature unreconstructed phase. Our simulated results show that a 7×7 island decays with its shape kept unchanged, and its area decay rate is a constant increasing with its initial area. LEEM experiments are explained naturally. Furthermore, we propose a two-dimensional phase-field-crystal (PFC) model for the (2×1)-(1×1) phase transitions of Si(001) and Ge(001) surfaces. Simulated periodic arrays are consistent with STM images. The calculated temperature dependence indicates that normal dimmers and broken ones coexist between two temperatures describing the characteristic temperature width of the phase transition, and a first-order phase transition takes place in between. This phase-field method is a reliable approach to modeling nanoscale phase dynamics and thermodynamics

of surface phase transitions. See Y-C Xu and B-G Liu: Phys. Rev. Lett. 100, 056103 (2008); J. Phys. D 42, 035402 (2009); and references therein.

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A Finite Element Discretization of the Cahn-Hilliard Equation with Dynamic Boundary Conditions

Morgan Pierre
University of Poitiers, France
(Laurence Cherfils and Madalina Petcu)

We consider a finite element space semi-discretization of the Cahn-Hilliard equation with dynamic boundary conditions. We prove optimal errors estimates in energy norms and weaker norms, assuming enough regularity on the solution. When the solution is less regular, we prove a convergence result in some weak topology. We also prove the strong stability of a fully discrete problem based on the backward Euler scheme for the time discretization. Some numerical results with the FreeFem++ software show the applicability of the method.

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Irreversible Solidification with Fluid Flow

Gabriela Planas
Universidade Estadual de Campinas, Brazil
(J. L. Boldrini and L. H. de Miranda)

We analyze a mathematical model for the evolution of the process of irreversible solidification of certain pure materials by taking into account the effects of fluid flow in the molten regions. The model consists of a system of highly non-linear free-boundary parabolic equations and inclusion: a heat equation, a doubly nonlinear inclusion for the phase-field and Navier-Stokes equation singularly perturbed by a Carman-Kozeny term to take care of the flow in the mushy regions and a Boussinesq term for the buoyancy forces due to thermal differences. Our approach to show existence of generalized solutions of this system involves time-discretization, a suitable regularization procedure and fixed point arguments.

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Stationary Solutions to Phase Field Crystal Equations

Arnaud Rougirel
University of Poitiers, France

We will present analytical and numerical results of the stationary solutions of the one dimensional Phase Field Crystal Equation. This new model recently introduced by K. Elder and M. Grant describes phase transformations at atomistic level on large time scales. By using bifurcation methods, we investigate quantitative and qualitative properties of these solutions: multiplicity, stability, periodicity. Quite unusual bifurcation diagrams are obtained by numerical simulations.

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Global Solutions for the Cahn-Hilliard-Type Equation

Vladimir Varlamov
University of Texas - Pan American, USA

For the forced Cahn-Hilliard-type equation in a disc global-in-time solutions are constructed. The method is based on using the expansion into the eigenfunctions of the Laplace operator in a disc. The mechanism of nonlinear smoothing is revealed. New special functions are employed, convolutions of Rayleigh functions with respect to the Bessel index. The study of this family was initiated in the earlier works of the author.

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Global Solutions in Higher Dimensions to a Fourth Order Parabolic Equation Modeling Epitaxial Thin Film Growth

Michael Winkler
Universität Duisburg-Essen, Germany

We consider the Neumann boundary value problem for the equation $u_t = -\Delta^2 u - \mu\Delta u - \lambda\Delta|\nabla u|^2 + f(x)$ used in the modeling of the evolution of a thin surface when exposed to molecular beam epitaxy. Correspondingly the physically most relevant spatial setting is obtained when $n = 2$, but previous mathematical results appear to concentrate on the case $n = 1$.

The goal of the talk is to outline the main mathematical difficulties arising in the spatially multi-dimensional setting, and to show how these can be overcome to a) assert global existence of weak solutions, b) see how these can be approximated numerically and c) establish some results on the large time behavior of solutions.

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Special Session 42: Interface Problems

Gunduz Caginalp, University of Pittsburgh, USA
James Glimm, Stony Brook University, USA
Alain Miranville, Université de Poitiers, France

Introduction: Interfaces arising from applications such as materials science pose important and interesting mathematical problems. This special session will feature a broad spectrum of research that includes mathematically rigorous work as well as modeling and large scale computations. The particular topics include phase field models, Cahn-Hilliard models, analysis of anisotropy in materials, turbulent mixing, interfaces in nanoscience, fluid dynamics in porous media, and nematic liquid crystals.

Analytical Results on Damaging in Domains and Interfaces

Elena Bonetti
University of Pavia, Italy

We present a model describing damage processes in a (nonlinear) elastic body which is in contact with adhesion with a rigid support. On the basis of phase transitions theory, we detail the derivation of the model written in terms of a PDE system, combined with suitable initial and boundary conditions. Some non-smooth internal constraints on the variables are introduced in the equations and on the boundary to get physical consistency and unilateral conditions. Existence of global in time solutions is proved for a weak formulation. Hence, we exploit an asymptotic analysis considering the interfacial damage energy (between the body and the contact surface) going to $+\infty$. At the limit we get a weak formulation of a damage problem with dissipative boundary conditions.

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Cahn-Hilliard Equations with Memory and Dynamic Boundary Conditions

Cecilia Cavaterra
Universitá degli Studi di Milano, Italy
(M. Grasselli, C. Gal)

We consider a Cahn-Hilliard equation where the velocity of the order parameter depends on the past history of the Laplacian of the chemical potential. This dependence is expressed through a time convolution integral characterized by a smooth nonnegative exponentially decreasing memory kernel. The chemical potential is subject to the no-flux condition, while the order parameter satisfies a (nonlinear) dynamic boundary condition. The latter accounts for possible interactions with the container walls. This equation has already been analyzed in the case of standard boundary conditions (e.g. Dirichlet type). However, the present case requires a different approach since, for instance, higher-order estimates cannot be obtained by multiplication with a suitable power of a linear operator. Here we confine ourselves to illustrate the results we have obtained in the viscous case, namely, well-posedness, existence of a smooth global attractor, existence of exponential attractors.

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Numerical and Theoretical Results for the Caginalp System with Singular Potentials and Dynamic Boundary Conditions

Laurence Cherfils
University of La Rochelle, France

This talk deals with the Caginalp system, which models melting-solidification phenomena in certain classes of materials. This system is endowed with a singular potential (e.g. a logarithmic potential) and dynamic boundary conditions. The combination of these two features makes the study of the system quite difficult: even the existence of global solutions is not assured. We will first recall the known results concerning this problem. In particular, we will focus on sufficient conditions which ensure the existence of global solutions. Then we will compare these theoretical results with numerical simulations, corresponding to a finite element space discretization and a semi-implicit time discretization.

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On a Paradox Within the Phase Field Modelling of Storage Systems and Its Resolution

Wolfgang Dreyer
WIAS Berlin, Germany
(Clemens Guhlke)

We study the reversible storage of lithium in electrodes of rechargeable lithium-ion batteries. During loading and unloading of foreign atoms into a crystal, there is a regime where two coexisting phases occur with low and high concentration of the foreign atoms. Furthermore hysteretic behavior can be observed, i.e. the processes of loading and unloading follow different paths.

In the regime of slow loading we apply a viscous Cahn-Hilliard model with mechanical coupling to

calculate the voltage-charge diagram of the battery, which reveals phase transition and hysteresis. We show that such a model is in principle not capable to describe the observed phenomena.

We relate the reason for failure to the microstructure of modern electrodes of lithium-ion batteries. The electrodes consists of an ensemble of nano-sized interconnected storage particle. Each particle is described by a non-monotone chemical potential function but on the time scale of slow loading of the ensemble, coexisting phases are unstable within an individual particle and thus cannot be observed.

However, the many-particle ensemble does exhibit two coexisting phases whose evolution is embodied by a nonlocal conservation law of Fokker-Planck type. Two small parameter control whether the ensemble transits the 2-phase region along a Maxwell line or along a hysteresis path or if the ensemble shows the same non-monotone behavior as its constituents.

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Higher Order Phase Field Equations with General Anisotropy

Emre Esenturk
University of Pittsburgh, USA

Generalizing the ideas in [G. Caginalp, Annals. Of Physics, 172, 136-155 (1986); Journal of Statistical Physics, 59, 869-884, (1990)] we present a derivation (in 2-d) of higher order phase field equations that is applicable to interfaces with (finite fold) anisotropy. Macroscopic quantities such as surface tension, entropy between phases are obtained in terms of phase variable. For small anisotropy, it is shown that, in the asymptotic limit that interface thickness approach to zero, the Gibbs-Thomson-Herring relation is approximately satisfied at the interface. Applications to the equilibrium surfaces of solidifying anisotropic materials are discussed.

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Diffuse-Interface Model for Fast Phase Transitions

Peter Galenko
German Aerospace Center (DLR), Germany

A thermodynamic approach to fast phase transitions within a diffuse interface in a binary system is presented. Assuming an extended set of independent thermodynamic variables formed by the union of the classic set of slow variables and the space of fast variables, we introduce finiteness of the heat and solute diffusive propagation at the finite speed of the interface advancing. To describe the transition within the diffuse interface, we use the phase-field model which allows us to follow the steep but smooth change of phases within the width of diffuse interface. The governing equations of the phase-field model are derived for the hyperbolic model, model with memory, and for a model of nonlinear evolution of transition within the diffuse-interface. The consistency of the model is proved by the condition of positive entropy production and by the outcomes of the fluctuation-dissipation theorem. A comparison with the existing sharp-interface and diffuse-interface versions of the model is given together with examples.

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Mathematical, Physical and Numerical Principles for Turbulent Mixing

James Glimm
Stony Brook University, USA

Numerical mass diffusion is a characteristic problem in most simulation codes. In fluid mixing flows, numerical mass diffusion has the effect of over regularizing the solution.

A number of startling conclusions have recently been observed. For a flow accelerated by multiple shock waves, we observe an interface occupying a constant fraction of the available mesh degrees of freedom. This result suggests (a) nonconvergence for the unregularized mathematical problem, (b) nonuniqueness of the limit if it exists, and (c) limiting solutions only in the very weak form of a space time dependent probability dstribution.

The cure for this pathology is a regularized solution, in other words inclusion of all physical regularizing effects, such as viscosity and physical mass diffusion.

In other words, the amount of regularization of an unstable flow is of central importance. Too much regularization, with a numerical origin, is bad, and too little, with respect to the physics, is also bad.

At the level of numerical modeling, the implication from this insight is to compute solutions of the Navier-Stokes, not the Euler equations. In the language of computational physics, the ILES (Implicit Large Eddy Simulation) formulation, which omits regularization, is scientifically incorrect.

Resolution requirements for realistic problems make this solution impractical in most cases. Thus subgrid transport processes must be modeled, and for this we use dynamic models of the turbulence modeling community. In the process we combine ideas of the capturing community (sharp interfaces or numerically steep gradients) with conventional turbulence models, usually applied to problems relatively smooth at a grid level. With use of Front Tracking, we improve on this combination by reduction of numerical mass diffusion. Thereby, numerically steep gradients are achieved.

These ideas are developed in the context of nu-

merical solution of turbulent mixing problems, some of long standing and some with applications to chemical processing and to turbulent combustion.

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Stochastic Evolution of 2D Crystals

Przemyslaw Górka
Warsaw Universities and Univ. de Talca, Poland
(Marcin Dudzinski)

We study a crystalline version of the modified Stefan problem in the plane. The feature of our approach is that, we consider a model with stochastic perturbations and assume the interfacial curve to be a polygon. Using Galerkin method and Schauder fixed point theorem we are able to show the existence of weak solution to our stochastic system.

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Colloidal and Fluid Dynamics in Porous Media Including Electrostatic Interaction

Peter Knabner
University Erlangen-Nuremberg, Germany
(Christof Eck, Nadja Ray, Kai-Uwe Totsche)

We present a pore scale model for transport of charged colloidal particles within a charged porous medium, e.g., the soil. In our model, transport is caused by diffusion, convection and, in addition, electric drift. The number density of colloidal particles is therefore computed by a modified convection-diffusion equation which is also known as Nernst-Planck equation. The electrostatic potential is given by Poisson's equation with inhomogenous boundary condition due to the charged porous medium. Since we pay special attention to the electrostatic effects, fluid flow is described by a modified incompressible Stokes' equations with electric force density as right hand side. Furthermore, we assume the viscosity to increase (linearly) with increasing number density of colloidal particles due to the strong electrostatic field. Depending on the size of the viscosity, we may split our computational domain up in solid and liquid phase, respectively. This models only indirectly the growth and shrinkage of the solid matrix, but can also be interpreted as free boundary. Applying formal two-scale asymptotic expansion we obtain Darcy's law and an averaged convection-diffusion equation on the macro scale. These equations are supplemented by microscopic cell problems determining averaged coefficient functions and the electric potential.

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Coupled PDEs and Bandstructure Calculations in Quantum Dots

Roderick Melnik
Wilfrid Laurier University, Waterloo, Canada
(Sunil Patil, Olena Tsviliuk)

Bandstructure calculations provide a key to properties of low dimensional semiconductor nanostructures (LDSNs). In a series of recent papers we demonstrated that coupled electro-mechanical effects can lead to pronounced contributions in bandstructure calculations of LDSNs such as quantum dots, wires, and even wells (e.g., [1] and references therein). Some such effects are essentially nonlinear. Mathematical models for the description of such effects are usually based on strongly coupled systems of Partial Differential Equations (PDEs). Note that from a physical point of view, both strain and piezoelectric effects have been used as tuning parameters for the optical response of LDSNs in photonics and band gap engineering applications. However, thermal effects have been largely neglected. At the same time, thermal effects coupled with electric and mechanical fields in LDSNs are becoming increasingly important in many applications of LDSN-based optoelectronic devices, in particular where such devices face challenges for thermal management. Indeed, as thermoelectric and thermoelastic effects are often significant in LDSNs, it is reasonable to expect that the temperature may also be used as a tuning parameter in photonics and band gap engineering applications. In this contribution, by using the fully coupled model systems of PDEs, we build on our previous results while analyzing the influence of these effects on optoelectronic properties of quantum dots.

[1] Patil, S. R., Melnik, R. V. N., 2009, Coupled electromechanical effects in II-VI group finite length semiconductor nanowires, Journal of Physics D - Applied Physics, 42 (14), Art. No. 145113.

⟶ ∞ ⋄ ∞ ⟵

Some Generalizations of the Caginalp Phase-Field System

Alain Miranville
Université de Poitiers, France
(R. Quintanilla)

Our aim in this talk is to discuss generalizations of the phase-field model proposed by G. Caginalp. In particular, we consider models in which the usual Fourier law is replaced by more general laws.

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Liquid-Solid Phase Transitions in a Deformable Container

Elisabetta Rocca
University of Milan, Italy
(Pavel Krejci, Juergen Sprekels)

We propose a model for water freezing in an elastic container, taking into account differences in the specific volume, specific heat and speed of sound in the solid and liquid phases. In particular, we discuss the influence of gravity on the equilibria of the system.

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On the Cahn-Hilliard Equation with a Chemical Potential Dependent Mobility

Riccarda Rossi
University of Brescia, Italy
(Maurizio Grasselli, Alain Miranville, Giulio Schimperna)

We analyse a family of (generalized) Cahn-Hilliard equations, with a mobility depending on the chemical potential. Such models arise from generalizations of the (classical) Cahn-Hilliard equation due to M. Gurtin. We present existence results and, in the case of viscous (generalized) Cahn-Hilliard equations, address the long-time behaviour of the system, proving the existence of the global and of the exponential attractor under suitable assumptions.

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A Nonisothermal Model for Nematic Liquid Crystals

Giulio Schimperna
University of Pavia, Italy
(Eduard Feireisl, Michel Frémond, Elisabetta Rocca)

In this talk we propose a new model describing the evolution of a liquid crystal substance in the nematic phase in terms of three basic state variables: the absolute temperature T, the velocity field u, and the director field d, representing preferred orientation of molecules in a neighborhood of any point of a reference domain. The time evolution of the velocity field is governed by the incompressible Navier-Stokes system, with a non-isotropic stress tensor depending on the gradients of the velocity and of the director field, where the transport (viscosity) coefficients vary with temperature. The dynamics of d is described by means of a parabolic equation of Ginzburg-Landau type, with a suitable penalization term to relax the constraint $|d| = 1$. The system is supplemented by a heat equation, where the heat flux is given by a variant of Fourier's law, depending also on the director field d. The proposed model is shown compatible with First and Second laws of Thermodynamics, and the existence of global-in-time weak solutions for the resulting PDE system is established, without any essential restriction on the size of the data.

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On a Surface Phase Field Model for Two-Phase Biological Membranes

Björn Stinner
University of Warwick, England

Vesicles formed by lipid bilayers (biomembranes) show a variety of interesting shapes that can be explained by their elastic energy. Due to inhomogeneities the lipids may separate and form different phases which results in an energy contribution from the phase interfaces. In order to compute equilibrium shapes a gradient flow dynamics has been defined where the membrane is represented by an evolving hypersurface and intra-membrane domains are described using the phase field method. The governing equations consist of a geometric evolution law of Willmore flow type coupled to a surface Allen-Cahn equation. The field variables are the positions of material points of the surface, the mean curvature vector, and the surface phase field function. Using asymptotic expansions the sharp interface limit of the stationary equations has been computed and results in a free boundary problem on the membrane surface which is unknown as well but given as the solution to a geometric equation. Numerical investigations with a method based on surface finite elements support the formal result.

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Phase Field Approximation of a Kinetic Moving-Boundary Problem Modelling Dissolution and Precipitation

Tycho van Noorden
Eindhoven University of Technology, Netherlands
(C. Eck)

We present a phase field model which approximates a one-phase Stefan-like problem with a kinetic condition at the moving boundary, and which models a dissolution and precipitation reaction. The concentration of dissolved ions is variable on one side of the free boundary and jumps across the free boundary to a fixed value given by the constant density of the precipitate. Using a formal asymptotic analysis we show that the phase field model approximates the appropriate sharp interface limit. The existence and uniqueness of solutions to the phase-field model is studied. By numerical experiments the approxi-

mating behaviour of the phase field model is investigated.

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Special Session 43: Recent Advances on Bifurcation Problems

Andre Vanderbauwhede, Ghent University, Belgium
Wolf-Jürgen Beyn, Universität Bielefeld, Germany
Reiner Lauterbach, Universität Hamburg, Germany

Introduction: The aim of this special session is to allow both established and junior researchers working on a broad spectrum of bifurcation-related problems to present their recent results, ideas and techniques. Topics covered include (but are not limited to): bifurcations in reversible and Hamiltonian systems, bifurcations in networks, homoclinic and heteroclinic bifurcations, symmetry-related bifurcation problems, bifurcations in nonsmooth systems and numerical techniques for studying bifurcations. The goal is to obtain a healthy mix of theory and application oriented presentations.

Hyperbolic Planforms in Relation to Visual Edges and Texture Perception

Pascal Chossat
Lab J-A Dieudonné, CNRS - UNSA, France
(Olivier Faugeras)

It has been established experimentally that local areas of images are in correspondance with small areas of the visual cortex named hypercolumns. Neurons in each hypercolumns are sensitive to texture (like edges for example), hence in some way, to the "structure tensor", a 2×2 tensor formed with the gradient of the intensity function of the image, which is widely used in image processing. It is natural to assume that the averaged membrane potential in an hypercolumn is a function of the structure tensor and time, and that it is governed by integro-differential equations of Wilson-Cowan type. Moreover it is natural to assume that these equations are invariant by the isometries of the space of structure tensors which is a hyperbolic manifold. In principle, comparing prediction vs experiments on the pattern self-generated by these equations for an isolated hypercolumn would allow to check the validity of the model. A first step towards this direction is to look at instabilities of the basic homogeneous state of the system without external input, hence to look for bifurcations in a system with "hyperbolic symmetries". I shall detail the model and give some preliminary results.

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Spike-Adding in the Morris-Lecar Burster

Mathieu Desroches
University of Bristol (UK), England
(Bernd Krauskopf and Hinke M Osinga)

We consider the Morris-Lecar neuronal bursting model, which is a slow-fast system in $\mathbb{R}^3$ with one slow and two fast variables. This system features an interesting spike-adding phenomenon related to the existence of saddle-type canard orbits. To understand this phenomenon, we first consider the classical planar Van der Pol oscillator, and look at its canard explosion from a numerical perspective. In particular, we emphasise the advantage of using a numerical continuation scheme, instead of simply integrating an initial condition, in order to compute canard orbits reliably. The same numerical continuation approach allows us to compute with good accuracy canard orbits in the Morris-Lecar burster. In this way, we obtain valuable insight in how a new spike is added to a burst, which is a subtle transition involving saddle-type canard orbits.

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Computation of Normal Form Coefficients by Automatic Differentiation in MatCont and MatContM

Virginie De Witte
Ghent University, Belgium
(Willy Govaerts and Reza Khoshsiar Ghaziani)

MatCont and MatContM are numerical continuation and bifurcation packages in Matlab for ODEs and maps, respectively.

Among other things, MatCont and MatContM compute the normal form coefficients of codimension-1 and -2 bifurcation points. These coefficients need the computation of multilinear forms in which derivatives up to the fifth order are used.

In the present version of MatCont the multilinear forms can either use symbolic derivatives or finite differences. However, finite differences are often inaccurate and symbolic derivatives require the Matlab symbolic toolbox, which is expensive.

Therefore, MatContM provides an alternative in

the form of automatic differentiation. Interestingly, automatic differentiation is even faster in the case of high period cycles.

We compare speed and accuracy of the three methods and give an overview of the advantages and disadvantages of automatic differentation, both for the version for maps and the version for ODEs, and discuss possible further developments in the software.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Orbit Continuation for Computing Stable/Unstable Manifolds, with Applications

Eusebius Doedel
Concordia University, Montreal, Canada

We demonstrate the remarkable effectiveness of numerical continuation and boundary value formulations of the problem of computing stable and unstable manifolds of equilibria and periodic orbits in ODE systems. The main example will be the Circular Restricted Three-Body Problem, which models the motion of a satellite in an Earth- Moon-like system. In particular we compute the unstable manifold of periodic orbits known as Halo orbits, which have been used in actual space missions. Our calculations lead to the detection of heteroclinic connections from Halo orbits to invariant tori. Subsequent continuation of such connections (as the Halo orbit is allowed to change) leads to a variety of connecting orbits that may be of interest in space-mission design.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Stability of Solutions of a PDE by Numerical Continuation: Application to the Dynamics and Bifurcation of a Partially Follower Loaded Viscoelastic Beam

Jorge Galan-Vioque
University of Sevilla, Spain
(M. A. Lago and J. Valverde)

The computation of an eigenvalue and the corresponding eigenvector of a matrix can be formulated as a combination of a continuation process and a bifurcation problem. The eigenvalue equation $Av - \lambda v = 0$ is satisfied for the trivial solution $v = 0$ for any value of λ. For the case of a simple real eigenvalue we can start from a value of λ lower than the leftmost intersection of the Gershgorin circle with the real axis and continue the trivial solution in λ. Precisely at the value of the eigenvalue the rank of the continuation marix will not be maximum and a branching will occur. The emanating branch can be computed and gives rise to the non vanishing eigenvector than can be easily obtained up to a normalization value. The case of purely imaginary or complex eigenvalues and eigenvectors and multiplicity greater than one can also be treated.

This expensive way of computing the eigenvalues is, in general, not advisable for ODEs but can be used in certain solutions of PDEs to determine its stability, analysing the behavior of the eigenvalues close to the imaginary axis [1]. In this contribution we illustrate this method analyzing the bifurcations of a beam subjected to a follower force.

We make use of a continuous Cosserat model of a visco-elastic beam subjected to a combination of a conservative force (load) and a follower term in one of the ends. The formalism takes into account the geometric nonlinearity that appears for large deformations from the straight solution. The result is a PDE whose solution can be analyzed for special cases and the eigenvalues of the linearization can be computed with the abovementioned

keywords: stability of PDE solutions, Cosserat modeling, viscoelatic beam, follower pendulum, bifurcations, flutter instability.

AMS: 37M20, 65P30, 35Q74, 37N15

[1] J. Valverde and G. H. M. Van der Heijden, Magnetically-induced buckling of a whirling conducting rod with applications to electrodynamic space tethers. *Journal of Nonlinear Science*, (in press), 2009.
[2] J. J. Thomsen, Chaotic dynamics of the partially follower-loaded elastic double pendulum. *Journal of Sound and Vibration*, 188:385-405 (1995).

$\longrightarrow \infty \diamond \infty \longleftarrow$

Two-Dimensional Attractors of the Border Collision Normal Form

Paul Glendinning
University of Manchester, England
(Chi Hong Wong)

We consider the dynamics of discrete time hybrid systems which are continuous across a switching surface, but where the Jacobian matrix jumps. The simple bifurcations of two dimensional systems of this type are described by a piecewise affine normal form due to Nusse and Yorke. In the early 1990s Gardini and others showed numerically that there can be two-dimensional attractors in this normal form and proved the existence of two-dimensional absorbing regions, but we are not aware of any rigorous proofs that the attractor itself is two-dimensional except in one special case. We provide proofs which can be applied at many parameter val-

ues, and also show numerically that two of these attractors can co-exist.

⟶ ∞ ⋄ ∞ ⟵

Study of a Dynamical Model for the Cell Cycle of Budding Yeast

Willy Govaerts
Ghent University, Belgium
(Charlotte Sonck)

Basic knowledge of the cell cycle of various types of cells consists of the substances involved in the process (cyclins, APC's etcetera) and the regulatory network.

Dynamical systems models based on this regulatory network are fairly recent, and they are not the only possible approach. Competing models use logical dynamic modeling, a simpler approach that in some cases is quite successful by showing that most trajectories funnel into a path which steps through the cell cycle in a robust way.

On the other hand, models based on differential equations allow to use mathematical tools such as bifurcation theory and numerical continuation under parameter variation. In this way, multistability, periodic behaviour etc. can be derived and studied in a fairly standard way. In most cases, these models exhibit a rich bifurcation structure though it is not always obvious how much of this is relevant to the actual cell cycle.

A survey of the already impressive literature was recently published by A. Csikasz-Nagy ("Computational systems biology of the cell cycle", Briefings in Bioinformatics, Vol. 10 no 4, 424-434). The leading group appears to be that of John. J. Tyson and Bela Novak. Among other things, they built the most detailed model so far of cell-cycle regulation by describing the control network of budding yeast Saccharomyces cerevisiae.

We discuss a computational and bifurcation study of this model that includes a few phenomena that were so far not discussed in this context. This includes the existence of different limit cycles, born at different Hopf points that (approximately) merge in a single bigger limit cycle and a rather unexpected relation between the growth rate of the cell and the mass increase after DNA-replication.

We further discuss the implications of the funneling effect for the cell cycle as a boundary value problem, and the computation of this cycle as the fixed point of a map.

⟶ ∞ ⋄ ∞ ⟵

Abundance of Periodic Orbits without Hyperbolicity

Roland Gunesch
University Hamburg, Germany

We present a new method for showing the abundance of periodic orbits. The method works both for flows and maps. It does not make the usual requirement of strong hyperbolicity.

Applications include geodesic flows, where we can make very precise quantitative asymptotic statements about their orbits (and about other quantities), flows with suitable cone conditions, and magnetic flows.

⟶ ∞ ⋄ ∞ ⟵

Bifurcations from Homoclinic Networks

Ale Jan Homburg
University of Amsterdam, Netherlands

I will discuss a number of recent results on the dynamics near and bifurcations from homoclinic networks, as they can occur in differential equations with discrete symmetry.

⟶ ∞ ⋄ ∞ ⟵

Computing Dichotomy Projectors and Sacker-Sell Spectra in Discrete Time Dynamical Systems

Thorsten Huels
Bielefeld University, Germany

We introduce an algorithm for computing approximations of dichotomy projectors with high accuracy. Our method is based on solving linear boundary value problems.

Extensions of these techniques are introduced, allowing the detection of Sacker-Sell spectra in discrete time dynamical systems. The first method computes the projector residual $PP - P$. If this residual is large, then the difference equation has no exponential dichotomy. A second criterion for detecting Sacker-Sell spectral intervals is the norm of end- or midpoints of the solution of a specific boundary value problem.

Error estimates for the underlying approximation process are given and the resulting algorithms are applied to an example with known continuous Sacker-Sell spectrum, as well as to the variational equation along orbits of Hénon's map.

⟶ ∞ ⋄ ∞ ⟵

Nonreversible Homoclinic Snaking

Jürgen Knobloch
TU Ilmenau, Germany

Homoclinic snaking refers to the sinusoidal "snaking" continuation curve of homoclinic orbits near a heteroclinic cycle connecting an equilibrium and a periodic orbit. Along this curve the homoclinic orbit performs more and more windings about the periodic orbit. Typically this behavior appears in reversible Hamiltonian systems. Here we discuss this phenomenon in systems without particular structure.

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Bifurcations of Global Manifolds in the Lorenz System

Bernd Krauskopf
University of Bristol, England
(Eusebius J Doedel and Hinke M Osinga)

The stable manifold of the origin of the Lorenz system, also known as the Lorenz manifold, plays a crucial role for the organisation of the overall dynamics. This two-dimensional manifold and associated manifolds of saddle periodic orbits can be computed accurately with numerical methods based on the continuation of orbit segments, defined as solutions of suitable boundary value problems. This allows us to study bifurcations of these global manifolds as the Rayleigh parameter ϱ is changed. In this way, we obtain new insight into how the the entire phase space of the Lorenz system is organised and changes dramatically during the transition to chaotic dynamics with increasing ϱ.

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The Tumbling Universe: Dynamics of Bianchi Models in the Big-Bang Limit

Stefan Liebscher
Free University Berlin, Germany
(Joerg Haerterich)

We consider cosmological models of Bianchi type. They yield spatially homogeneous, anisotropic solutions of the Einstein field equations. In particular, we are interested in the alpha-limit dynamics of the Bianchi model corresponding to the big-bang singular limit of the Einstein equations.

Emphasis is on transient behavior of solutions near the (backward) Bianchi attractor composed of the Kasner circle of equilibria and attached heteroclinic connections. The heteroclinic orbits in the Bianchi attractor form formal sequences of shift type. Along the Kasner circle we encounter bifurcations without parameters, i.e. the Kasner circle is not normally hperbolic.

We prove the existence of submanifolds of Bianchi solutions converging to given sequences of heteroclinics in the big-bang limit.

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Numerical Explorations for Subharmonic Bifurcations in Reversible Systems

Francisco Javier Muñoz-Almaraz
Universidad CEU—Cardenal Herrera, Spain
(E. Freire, J. Galán-Vioque and A. Vanderbauwhede)

Subharmonic bifurcations in reversible systems can be located numerically as bifurcations of boundary values problems. With these numerical tools, we study in detail a degenerate case of the banana scenario where this non-genericity is a consequence of the existence of a first integral which does not vary along the family with the subharmonic bifurcation. Finally, we apply these techniques to some families of periodic orbits for the N-body problem.

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Hopf Bifurcation in One-Dimensional Symmetric Coupled Cell Networks

Rui Paiva
Oporto, Portugal
(Ana Dias)

We consider symmetric coupled cell networks of differential equations. We show that already at the level of Abelian symmetry, very degenerate codimension-one bifurcations can occur. This degenerate behaviour occurs due to the restrictions that the symmetry group of the network and the network structure impose at the associated coupled cell networks of differential equations.

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Continuation of Sets of Constrained Orbit Segments

Frank Schilder
DTU, Denmark
(Harry Dankowicz)

Sets of constrained orbit segments of time continuous flows are collections of trajectories that represent a whole or parts of an invariant set. A nontrivial but simple example is a homoclinic orbit. A typical representation of this set consists of an equilibrium point of the flow and a trajectory that starts close and returns close to this fixed point within finite time. More complicated examples are hybrid periodic orbits of piecewise smooth systems or quasiperiodic invariant tori. Even though it is possible to define generalised two-point boundary value problems for computing sets of constrained orbit segments, this is very disadvantageous in practice. In this talk we will present an algorithm that allows

the efficient continuation of sets of constrained orbit segments together with the solution of the full variational problem.

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Multipulses in the Swift-Hohenberg Equation

Thomas Wagenknecht
University of Leeds, England

(Steve Houghton, Juergen Knobloch, Bjorn Sandstede)

The Swift-Hohenberg equation is one of the standard examples, for which localised roll structures have been found to lie on snaking curves. Along those curves infinitely many fold bifurcations occur, with the corresponding solutions developing more and more oscillations about their centre.

Recently an analytical approach, yielding a complete description of this homoclinic snaking scenario for 1-pulse solutions, has been developed. Following this, we analyse the behaviour of 2-pulse solutions, which are found to lie on isolas in the bifurcation diagram and we discuss their behaviour under symmetry-breaking perturbations of the equation.

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Numerical Relative Lyapounov Centre Bifurcation

Claudia Wulff
University of Surrey, England

(Frank Schilder)

Relative periodic orbits (RPOs) are ubiquitous in symmetric Hamiltonian systems and occur for example in celestial mechanices, molecular dynamics and rigid body motion. RPOs are solutions which are periodic orbits of the symmetry-reduced system.

In this talk we analyze relative Lyapounov centre bifurcations under conditions which are generically satisfied. Moreover we show how they can be detected numerically along branches of RPOs and present an efficient method for their numerical computation.

We apply our methods to the family of rotating choreographies which bifurcates from the famous Figure Eight solution of the three body problem as angular momentum is varied and connects to the Lagrange relative equilibrium.

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Special Session 44: Inverse Problems

Lassi Paivarinta, University of Helsinki, Finnland
Gunther Uhlmann, University of Washington, USA

Reconstruction of Small Perturbations of an Elastic Inclusion from Spectral Measurements

Elena Beretta
Universitá "La Sapienza" Roma, Italy
(H. Ammari, E. Francini, H. Kang and M. Lim)

In order to reconstruct small changes in the interface of an elastic inclusion from modal measurements, we derive rigorously an asymptotic formula which is in some sense dual to the leading-order term in the asymptotic expansion of the perturbations in the eigenvalues due to interface changes of the inclusion. Based on this (dual) formula we propose an algorithm to reconstruct the interface perturbation.

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4D Reconstruction in Molecular Imaging

Martin Burger
University (WWU) Münster, Germany

In this talk we shall discuss some aspects of dynamics in image reconstruction, in particular in Positron Emission Tomography. In particular we shall discuss simultaneous motion estimation and reconstruction, which is a problem of central importance in cardiovascular applications, leading to variational problems in space-time similar to optimal transport problems, and their numerical solution.

Moreover, we discuss model-based reconstruction methods, which incorporate prior information into the reconstruction process via kinetic models for tracer behaviour. The reconstructed quantities are spatially or time-dependent parameters in these models, which can be obtained also for low SNR due to the reduction of degrees of freedom. On the other hand, these reconstruction problems are nonlinear parameter identification problems in differential equations and appropriate methods for such problems will be presented.

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Stability Estimates for Evolution Equations from the Dirichlet to Neumann Map

David Dos Santos Ferreira
Université Paris 13, France
(Mourad Bellassoued)

We are interested in the following inverse problem for evolution equations in an anisotropic media: in a compact Riemannian manifold with boundary, find the potential or the conformal factor of the metric from the knowledge of the Dirichlet-to-Neumann map. For instance, for the wave equation, the question of identifiability has been settled by Belishev and Kurylev using the boundary control method, introduced by Belishev. This method however does not seem to provide suitable stability estimates. Following ideas of Stefanov and Uhlmann and inspired by a recent work of DSF, Kenig, Salo and Uhlmann (concerned with the anisotropic Calderón problem), we derive stability estimates in simple geometries for potentials and close conformal factors from the Dirichlet-to-Neumann map associated to the dynamical Schrödinger equation (or the wave equation). This a joint work with Mourad Bellassoued (Faculté des Sciences de Bizerte).

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Partial Reconstruction of a Source Term in a Parabolic Problem

Davide Guidetti
University of Bologna, Italy

We consider a linear parabolic problem of the form

$$\begin{aligned}
&D_t u(t,x,y)\\
&\quad = A(t,x,D_x)u(t,x,y) + B(t,y,D_y)u(t,x,y)\\
&\qquad = g(t,x)f(t,x,y),\ t\in[0,T],\ x\in U,\ y\in V,\\
&u(0,t,x,y) = u_0(x,y),\ x\in U,\ y\in V,
\end{aligned}$$

with Dirichlet or first order boundary conditions, in a cylindrical domain $U\times V$. $g(t,x)$ is unknown, together with u. We want to reconstruct both u and g, from supplementary informations of the form

$$\int_W u(t,x,y)d\mu(y),$$

with μ complex Borel measure in the closure W of V.

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Transmission Eigenfrequencies for Dielectrics and Their Use in the Identification Problem

Houssem Haddar
INRIA / Ecole Polytechnique France

We shall address in this talk recent progress in the analysis of so called transmission eigenfrequencies

for dielectric (anisotropic) inclusions and their use in the identification problem, where one is interested in getting qualitative information on the inclusion physical parameters from measurements of electromagnetic scattered waves. We shall also outline possible applications in non-destructive testing of complex media, where no a priori knowledge on the structure of the media is available. Exploiting these special frequencies in the identification problem is a quite recent idea that seems promising and that also arises many numerical and theoretical questions that will be discussed in the talk.

The presented material is extracted from joint works with Fioralba Cakoni, David Colton and Drossos Gintides.

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Localized Potentials for Elliptic Inverse Coefficient Problems

Bastian Harrach
Joh. Gutenberg University Mainz, Germany

We consider the famous Calderón problem: Is it possible to determine the diffusivity coefficient a of an elliptic PDE

$$\operatorname{div}(a \operatorname{grad} u) = 0$$

from the set of all possible Dirichlet and Neumann boundary values on (a part of) the boundary of the considered domain?

Based on simple functional analytic arguments, we will construct special solutions (the so-called localized potentials), and use them to derive new identifiability results. We also show how to extend these arguments to the problem of determining two coefficients which arises, e.g., in diffuse optical tomography.

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Convex Backscattering Support in Electrical Impedance Tomography

Nuutti Hyvonen
Aalto University, Finland

(Martin Hanke and Stefanie Reusswig)

This work studies a recently introduced notion of backscattering for the inverse obstacle problem of electrical impedance tomography. It is shown that the corresponding backscatter data is the boundary value of a function that is holomorphic in the exterior of the inclusions, which makes it possible to reformulate the problem as an inverse source problem for the Poisson equation. The convex backscattering support is then defined to be the smallest convex set that carries an admissible source, i.e., a source that yields the given backscatter data as the Dirichlet trace of the associated potential. The convex backscattering support can be computed numerically; reconstructions are presented to illustrate the usefulness of the method.

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Maximum Compatibility Estimate for Optimal Combination of Multiple Data Modalities in Inverse Problems

Mikko Kaasalainen
Tampere University of Technology, Finland

We present an optimal strategy for the relative weighting of different data modalities in inverse problems, and derive the maximum compatibility estimate (MCE) that corresponds to the maximum likelihood or maximum a posteriori estimates in the case of a single data mode. MCE is not explicitly dependent on the noise levels, scale factors or numbers of data points of the complementary data modes, and can be determined without the mode weight parameters. As a case study, we consider the problem of reconstructing the shape of a body in $\mathbb{R}^3$ from the boundary curves (profiles) and volumes (brightness values) of its generalized projections.

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A Parameter Identification Problem in a Nonlinear Hyperbolic PDE: Uniqueness, Stability and a Numerical Solution Approach

Barbara Kaltenbacher
University of Graz, Austria

The subject of this talk is motivated by applications in piezoelectric material characterization, which leads to the problem of identifying coefficient functions in nonlinear hyperbolic PDEs.

By means of the model problem of determining c in

$$u_{tt} - (c(u_x)u_x)_x = f \text{ in } \Omega = (0,1)$$

from overdetermined boundary measurements $c(u_x)u_x = g\,,\ u = m$ on $\partial\Omega = \{0,1\}$ of u, in this talk we will discuss several approaches for showing uniqueness and stability.

The first one is to adapt the idea of integration along characteristics of the hyperbolic PDE [1], cf. [2].

The second approach works with a neighboring identification problem for which a sufficient condition for identifiability can directly be formulated and checked in terms of the given initial and boundary data, cf. [3].

A third possibility arises when excitations at different intensities can be carried out and corresponding measurements can be taken. In that case and

with a polynomial coefficient function c, an asymptotic expansion of u in therm of the excitation intensity enables a uniqueness result, cf. [4]. The latter also leads to a possible numerical solution strategy.

Additionally, we will present a numerical solution approach that is based on a multiharmonic expansion of the PDE solution, and show numerical results.

[1] V. Isakov, *Inverse Problems for Partial Differential Equations*, Springer, New York, 1998.
[2] B. Kaltenbacher, Identification of nonlinear parameters in hyperbolic PDEs, with application to piezoelectricity, in: K. Kunisch, G. Leugering, J. Sprekels, F. Tröltzsch, eds., *Optimal Control of Coupled Systems of PDEs.* Springer, 2006. (Oberwolfach Workshop, 2005).
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[4] G. Nakamura, M. Watanabe, and B Kaltenbacher, On the identification of a coefficient function in a nonlinear wave equation, *Inverse Problems*, 25 035007 (16pp), 2009.
[5] B. Kaltenbacher Determination of parameters in nonlinear hyperbolic PDEs via a multiharmonic formulation, used in piezoelectric material characterization *Math. Models Methods Appl. Sci.* 16:869–895, 2006.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Direct Non-Linear Reconstruction of Conductivities in 3D

Kim Knudsen
Technical University of Denmark
(Jutta Bikowski and Jennifer Mueller)

The inverse conductivity problem (or the so-called Calderón problem) was formulated mathematically in Calderóns seminal paper from 1980. The problem is concerned with the unique determination and reconstruction of a conductivity dstribution in a bounded domain from knowledge of the Dirichlet-to-Neumann map on the boundary of the domain. In three dimensions (for smooth conductivities) the uniqueness and reconstruction problems were addressed and solved in theory in the 1980s in a series of papers authored by A. Nachman, J. Sylvester and G. Uhlmann. The main ingredients are complex geometrical optics solutions to the conductivity equation and a (non-physical) scattering transform. The resulting reconstruction algorithm is in principle direct and adresses the full non-linear problem immediately.

Only recently the numerical implementation of the algorithm was initiated. In this talk I will describe the algorithm and its numerical realization, and evaluate the performance of the method on several examples.

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Transmission Eigenvalues for Perturbed Operators with Constant Coefficients

Katya Krupchyk
University of Helsinki, Finland

We study transmission eigenvalues for multiplicative sign-definite perturbations of hypoelliptic partial differential operators with constant coefficients. The discreteness of the set of transmission eigenvalues is shown and sufficient conditions for the existence of transmission eigenvalues, real or complex, are derived. A link to the scattering theory is also established. This is a joint work with M. Hitrik, P. Ola, and L. Päivärinta.

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Collapsing Riemannian Manifolds and Stability of Inverse Spectral Problems

Yaroslav Kurylev
UCL, London, England
(M. Lassas and T. Yamaguchi)

We consider a class of n-dimensional Riemannian manifolds of bounded geometry which allows the 1-dimensional collapse. This collapse gives rise, in particular, to $(n-1)$-dimensional Riemannian orbifolds with non-trivial weight function. We show the uniquesness of the inverse spectral problem for the associated weighted Laplacians on Riemannian orbifolds. We also show that the stability, in the Gromov-Hausdorff metric, of the inverse spectral problem for the original class of Riemannian manifolds.

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Statistical Inverse Problems and Discretization of Continuous Inverse Problems

Matti Lassas
University of Helsinki, Finland
(Samuli Siltanen, Eero Saksman)

In this talk we consider the question how inverse problems posed for continuous objects, for instance for continuous functions, can be discretized. This means the approximation of the problem by infinite dimensional inverse problems.

We will consider a linear inverse problem $m = Af + \varepsilon$. Here function m is the measurement, A is a ill-conditioned linear operator, u is an unknown function, and ε is random noise. The inverse problem means determination of u when m is given.

The traditional solutions for the problem include the generalized Tikhonov regularization and the estimation of u using Bayesian methods. To solve the problem in practise u and m are discretized, that is, approximated by vectors in an infinite dimensional vector space. We show positive results when this approximation can successfully be done and consider problems that can surprisingly appear. As an example, we consider the total variation (TV) regularization and the Bayesian analysis based on total variation prior.

[1] M. Lassas, S. Siltanen: Can one use total variation prior for edge preserving Bayesian inversion, Inverse Problems 20 (2004), 1537-1564.
[2] M. Lassas, S. Saksman, S. Siltanen: Discretization invariant Bayesian inversion and Besov space priors. Inverse Problems and Imaging 3 (2009), 87-122.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Linear Sampling for Inverse Problems in the Time Domain

Armin Lechleiter
INRIA Saclay / CMAP Ecole Polytechn., France

We consider a near field inverse scattering problem for the wave equation: find the shape of a Dirichlet scattering object from time domain measurements of scattered waves. For this time domain inverse problem, we propose a linear sampling method, a well-known technique for corresponding frequency domain inverse scattering problems. The problem setting and the algorithm incorporate two basic features: First, the data for the method consists of measurement of causal waves, that is, of waves that vanish before some moment in time. Second, the inversion algorithm directly works on the time domain data without using a Fourier transformation. Applications we have in mind include for instance ground penetrating radar imaging. The technique naturally incorporates a continuum of frequencies in the inversion algorithm. Consequently, it induces the potential to improve the quality of the reconstruction compared to frequency domain methods working at one single frequency.

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An Inverse Problem for a Wave Equation

Shitao Liu
University of Virginia, USA
(Roberto Triggiani)

We consider a second-order hyperbolic equation on an open bounded domain Ω in $\mathbb{R}^n$ for $n \geq 2$, with C^2-boundary $\Gamma = \partial\Omega = \overline{\Gamma_0 \cup \Gamma_1}$, $\Gamma_0 \cap \Gamma_1 = \emptyset$, subject to Neumann boundary conditions on the entire boundary Γ. $w_{tt}(x,t) - \Delta w(x,t) - q(x)w(x,t) = 0$ in $\Omega \times [0,T]$ and $\frac{\partial w}{\partial \nu}(x,t) = 0$ on $\Gamma \times [0,T]$ with given $w(\cdot,\frac{T}{2})$ and $w_t(\cdot,\frac{T}{2})$. We consider an inverse problem of determining $q(x)$, $x \in \Omega$, from Dirichlet data on the subboundary Γ_1. Under suitable conditions on Γ_0, Γ_1 and $T > 0$, we prove the uniqueness and the stability of $q(x)$. The proof is based on the Carleman estimate and continuous observability inequality.

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A Variational Approach to an Inverse Problem in Photolithography

Luca Rondi
Università di Trieste, Italy
(Fadil Santosa (University of Minnesota))

Photolithography is an important industrial process for the construction of integrated circuits. A light ray passes through a mask and then hits a photosensible material, determining in such a way the shape of the circuit. The aim is to find the optimal mask to build a given circuit.

We use a penalization to the perimeter of the mask to regularize the problem and phase-field functions for a formulation amenable to numerical implementation. We also address the meaning of optimality, that is how to measure the difference between the desired circuit and the one obtained with a certain mask.

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Borg-Levinson Theorem for Magnetic Schrödinger Operator

Valeriy Serov
University of Oulu, Finland

It is proved that the boundary spectral data, i.e. the Dirichlet eigenvalues and modified normal derivatives of the normalized eigenfunctions at the boundary of smooth bounded domain uniquely determines an electric potential V in L^∞ and $\operatorname{rot} A$ of a magnetic vector potential A in W^1_∞ from the magnetic Schrödinger operator. This result is based on the recent publication of L. Päivärinta, M. Salo and G. Uhlmann-Inverse scattering for the magnetic Schrödinger operator, Preprint (2009).

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Regularized D-Bar Method for the Inverse Conductivity Problem

Samuli Siltanen
University of Helsinki, Finland
(Kim Knudsen, Matti Lassas, Jennifer Mueller)

The inverse conductivity problem formulated by Calderón is the mathematical model of the emerging medical imaging method called Electrical Impedance Tomography (EIT). The idea of EIT is to feed harmless electric currents to the patient through electrodes attached to the skin, measure the resulting voltages at the electrodes, and form an image of the inner organs of the patient based on the measurement data. This image reconstruction task is a nonlinear and severely ill-posed inverse problem.

A strategy for regularizing the inversion procedure for the two- dimensional D-bar reconstruction algorithm for EIT is presented. The strategy utilizes truncation of a boundary integral equation and a scattering transform. It is shown that this leads to a stable reconstruction of the conductivity; an explicit rate of convergence in appropriate Banach spaces is derived as well. Numerical results are also included, demonstrating the convergence of the reconstructed conductivity to the true conductivity as the noise level tends to zero.

The results provide a link between two traditions of inverse problems research: theory of regularization and inversion methods based on complex geometrical optics. Also, the procedure is a novel regularized imaging method for electrical impedance tomography.

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Special Session 45: Evolution Equations and Mathematical Biology

Jozsef Z. Farkas, University of Stirling, UK
Peter Hinow, University of Wisconsin - Milwaukee, USA

Introduction: Mathematical biology is an ever growing field. The use of mathematical models helps scientists to shed light on natural phenomena in the life sciences. The application of mathematical methods in the biological and medical sciences are particularly relevant since they are rigorous and provide a much more feasible tool than in-vivo experiments. Evolution equations, and in more general differential equations, are among the key tools since they are amenable to both analytical and numerical investigations. Our symposium brings together scientists from a variety of areas to present and discuss recent trends and progress in their specific field of interest. Our speakers expertise will bridge a wide range of topics from mathematical analysis, modelling and simulation to real world applications.

A Structured Juvenile-Adult Model with Application to Amphibians

Azmy Ackleh
University of Louisiana at Lafayette, USA
(Keng Deng and Qihua Huang)

In 2004 we began a monitoring program of an urban population of Green Tree Frogs (*Hyla cinerea*). Using a capture-mark-recapture technique and statistics we derived estimates of the total population during the breeding season. In this talk, we present a structured juvenile-adult model which describes the dynamics of this population. We discuss the long time behavior of this model for simple cases. We also discuss a least squares approach to estimate model parameters from field data. We apply this approach to compare the model output to the population estimates obtained from field data. Finally, we present stochastic models developed to understand the effect of demographic stochasticity on the long time behavior of this population.

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Compartmental Disease Transmission Models for Smallpox

Burcu Adivar
Izmir University of Economics, Turkey
(Ebru Selin Selen)

Mathematical models become important tool to develop emergency response plans and mitigating strategies for any anticipated epidemic or pandemic attack. With the objective of minimizing total number of deaths, total number of infected people, and total health care costs, mitigation decisions may include early response, vaccination, hospitalization, quarantine or isolation. In this study, smallpox disease transmission is analyzed with a compartmental model consisting of the following disease stages: susceptible, exposed, infectious, quarantined and recovered. Considering natural birth and death rates in a population, two models are developed to study the control and intervention of smallpox under the assumption of imperfect quarantine. First model is a ordinary differential equation (ODE) model assuming exponential dstribution function and the latter is a general integral equation model with general dstribution. Numerical results are provided and model results are compared to demonstrate the dif-

ferences in the reproduction numbers, which can be described as the threshold for stability of a disease-free equilibrium related to the peak and final size of an epidemic.

⟶ ∞ ⋄ ∞ ⟵

Structured Population Equation with Fragmentation and Coagulation: The Case of Strong Fragmentation

Jacek Banasiak
University of KwaZulu-Natal, So Africa

We consider a population of organisms which can grow and form clusters which, in turn, can break up and join each other. The smallest ('zero' size) organisms can break from the mother cluster and start forming new clusters. Such processes can be modelled by the classical McKendrick equation of population theory with fragmentation and coagulation terms and with renewal boundary conditions. A natural space in which to seek solutions to this equation is the space $X_{0,1} = L_1(\mathbb{R}_+, (1+x)dx)$ which controls both the number of particles and the size of the ensemble. The talk by Wilson Lamb will show that it is possible to analyse this equation in $X_{0,1}$, using results from the theory of semigroups of operators, provided that the fragmentation rate, the growth rate and the renewal coefficient all grow sublinearly as $x \to \infty$. While the assumption on the growth rate is dictated by properties of the transport part of the equation in $X_{0,1}$, the other two restrictions are only required for technical reasons. In this talk we shall show that this equation is well posed in $X_{0,m} = L_1(\mathbb{R}_+, (1 + x^m)dx)$ for some m provided the fragmentation rate and the renewal coefficient are also $O(x^k)$ for some k (in general different from m). A crucial property for establishing this is the recently proved regularity of solutions to the pure fragmentation equation obtained in [2].

This talk is partly based on joint work with Wilson Lamb (Strathclyde).

[1] J. Banasiak and W. Lamb, Coagulation, fragmentation and growth processes in a size structured population, *Discrete and Continuous Dynamical Systems-B*, 11(3), (2009), 563—585.
[2] J. Banasiak and W. Lamb, Global strict solutions to continuous coagulation-fragmentation equations with strong fragmentation, submitted.

⟶ ∞ ⋄ ∞ ⟵

Modeling and Mathematical Analysis of Metastatic Growth Including Angiogenesis

Sebastien Benzekry
Universite de Provence, France
(Assia Benabdallah)

Tumoral angiogenesis is a key process in the tumoral growth which allows the tumor to impact on its vasculature in order to improve the nutrient's supply and the metastatic process. We developped a model for the density of metastasis which takes into account for this process. It is a renewal-type equation structured both in size and so-called angiogenic capacity. In this talk I will present the mathematical analysis of this model, namely the well-posedness of the equation and the asymptotic behavior of the solutions.

⟶ ∞ ⋄ ∞ ⟵

Using Quasi-Normal Form for the Examination of Tumor-Free Equilibrium Stability in a Bcg Mathematical Model

Svetlana Bunimovich
Lecturer in the Ariel University Center, Israel
(Yakov Goltser)

Understanding the dynamics of human hosts and tumors is of critical importance. A mathematical model was developed by Bunimovich-Mendrazitsky et al. (2007) that explored the immune response in bladder cancer as an effect of BCG treatment. This treatment exploits the host's own immune system to boost a response that will enable the host to rid itself of the tumor. Although this model was extensively studied using numerical simulation, no global analytical results were originally presented. In this work we analyze fixed point stability in a BCG-treatment mathematical model for bladder cancer based on the use of Quasi-Normal form and theory of stability. These tools are employed in critical cases, especially when an analysis of the linear system is insufficient. Our goal is to gain a deeper insight into this treatment, which is based on a mathematical model for bladder cancer and on biological considerations; and thereby to bring us one step closer to a protocol for the use of BCG treatment in bladder cancer.

⟶ ∞ ⋄ ∞ ⟵

Modeling Cell Migration in the Extracellular Matrix

Arnaud Chauviere
University of Texas, Houston, USA
(Helen Byrne, Thomas Hillen, Luigi Preziosi)

Cell migration is an essential feature of both normal and pathological biological phenomena. Cells interact with both other cells and the surrounding tissue (the extracellular matrix or ECM). The ECM serves many functions, such as providing support and anchorage for cells. It is composed of an interlocking mesh of fibrous proteins as collagen and provides directional information through the fibers

along which cells tend to align. Various cell migratory behaviors in the ECM have been identified. In this presentation, we focus on the amoeboid migration, i.e. when cells migrate by using the ECM as a scaffold and squeeze into free spaces of the matrix, frequently changing direction and leaving the fibers almost unaltered. We present a mesoscopic framework of transport equations for velocity-jump processes including the alignment process along the fibers and cell-cell interactions leading to random movement reorientation. The model is extended to account for the influence of environmental factors (e.g. chemotaxis). Changes in migratory behavior, associated with phenotypic switching, are also included. The corresponding macroscopic continuum models are derived by appropriate rescaling, which leads to the so-called diffusive approximation. We present numerical simulations of these models and evidence the influence of the ECM heterogeneity and anisotropy on cell migration.

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Cell Proliferation and Circadian Rhythm

Jean Clairambault
INRIA, France
(Stéphane Gaubert, Thomas Lepoutre)

Molecular circadian clocks, that are found in all nucleated cells of mammals, are known to dictate rhythms of approximately 24 hours (*circa diem*) to many physiological processes, including metabolism (e.g., temperature, hormonal blood levels) and cell proliferation. It has been observed in tumour-bearing laboratory rodents that a severe disruption of these physiological rhythms results in accelerated tumour growth. The question of accurately representing by physiologically based models of population dynamics the control exerted by circadian clocks on healthy and tumour tissue proliferation to explain this phenomenon has given rise to mathematical developments, which we will review in this talk. The main goal of these works is to examine the influence of a periodic control on the cell division cycle in physiologically structured cell populations, comparing the effects of periodic control with no control, and of different periodic controls between them. We will furthermore state a new and general convexity result that leads us to hypothesize that the above mentioned effect of disruption of circadian rhythms on tumour growth enhancement is indirect, that is, this enhancement is likely to result from the weakening of healthy tissues that are at work fighting tumour growth. Our result may also yield a theoretical justification to the concept of cancer chronotherapeutics.

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Existence and Stability of Steady States in Hierarchical Structured Populations with Infinite States at Birth

Jozsef Farkas
University of Stirling, Scotland
(P. Hinow)

In the present talk I will address the existence and stability of steady states of a quasilinear first order hyperbolic partial integro-differential equation. The model describes the evolution of a hierarchical structured population with distributed states at birth. Hierarchical size-structured models describe the dynamics of populations when individuals experience size-specific environment. This is the case for example in a population where individuals exhibit cannibalistic behaviour. The other distinctive feature of the model is that individuals may be recruited into the population at arbitrary size. This assumption amounts to a general integral operator describing the recruitment process. Using finite rank approximations of the recruitment process we establish conditions for the existence of a positive steady state of the model. Our method utilizes a fixed point result of nonlinear maps in conical shells of Banach spaces. We also study the stability of steady states utilizing results from the theory of positive operators on Banach lattices.

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Modelling and Analysis of Stem Cell Differentiation

Philipp Getto
Basque Center for Applied Mathematics, Spain
(A. M. Marciniak-Czochra)

We are developing new models to describe maturation processes of stem cell populations and in particular the mechanisms that regulate these processes. We formulate such models as population models with maturity structure. Due to regulation mechanisms such models are quasi-linear and thus of great challenge. We are analyzing these models in a recently developed framework of Delay Equations, meaning integral equations with a continuous history dependence involved. The aim is to analyze, via linearization, mechanisms that can explain the stability of population-dynamical equilibria.

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Mathematical Models for Pneumococcal Spread Taking into Account Serotypes and Sequence Types

David Greenhalgh
University of Strathclyde, Scotland
(K. Lamb and C. Robertson)

Streptococcus pneumoniae is a bacterium commonly found in young children. Pneumococcal serotypes can cause diseases such as meningitis and pneumonia. In 2000 a vaccine was introduced in the USA that eliminates carriage of the vaccine serotypes. One problem is that the same sequence types (genetic material found in the serotypes) are able to manifest in more than one serotype. This is a problem if sequence types associated with invasive disease are able to manifest themselves in non-vaccine type serotypes.

By using differential equation models we attempt to explore the relationship between sequence types and serotypes, where a sequence type is able to manifest itself in one vaccine serotype and one non-vaccine serotype. A variety of different models were explored with different transmission assumptions and relationships between sequence types and serotypes. For each the basic reproduction number was found and equilibrium, local and global stability analyses were performed.

When there are two or more sequence types circulating, there are three possible equilibria: the carriage-free equilibrium and two possible carriage equilibria. Global stability analyses show elimination of one or other sequence type and the sequence type eliminated depends on the pre-vaccination transmission rates and serotype proportions within sequence type.

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Modelling Radiation Damage

Michael Grinfeld
University of Strathclyde, Glasgow, Scotland
(M. Grinfeld and F. Mohd Siam, University of Strathclyde)

We will discuss modelling both targeted effects of large dose ionizing radiation (which leads empirically to the linear-quadratic dose response) and the much more mysterious non-targeted low dose radiation effects, such as the bystander effects and chromosomal instability. In particular, we will consider the crucially important ATM pathway of damage information transduction.

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Mathematical Modelling of Forest Fire Spread

Thomas Hillen
University of Alberta, Canada
(Jon Martin, Petro Babak)

In this talk I will discuss various models for the prediction and control of forest fire spread. (i) A first class of models bases on curve evolution equations. These have been implemented into powerful numerical packages, which are used in real time forest fire simulations. (ii) Alternatives are based on classical combustion models of reaction-diffusion type. These models are derived from physical properties but they are, so far, unsolvable for a real sized forest fire. (iii) A third, and new, class of models is based on a probabilistic description of the fire. It does not attempt to predict the exact location of the fire, it rather projects a probability profile of finding a fire in a given landscape. I will present the basic ideas of these models and discuss their advantages and shortcomings. I will also present some of our new results on invariant shapes for the curve evolution models, on traveling invasion waves for the physical combustion models and on simulations of the probabilistic model.

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Multi-Species Models of Size-Structured Population with Nonlinear Growth Rate

Nobuyuki Kato
Shimane University, Japan

We intend to develop the theory of a general model of size-structured population dynamics. Applications to species interaction models and subpopulation models are considered. The former models are shown to have at least one solution while the latter models are shown to have a unique solution. We also give characterizations of stationary solutions and give an existence result of nonzero stationary solutions for a two-species model.

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The Impact of Information Transmission on Epidemic Outbreaks

Istvan Kiss
University of Sussex, UK, England
(Jackie Cassell, Mario Recker and Péter L. Simon)

For many diseases (e.g., sexually transmitted infections, STIs), most individuals are aware of the potential risks of becoming infected, but choose not to take action ('respond') despite the information that aims to raise awareness and to increases the responsiveness or alertness of the population. We pro-

pose a simple mathematical model that accounts for the diffusion of health information disseminated as a result of the presence of a disease and an 'active' host population that can respond to it by taking measures to avoid infection or if infected by seeking treatment early. In this model, we assume that the whole population is potentially aware of the risk but only a certain proportion chooses to respond appropriately by trying to limit their probability of becoming infectious or seeking treatment early. The model also incorporates a level of responsiveness that decays over time. We show that if the dissemination of information is fast enough, infection can be eradicated. When this is not possible, information transmission has an important effect in reducing the prevalence of the infection. We derive the full characterisation of the global behaviour of the model, and we show that the parameter space can be divided into three parts according to the global attractor of the system which is one of the two disease-free steady states or the endemic equilibrium.

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A Mathematical Model for Heat Damage

Richard Kollár
Comenius University, Bratislava, Slovakia
(Sabino Pietrangelo)

We investigate damage to living cells due to external heating particularly relevant to skin burns due to a contact with a hot surface but also applicable to other types of heat damage. Along with a criticism of the traditional approach to this problem we derive a very novel model for heat damage for a general class of chemical kinetic systems. Based on this results, we identify a fundamental underlying dynamical system and build a simple phenomenological model that shows a good agreement with experimental data. We will discuss applications of the results in daily life, fire protection or hyperthermic cancer treatment.

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Coagulation, Fragmentation and Growth Processes in a Size Structured Population

Wilson Lamb
University of Strathclyde, Scotland
(Jacek Banasiak (UKZN, Durban))

We consider a mathematical model that describes the dynamical behaviour of aggregrates of phytoplankton cells. Changes in the size dstribution of the aggregrates are assumed to be caused by three processes; coagulation, fragmentation and cell division. The effects of these processes are incorporated in the model by coupling the classical McKendrick-von Foerster equation with standard fragmentation and (Smoluchowski-type) coagulations terms. This leads to an integro-differential equation with renewal boundary conditions for the aggregrate density function $u(x,t)$ (the number density of aggregates of size x at time t). A mathematical analysis of this equation is carried out using perturbation results involving semigroups of operators. When the model parameters representing fragmentation, growth and renewal are assumed to grow sublinearly, and the coagulation rate kernel is restricted to be bounded, the associated initial-boundary value problem is shown to be well posed in a physically relevant Banach space.

This is joint work with Jacek Banasiak (UKZN) and in his talk he will describe some more recent developments in which it is possible to relax the conditions on the fragmentation rate and renewal coefficient.

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Mathematical Models of Discrete and Continuous Cell Differentiation

Anna Marciniak-Czochra
University of Heidelberg, Germany

Cell differentiation is a multi-step process, in which a relatively small population of stem cells undergo asymmetric cell divisions leading to formation of more mature cells (differentiation process) and subsequent replenishment of cells at different maturation stages. Understanding of the mechanisms governing cell differentiation is of central interest for stem cell biology, especially because of its clinical impact. One established method of modeling of such hierarchical cell systems is to use a discrete collection of ordinary differential equations, each of which describes a well-defined differentiation stage. However, there are indications that the differentiation process is less rigid and that it involves transitions which are continuous, along with discrete ones.

In this talk we compare the applicability of both the discrete and continuous framework to describe dynamics of cell differentiation. Multi-compartmental and structured populations models are formulated to investigate the role of regulatory feedbacks in the process of blood regeneration. Model results are compared to the clinical data. Analysis of the model equations leads to a generalization of the concept of self-renewal potential, which might be helpful to define stem cells population.

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Well-Posedness, Stability Properties and Bifurcation Analysis for a Tumor Growth Model

Anca-Voichita Matioc
Leibniz University Hannover, Germany
(Prof. Joachim Escher)

We study a moving boundary problem describing the growth of nonnecrotic tumors in different regimes of vascularisation. This model consists of two decoupled Dirichlet problems, one for the rate at which nutrient is added to the tumor domain and one for the pressure inside the tumor. These variables are coupled by a relation which describes the dynamic of the boundary. By re-expressing the problem as an abstract evolution equation, we prove local well-posedness in the small Hoelder spaces context. Further on, we use the principle of linearised stability to characterise the stability properties of the unique radially symmetric equilibrium of the problem. Finally, a bifurcation argument shows that there exist also other stationary solutions of the problem, which are no longer radially symmetric.

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An Existence and Uniqueness Result for the Dynamics of Micro-Swimmers

Marco Morandotti
SISSA - Trieste, Italy
(Gianni Dal Maso, Antonio DeSimone)

We present a model for the motion of micro-swimmers in a viscous fluid. Our result is that, under very mild regularity assumptions, the change of shape determines uniquely the motion of the swimmer. We assume that the Reynolds number is very small, so that the velocity field of the surrounding, infinite fluid is governed by the Stokes system and all inertial effects can be neglected. Moreover, we assume the self propulsion constraint (no external forces an momenta). Therefore, the Newton's equations of motion reduce to the vanishing of the viscous drag force and torque acting on the body. By exploiting an integral representation of viscous force and momentum, the equations of motion can be manipulated and turn out to be a system of six ordinary differential equations. Variational techniques are used to prove the continuity in time of its coefficients, so that the classical results on ordinary differential equations can be invoked to get the existence and uniqueness of the solution.

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A Delay Differential Equation Model for the Cell-Cycle Specificity of Vesicular Stomatitis Virus Efficacy

Joanna Pressley
Vanderbilt University, USA
(Joseph Crivelli, Peter Kim)

Despite recent efforts to develop new therapies for cancer treatment, few methods ever make it from the laboratory to the bedside. One promising form of treatment is oncolytic virotherapy. Oncolytic viruses preferentially infect and replicate in cancerous cells, leading to the destruction of tumor populations while sparing normal cells. We develop a mathematical model of virotherapy treatment with vesicular stomatitis virus (VSV), an oncolytic virus which selectively kills a wide range of human tumor cell types. VSV can infect cells during all parts of the cell cycle except during the resting phase. Since the interphase of the cell cycle has a minimum biological time course, we use delay differential equations to model the cell cycle-specific nature of the VSV system. We investigate how the length of the time delay and other parameter values affect the stability of the equilibria, and therefore, the efficacy of the treatment.

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A Mathematical Analysis of Prion Proliferation, Application to Models in Epidemiology

Laurent Pujo-Menjouet
University of Lyon, France

How do the normal prion protein PrPC and infectious prion protein PrPSc populations interact in an infected host? To answer this question, we analyze the behavior of the two populations by studying a system of differential equations. The system is constructed under the assumption that PrPSc proliferates using the mechanism of nucleated polymerization. We prove that with parameter input consistent with experimentally determined values, we obtain the persistence of PrPSc. We also prove local and global stability results for steady states. We also give numerical simulations, which are confirmed by experimental data. We show the link between this theoretical work and some epidemics theory for SEIS and HIV models. Finally, a close collaboration with biologists will be pointed out through several experiments and new theoretical approaches.

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The Detrimental Effect of Parametric Noise in Models of Population Growth

Ami Radunskaya
Claremont Colleges, Pomona College, USA

Dynamical systems arising from models of self-regulating growth often contain a stochastic component representing noise in the environment, or "parametric" noise. What is the effect of this noise on the long-term behavior of the system? How does this answer depend on the dstribution of the random variable? In order that the question make sense, the system must have a well-defined long-term average, i.e. it must be ergodic. In this talk we prove ergodicity for a class of growth models, and show that the randomness is harmful to the population in the sense that the long-term average is decreased by the presence of noise.

When systems obeying noisy growth laws are connected together as a coupled lattice, the long-term effects of the noise can thus have damaging effect on the organism as a whole, even though local interactions might favor growth in a particular area. It has been suggested that cancer growth might be modeled as this type of lattice. We will present simulations that highlight the effect of both the noise and the local coupling on the survival of the organism.

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Exact Epidemic Models on Graphs Using Graph Automorphism Driven Lumping

Peter Simon
Eotvos Lorand University, Budapest, Hungary
(Michael Taylor, Istvan Z. Kiss)

The dynamics of disease transmission strongly depends on the properties of the population contact network. Pair-approximation models and individual-based network simulation have been used extensively to model contact networks with non-trivial properties. We start from the exact Markov chain formulation of a simple epidemic model on an arbitrary contact network and rigorously derive and prove some known results that were previously mainly justified based on some biological hypotheses. The main result of the paper is the illustration of the link between graph automorphisms and the process of lumping whereby the number of equations in a system of linear differential equations can be significantly reduced. The main advantage of lumping is that the simplified lumped system is not an approximation of the original system but rather an exact version of this. For a special class of graphs, we show how the lumped system can be obtained by using graph automorphisms, and we use lumping on a complete graph to show that, in the limit of large networks, the solution of the exact Markov chain model converges to that of the ODE-based mean-field model.

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Spectral Bound and Reproduction Number for Infinite Population Structure and Time-Heterogeneity

Horst Thieme
Arizona State University, USA

Spectral bounds of matrices are crucial mathematical threshold parameters in population models that are formulated as systems of ordinary differential equations: the sign of the spectral bound of the variational matrix at a particular population state decides about whether, cum grano salis, the population size increases or decreases.

Another threshold parameter is the reproduction number $\mathcal{R}$ which is the spectral radius of a related matrix: ideally the set up of the model allows interpreting it as the total number of offspring an average individual would produce during its lifetime if the population were to remain at that particular population state.

As it is well-known, the spectral bound and $\mathcal{R}-1$ have the same sign provided that the matrices have a particular form.

The relation between spectral bound and reproduction number can be extended to models with infinite dimensional state space: it will now hold between the spectral bound of a resolvent positive closed linear operator and the spectral radius of a related positive bounded linear operator. The infinite dimensional character of these systems creates a problem, however: it is no longer the spectral bound of the variational operator that decides about population increase or decrease, but the exponential growth bound (or type) of the operator semigroup it generates: the exponential growth bound may be larger but is never smaller than the spectral bound. So conditions for the equality of the two become crucial.

We illustrate the general theory by applying it to time-heterogenous population models using (Howland) evolution semigroups.

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A Delay Differential Equations Model for a Crocodilian Population

David Uminsky
UCLA, USA
(Angela Gallegos, Tenecia Plummer, Cinthia Vega, Clare Wickman, Michael Zawoiski)

The crocodilia have multiple interesting characteristics that affect their population dynamics. They

are among several reptile species which exhibit temperature-dependent sex determination in which the temperature of egg incubation determines the sex of the hatchlings. Their life parameters, specifically birth and death rates, exhibit strong age-dependence. We develop delay-differential equation models describing the evolution of a crocodilian population. In our single-delay model we prove local asymptotic stability for the equilibrium population values. For all delay models we obtain very strong agreement with the age structure of crocodilian population data as reported in Smith and Webb, 1985. We also obtain reasonable values for the sex ratio of the simulated population.

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Weak Solutions to a Population Model for Proteus Mirabilis

Christoph Walker
Leibniz Universität Hannover, Germany

(Ph. Laurençot (Toulouse))

Proteus mirabilis are bacteria that make strikingly regular spatial-temporal patterns on agar surfaces. In this talk we consider a mathematical model that has been shown to display these structures when solved numerically. The model consists of an ordinary differential equation coupled with a partial differential equation involving a first-order hyperbolic aging term together with nonlinear degenerate diffusion. We prove that the system admits global weak solutions. This is joint work with Ph. Laurençot (Toulouse).

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Special Session 46: Topological Methods for Boundary Value Problems

John Graef, University of Tennessee at Chattanooga, USA
Jeff Webb, University of Glasgow, UK

Introduction: Boundary value and related problems for ordinary and partial differential equations that make use of various topological methods are included in this session. This covers a broad spectrum of problems including singular problems and those with multipoint conditions.

Asymptotic Properties of Solutions of Second Order Differential Equations with Delay

Miroslav Bartusek
Masaryk University, Brno, Czech Republic

The lecture is devoted to a second order delayed differential equation $(ay')' = rf(y(h(t)))$ in case a, r are positive. The continuability of solutions to the infinity is investigated and asymptotic behaviour of oscillatory solutions is studied. Also applications to limit-point/limit-circle problem are given. The presented results are obtained by joint work with J. R. Graef.

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Higher Order Semipositone Multi-Point Boundary Value Problems on Time Scales

Abdulkadir Dogan
Nigde University, Turkey

(John R. Graef, Lingju Kong)

The authors obtain some existence criteria for positive solutions of a higher order semipositone multi-point boundary value problem on a time scale. Applications to some special problems are also discussed. This work extends and complements many results in the literature on this topic.

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Freholm Alternative Results for the Fucik Spectrum I

Pavel Drabek
University of West Bohemia, Pilsen, Czech Rep.

(S. Robinson)

Consider the problem

$$\begin{gathered} -u'' - \alpha u^+ + \beta u^- = f(x) \quad \text{in} \quad (0,1) \\ u(0) = u(1) = 0 \end{gathered}$$

where (α, β) is a point in the Fucik Spectrum of $-\frac{\partial^2}{\partial x^2}$ and $f \in L^2(0,1)$. Using variational arguments we prove extensions of the Fredholm Alternative to this nonlinear problem. Our theorems consider points on both the trivial branch and on the first nontrivial branch of the Fucik Spectrum. Our results generalize to the quasilinear case, and our

results lead to a variational characterization for the third curve, i.e. second nontrivial curve, in the Fucik Spectrum.

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Positive Solutions and Eigenvalues for a Coupled Nonlinear Fractional Differential System

Wenying Feng
Trent University, Canada

We study the nonlinear fractional differential system with two parameters. By applying a nonlinear alternative of Leray-Shauder type and Krasnoselskii's in a cone, existence of positive solutions and eigenvalues are obtained. Applications are shown by examples.

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Existence Results for Systems of Second Order Differential Equations

Daniel Franco
Universidad Nacional de Educ. a Dist., Spain

We present sufficient conditions for the existence of solutions for a system of second order ordinary equations with nonlocal boundary conditions. The results are based on recent developments on vector versions of cone fixed point theorems.

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Existence Results for Systems of Third Order Differential Equations

Marlene Frigon
University of Montreal, Canada

The notion of solution-tube of systems of third order differential equations is introduced. It extends naturally the notion of upper and lower solutions of third order differential equations. Using this notion, we present existence results. We consider different growth condition and in particular a Nagumo-Wintner growth condition. Since we consider systems of differential equations, an extra condition is needed such as Hartman-type condition.

[1] A. Cabada, *The method of lower and upper solutions for third-order periodic boundary value problems*, J. Math. Anal. Appl. 195 (1995), 568–589.
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[3] M. Frigon and H. Gilbert, *Existence theorems for systems of third order differential equations*, Dynamic Systems and Applications (to appear).
[4] M. Grossinho, and F. Minhós, *Existence result for some third order separated boundary value problems*, Proceedings of the Third World Congress of Nonlinear Analysts, (Catania, 2000). Nonlinear Anal. 47 (2001), 2407–2418.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Application of Monotonicity Method on Some Classes of $p(x)$-Laplacian Problems

Mohammad Bagher Ghaemi
IUST, Iran
(S. Saiedinezhad)

We study the boundary value problem

$$(\mathbf{P})\ \begin{cases} -\triangle_{p(x)}u(x) + K(x)|u(x)|^{p(x)-2}u(x) \\ \quad + f(x,u(x)) = h(x); \quad \text{in } \Omega \\ u \equiv 0; \quad \text{on } \partial\Omega; \end{cases}$$

where Ω is a bounded domain in $\mathbb{R}^N$ with smooth boundary $\partial\Omega$, $p \in \mathbf{C}(\overline{\Omega})$ and

$$1 < p^- := \inf_{x\in\Omega} p(x) \leq p^+ := \sup_{x\in\Omega} p(x) < N.$$

We prove for every $\lambda > 0$ there exist a weak solution. Our approach relies on the variable exponent theory of generalized Lebesgue-Sobolev spaces combined with adequate monotonicity method.

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Uniqueness and Parameter Dependence of Positive Solutions of Third Order Boundary Value Problems with p-Laplacian

John Graef
University of Tennessee at Chattanooga, USA
(Lingju Kong)

We consider the boundary value problem with nonhomogeneous three–point boundary condition

$$(\varphi_p(u'))'' + a(t)f(u) = 0,\ t \in (0,1),$$

$$u(0) = \xi u(\eta) + \lambda,\ u'(0) = u'(1) = 0.$$

Sufficient conditions are obtained for the existence and uniqueness of a positive solution. The dependence of the solution on the parameter λ is also studied. This work extends and improves some recent results in the literature on the above problem, especially those in the paper [L. Kong, D. Piao, and L. Wang, Positive solutions for third order boundary value problems with p-Laplacian, Result. Math. 55 (2009) 111–128].

$\longrightarrow \infty \diamond \infty \longleftarrow$

Positive Solutions of a Nonlocal BVP Arising in Chemical Reactor Theory

Gennaro Infante
University of Calabria, Italy
(Paolamaria Pietramala)

We discuss the existence of positive solutions of a nonlocal boundary value problem that models a tubular chemical reactor. Our main approach relies on the theory of fixed point index for compact maps.

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Periodic Solutions for Some Nonlinear Fully Fourth Order Differential Equations

Feliz Minhós
University of Évora, Portugal
(João Fialho)

In this talk we present sufficient conditions for the existence of solutions of the periodic fourth order boundary value problem

$$\begin{aligned} u^{(4)}(x) =& f(x, u(x), u'(x), u''(x), u'''(x)) \\ u^{(i)}(a) =& u^{(i)}(b),\ i = 0, 1, 2, 3, \end{aligned}$$

for $x \in [a, b]$, and $f : [a, b] \times \mathbb{R}^4 \to \mathbb{R}$ a continuous function.

To the best of our knowledge it is the first time where this type of general nonlinearities is considered in fourth order equations with periodic boundary conditions

The difficulties in the odd derivatives are overcome due to the following arguments: the control on the third derivative is done by a Nagumo-type condition; the bound on the first derivative is obtained by lower and upper solutions, not necessarily ordered.

By this technique, not only it is proved the existence of a periodic solution, but also, some qualitative properties of the solution can be obtained.

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Existence Results for Semilinear Impulsive Neutral Functional Differential Inclusions with Nonlocal Conditions

Sotiris Ntouyas
University of Ioannina, Greece

In this talk we present existence results for first and second order semilinear impulsive neutral functional differential inclusions with finite or infinite delay in Banach spaces with nonlocal conditions, via a Nonlinear Alternative for contractive maps. Our theory makes use of analytic semigroups and fractional powers of closed operators, integrated semigroups and cosine families.

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Fredholm Alternative Results for the Fucik Spectrum II

Stephen Robinson
Wake Forest University, USA
(Pavel Drabek)

This will be a continuation of part I.

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Existence of Non-Variational Eigenvalues of the p-Laplacian Operator

Bryan Rynne
Heriot-Watt University, Scotland

A standard technique for showing the existence of eigenvalues for nonlinear operators is the well known Ljusternik-Schnirelmann method, which constructs 'variational' eigenvalues as inf-sups of a suitable functional over sets of given genus. The question then arises as to whether the variational construction produces all the eigenvalues of the operator. For the p-Laplacian operator (with p not equal to 2) this has been a long-standing open question (for both ordinary and partial differential operators).

In the case of the ordinary differential p-Laplacian operator, with separated boundary conditions (e.g., Dirichlet or Neumann), it is known that the Ljusternik-Schnirelmann method does indeed yield all the eigenvalues. However, it is not known if this is true more generally, although, until now, no examples of non-variational eigenvalues have been constructed.

In this talk, non-variational eigenvalues will be constructed for the ordinary differential p-Laplacian operator with periodic boundary conditions. This construction is then extended to partial differential operators on annuli, with Neumann boundary conditions. Thus, in general, the Ljusternik-Schnirelmann method need not yield all the eigenvalues of the p-Laplacian operator.

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S-Shaped Bifurcation Curves in Ecosystems

Ratnasingham Shivaji
Mississippi State University, USA
(Eun Kyoung Lee, Sarath Sasi)

We consider the existence of multiple positive solutions to the steady state reaction diffusion equation with Dirichlet boundary conditions of the form:

$$\begin{cases} -\Delta u = \lambda[u - \dfrac{u^2}{K} - c\dfrac{u^2}{1+u^2}], \quad x \in \Omega \\ \quad u = 0, \quad x \in \partial\Omega. \end{cases}$$

Here $\Delta u = div(\nabla u)$ is the Laplacian of u, $\frac{1}{\lambda}$ is the diffusion coefficient, K and c are positive constants and $\Omega \subset \mathbb{R}^N$ is a smooth bounded region with $\partial\Omega$ in C^2. This model describes the steady states of a logistic growth model with grazing in a spatially homogeneous ecosystem. It also describes the dynamics of the fish population with natural predation. In this paper we discuss the existence of multiple positive solutions leading to the occurrence of an S-shaped bifurcation curve. We prove our results by the method of sub-super solutions.

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Positive Periodic Solutions of a Class of Singular Nonautonomous Systems

Haiyan Wang
Arizona State University, USA

Singular nonautonomous systems of ordinary differential equations arise in several applications. In this talk, we will investigate the existence and multiplicity of positive periodic solutions for a class of second order nonautonomous singular systems with superlinearity or sublinearity assumptions at infinity for an appropriately chosen parameter. The proof of our results is based on the Krasnoselskii fixed point theorem in a cone.

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A Nonlocal Problem Modelling a Thermostat

Jeff Webb
University of Glasgow, Scotland

We will discuss positive solutions of a nonlocal boundary value problem that models a thermostat. The sensor occupies some part (a sub-interval) of the heated bar and gives feedback to a controller at one of the endpoints. We use the theory of fixed point index and the discussion of problems that have Riemann-Stieltjes integral boundary conditions.

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Special Session 47: Time Decomposition Methods for Differential Equations: Theory and Application

Jürgen Geiser, Humboldt University of Berlin, Germany
Qin Sheng, Baylor University, USA

Introduction: In recent years, splitting, or decomposition, methods have become increasingly important for solving nonlinear and even singular partial differential equations. In this special session we will discuss the latest goals and results of the powerful numerical methods. Higher accuracy, efficiency and effectiveness will be among our topics in discussions. Iterative and non-iterative strategies associate with different adaptations will be incorporated. Novel decomposition schemes in time and space will be investigated and presented. Practical applications will be studied.

Aims of this special session also include to bring together researchers in the aforementioned field, to highlight the current developments both in theory and methods, to exchange latest research ideas, and to promote further collaborations in the community.

Multiphase Models Solved by Operator Splitting Methods

Meraa Arab
Humboldt University Berlin, Germany

We present a multiphase model applied in a CVD (chemical vapor deposition) process. Mobile and immobile phases are used to modeled the gaseous transport and kinetics of deposition species.

We decouple the underlying convection-diffusion-reaction equation into a transport and kinetic part. With finite volume methods we solve the transport part and with ODE solvers the kinetic part. To combine both parts we apply operator splitting methods of Lie-Trotter type. Numerical examples are discussed in considering real-life applications.

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Processed Splitting Methods in Path Integral Monte Carlo Simulations

Fernando Casas
Universitat Jaume I, Spain

Processed splitting methods are particularly well adapted to carry out path integral Monte Carlo (PIMC) simulations: since one is mainly interested in estimating traces of operators, only the kernel of the method is necessary to approximate the thermal density matrix. Unfortunately, they suffer the same drawback as standard, non-processed integrators: kernels of effective order greater than two necessarily involve some negative coefficients. This problem can be circumvented, however, by incorporating modified potentials into the composition, thus rendering schemes of higher effective order. In this work we analyse a family of fourth-order schemes recently proposed in the PIMC setting, paying special attention to their linear stability properties, and justify their observed behaviour in practice. We also conclude that effective order six can only be achieved within this family with some negative coefficients.

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Analysis of Parareal Operator Splitting Techniques for Multi-Scale Reaction Waves

Stéphane Descombes
Université de Nice Sophia Antipolis, France
(Max Duarte, Marc Massot)

In this talk, I present a new strategy based on the combination of time operator splitting and the parareal algorithm for the simulation of reaction-diffusion equations modeling multi-scale reaction waves, This type of problems induces peculiar difficulties and potentially large stiffness which stem from the broad spectrum of temporal scales in the nonlinear chemical source term as well as from the presence of large spatial gradients in the reactive fronts which are spatially very localized.

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Successive Approximation for Solving Time-Dependent Problems

Jürgen Geiser
Humboldt University of Berlin, Germany

We present a novel method to solve time-dependent differential equations. The method is based on iterative splitting schemes and generally known as successive approximation methods. While Magnus expansion has been intensely studied and widely applied for solving explicitly time-dependent problems. Due to its exponential character, it is rather difficult to derive practical algorithms beyond the sixth-order. An alternative method is based on successive approximation methods, that taken into account the temporally inhomogeneous equation (method of Tanabe and Sobolevski). In this work, we show that the recently derived ideas of the successive approximation method in a splitting method. Examples are discussed.

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Iterative Operator Splitting Method for the Capillary Formation Model in Tumor Angiogenesis Problem: Application and Analysis

Nurcan Gücüyenen
Izmir Institute of Technology, Turkey
(,Gamze Tanoğlu)

In this talk, iterative operator splitting method is used to obtain the numerical solution for the mathematical model which defines the role of the endothelial cells in capillary. The method is based on splitting the complex problem into simpler differential equations then combining each sub-equations with iterative schemes and each of them are solved efficiently with suitable integrators. The explicit error bounds are first time derived in terms of the operator norms. The stability is studied in terms of the system of equations by demonstrating the stability function obtained from predictor-corrector process. The method seems attractive in view of the consistency and accuracy. The implementation of the method is quite easy and done by matlab program.

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Krylov-Enhanced Parallel Integrators for Linear Problems

Stefan Güttel
Université de Genève, Switzerland
(M. Gander, M. Petcu)

The parareal algorithm is a numerical method to integrate evolution problems on parallel computers. The main components of this algorithm are a coarse integrator, which quickly propagates information on a coarse partition of the time interval, and a fine integrator, which solves the evolution problems more accurately on each subinterval. The performance of this algorithm is well understood for diffusive problems, but it can also have spectacular performance when applied to certain non-linear problems. In (Gander & Petcu 2008) the authors proposed a Krylov-enhanced version of the parareal algorithm, which for linear problems is equivalent to the modified PITA algorithm described in (Farhat et al 2006). Both of these algorithms can be successful for 2nd order ODEs. Refining the analysis in (Gander & Petcu 2008), we study the convergence of the Krylov-enhanced parareal algorithm and consider the particularly interesting special case when the coarse integrator is a polynomial or rational Krylov-based exponential or trigonometric integrator.

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Recent Progress in the Construction of Green's Functions for the Static Klein-Gordon Equation

Yuri Melnikov
Middle Tennessee State University, USA

In contrast to the cognate Laplace equation in two dimensions, for which a notable list of Green's functions is available, the field is not that developed for the static Klein-Gordon equation.

A variety of boundary-value problems is considered for which computer-friendly forms of Green's functions are obtained. Klein-Gordon equation represents a natural domain for the extension of some of the methods that are proven productive for the Laplace equation. The perspective appears especially attractive for the methods of images and eigenfunction expansion.

This study is based on our, gained in recent decades, experience on the construction of Green's functions for applied elliptic partial differential equations. An extensive list of boundary-value problems formulated for the static Klein-Gordon equation is considered. Compact readily implementable representations of their Green's functions are obtained, most of which have never been published before.

As an indirect outcome of the present investigation, some new multivariable identities have been derived for the modified Bessel functions. This became possible when different methods are used for the construction of Green's functions and the appearances of the latter are compared.

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Perspectives for the Numerical Solution of the Wave Action Equation on Unstructured Meshes

Aron Roland
Technical University Darmstadt, Germany

The solution of the WAE, which is a 5-dimensional PDE with nonlinear sources and sinks, is a computational intensive task since often more then 1000 realisations of linear waves for every geographical grid point are used to describe the sea surface. One major problem is the time scales, which had to be resolved when waves are entering shallow water. Here the spatial and temporal scales, in which the wave spectra change, become much smaller then in deep water. Already the adiabatic part of the WAE can be regarded in shallow water environments as a stiff problem. In particularly the discrete characteristic velocities of geographical and intra-spectral propagation can vary very strong in the different phase spaces in the presence of strongly varying bathymetry or in inhomogeneous media. The in-

clusion of the source terms makes the situation even more complicated, since the locally strong nonlinear terms, (e.g. depth induced wave breaking) enhance the stiffness of the WAE.

Actually the available numerical methods can be recast into two groups. In particularly this are OSM (Operator splitting Methods; e.g. WWIII, WAM, CREST, TOMAWAC, MIKE21SW and WWM) or implicit IDM (Iterative Direct Methods; e.g. SWAN) where the left hand side is discretized at ones and the source terms are linearized and solved iteratively following e.g. Patankar (1980). However, not all source terms can be linearized, especially problematic are the approximation of a 4-fold Bolzmann type nonlinear integral equations that describe the resonant wave-wave interaction in shallow water and other non-linear terms e.g. for the dissipation of waves in deep water.

OSM methods reduce memory consumption, vectorization and parallelization is applicable at the same time (e.g. Tolman, 2002) and the famous Lax-Theorem can be easily applied to prove the convergence of the whole scheme. The deficiency is that these methods are subject to so called “Splitting Errors” (e.g. Leveque 1990, Sportisse, 2002 and Lanser & Verwer, 1998). Different schemes have been invented in order to reduce the error of splitting (e.g. Strang, 1968) and increase the order of the whole splitting scheme. However, in the stiff case OSMs are usually undergoing a severe order reduction and make e.g. a simple “Strang-splitting” inefficient (e.g. Sportisse, 2002).

Implicit IDM lead to large equation systems and cumbersome memory management. Especially, when unstructured meshes are used and implicit time stepping, the efficient storage of the non-zero matrix entries and also the solution of the final equation system is a difficult mission. The prove of convergence is for the case of the WAE with sources terms extremely complicated within IDM. Furthermore, the use of nonlinear advection schemes (escaping the Godunov theorem) results within implicit methods to nonlinear equation systems for which convergence of the solution can hardly be guaranteed in the general case. This leads to linear first order monotone but rather diffusive schemes (e.g. BSBT) or non-monotone higher order linear schemes (see e.g. Rogers et al., 1999) which are rather doubtful and produce unphysical results in the vicinity of strong gradients in the solution.

The main motivation of this work is to present this very important and complicated problem to the “splitting community” and explore possibilities for interdisciplinary work between coastal engineers, oceanographers and mathematicians. The actually available solutions are not satisfactory and improved methods, like modified operator splitting methods, applying iterative algorithms, can improve the accuracy and reduce the order reduction problem (e.g. Geiser, 2007). Another way to go is to solve the multidimensional nonlinear problem using a fully implicit nonlinear solver, which was never done sine the computation complexity is very high and there is not guarantee for convergence using Newton type methods to solve the nonlinear system.

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Higher Order Operator-Splitting Methods Via Zassenhaus Product Formula: Theory and Applications

Gamze Tanoğlu
Izmir Institute of Technology, Turkey
(Jürgen Geiser, Nurcan Gücüyenen)

In this paper, we concentrate on how to improve the classical and iterative operator splitting methods via Zassenhaus product formula. In our approach, acceleration of the initial conditions and weighted polynomials ideas for each cases are individually handled and related algorithms are obtained. We restricted ourselves for bounded, linear

operators since partial differential equations can be easily converted to the system of ODE with the help of the method of lines. In this case, we discuss the acceleration of the operator splitting methods and their benefits of balancing time and spatial scales to respect the order reduction and stability. Finally, the verification of the improved splitting methods are done with numerical examples.

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Special Session 48: Differential, Difference, and Dynamic Equations

Martin Bohner, Missouri University of Science and Tech., USA
Stefan Hilger, Catholic University Eichstaett, Germany
Ağacık Zafer, Middle East Technical University, Turkey

Introduction: Going back to 1988, the study of dynamic equations on time scales is a fairly new area of mathematics. Motivating the subject is the notion that dynamic equations on time scales can build bridges between continuous and discrete mathematics. Time is considered to be an element of an arbitrary closed subset of the reals, the so-called time scale. Dynamic equations on the time scale of all real numbers are differential equations, while dynamic equations on the time scale of all integers are difference equations. But not only is this theory able to unify the continuous and the discrete, it also can help to extend these theories to cases in between and hence to other dynamic equations (e.g., q-difference equations). The study of time scales theory has led to several important applications, e.g., in the study of insect population models, neural networks, heat transfer, epidemic models and economic models. Talks in this session contain lectures on the topics of dynamic equations, differential equations, difference equations, and q-difference equations.

Principal Matrix Solutions and Variation of Parameters for Volterra Integro-Dynamic Equations on Time Scales

Murat Adıvar
Izmir University of Economics, Turkey

We introduce the principal matrix solution $Z(t,s)$ of the linear Volterra vector integro-dynamic equation

$$x^{\Delta}(t) = A(t)x(t) + \int_s^t B(t,u)x(u)\Delta u$$

and prove that it is the unique matrix solution of

$$Z^{\Delta_t}(t,s) = A(t)Z(t,s) + \int_s^t B(t,u)Z(u,s)\Delta u,$$
$$Z(s,s) = I.$$

We also show that the solution of

$$x^{\Delta}(t) = A(t)x(t) + \int_s^t B(t,u)x(u)\Delta u + f(t),$$
$$x(\tau) = x_0$$

is unique and given by the variation of parameters formula

$$x(t) = Z(t,\tau)x_0 + \int_\tau^t Z(t,\sigma(s))f(s)\Delta s.$$

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Oscillation Criteria for Fourth Order Nonlinear Dynamic Equations

Elvan Akin-Bohner
Missouri S&T, USA
(Shurong Sun)

We are concerned with oscillatory behavior of fourth order nonlinear dynamic equations.

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Spectral Analysis of a q-Difference Operator

Martin Bohner
Missouri S&T, USA
(M. Bekker, A. Herega, H. Voulov)

For a number q bigger than one, we consider a q-difference version of a second-order singular differential operator which depends on a real parameter. We give three exact parameter intervals in which the operator is semibounded from above, not semibounded, and semibounded from below, respectively. We also provide two exact pararameter sets in which the operator is symmetric and self adjoint, respectively. Our model exhibits a more complex behaviour than in the classical continuous case but reduces to it when q approaches one.

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Some Implications of a New Definition of the Exponential Function on Time Scales

Jan Cieśliński
University of Białystok, Poland

We present new defintions of exponential, hyperbolic and trigonometric functions on time scales, motivated by the Cayley transformation. These functions approximate more accurately the corresponding continuous functions and preserve most

of their qualitative properties. In particular, Pythagorean trigonometric identities hold exactly on any time scale.

Dynamic equations satisfied by the Cayley-motivated functions have a natural similarity to the corresponding differential equations but, in contrast to the standard approach, they are implicit. For instance, the Cayley-exponential function satisfies the equation

$$x^{\Delta}(t) = \alpha(t)\frac{x(t) + x(\sigma(t))}{2}$$

where $\alpha = \alpha(t)$ is a given function on a time scale. It suggests a new natural correspondence between differential equations and dynamic systems on time scales.

Our approach is strongly motivated by numerical methods. The delta calculus corresponds to the forward (explicit) Euler scheme, the nable calculus corresponds to the implicit Euler scheme, and the presented Cayley approach is related to the trapezoidal rule (and to the discrete gradient methods). An important conclusion is that there are no unique 'natural' time scales analogues of differential equations and it is worthwhile to consider and apply different numerical schemes in the context of the time scales approach.

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Boundary Data Maps for Schrödinger Operators on a Compact Interval

Stephen Clark
Missouri University of Science and Tech., USA
(F. Gesztesy and M. Mitrea)

A systematic study is described for boundary data maps, that is, 2×2 matrix-valued Dirichlet-to-Neumann and more generally, Robin-to-Robin maps, associated with one-dimensional Schrödinger operators on a compact interval $[0, R]$ with separated boundary conditions at 0 and R. Most of our results are formulated in the non-self-adjoint context.

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Fractional h-Difference Equations

Rui Ferreira
Lusophone University of Hum. and Tech., Portugal
(Delfim F. M. Torres)

We develop some properties for the fractional h-difference left and right operators. These will then be applied in finding solutions to some fractional h-difference equations, in particular, to equations arising from calculus of variations problems.

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Closed Form Solutions to Discrete Time Portfolio Optimization Problems

Mathias Goeggel
Missouri University of Science and Tech., Germany
(Martin Bohner)

We introduce a discrete time portfolio process, modeled by difference and stochastic difference equations. We give closed form solutions to different discrete time portfolio optimization problems. We also provide examples to illustrate the difference between these optimization problems.

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On Criticality of Higher Order Difference Operators

Petr Hasil
Masaryk University, Czech Republic

This is a joint work with Prof. Ondřej Došlý. We study the so-called p-critical difference operators, i.e., the non-negative operators that can be turned to negative ones by small (in a certain sense) negative perturbations. This research is motivated by [2], where the concept of critical operators is introduced for difference operators of the second order. We have generalized this concept in [1] for $2n$-order Sturm-Liouville difference operators, and we have suggested a criterion of p-criticality for one term $2n$-order difference operators $l(y)_k = \Delta^n(r_k\Delta^n y_k)$. This criterion turned out to be valid and we have proved it using a structure of the solution space of the equation $l(y)_k = 0$, see [3].

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$\longrightarrow \infty \diamond \infty \longleftarrow$

Orthogonal Polynomials in Quiver Representations

Stefan Hilger
Katholische Universität Eichstätt, Germany

After a short introduction into the general notion of quiver representations and a relevant specialization we discuss some conditions that ensure scalar operation of the so called intrinsic endomorphisms.

A couple of examples will illustrate the theory. Most of them show how ladders or quadratic quiver

structures are underlying the systems of classical orthogonal polynomials.

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Sturmian Theory for Linear Hamiltonian Systems without Controllability

Roman Šimon Hilscher
Masaryk University, Brno, Czech Republic

Sturmian theory for differential equations is a classical topic in the literature with active research and generalizations in various directions. Classical results of this type, the Sturmian comparison and separation theorems, relate the numbers of zeros or the focal points of two solutions of one or two differential equations. In this talk we present new results in the oscillation theory of linear Hamiltonian systems. In particular, we discuss Sturmian separation and comparison theorems for these systems when no controllability assumption is imposed, which generalizes the traditional results of W. T. Reid for controllable linear Hamiltonian systems. Our new theory is based on several recent results by W. Kratz, M. Wahrheit, V. Zeidan, and the author regarding the piecewise constant kernel for conjoined bases of the Hamiltonian system, the oscillation and eigenvalue theorems, and the Rayleigh principle for linear Hamiltonian systems without controllability.

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$\longrightarrow \infty \diamond \infty \longleftarrow$

Modeling Antibiotic Resistant Bacteria in Rivers

Bonita Lawrence
Marshall University, USA
(Anna Mummert and Charles Somerville)

Recent studies of surface waters have revealed large reservoirs of freshwater bacteria resistant to one or more antibiotics. Since these waterways, are multiple-use resources, including sources of drinking water, these discoveries are matters of public health interest. Concern over the spread of antibiotic resistant bacteria lead to the formation of the National Antimicrobial Resistance Monitoring System (NARMS) in the USA. However, this group does not monitor antimicrobial resistance in surface waters.

Inspired by data collected from water samples from the Mud and Ohio Rivers, we have developed a dynamical model for bacteria in the river that includes both antibiotic resistant and non-antibiotic resistant bacteria. Because of the strong correlation between the land use along the river and levels of antibiotic resistant bacteria, we have included a function that describes the influx of bacteria from the shore. In this talk we will report the latest results of our study.

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Higher-Order Calculus of Variations with Hahn Quantum Derivatives

Natalia Martins
University of Aveiro, Portugal
(Delfim F. M. Torres and Artur Miguel Cruz)

We prove a necessary optimality condition of the Euler-Lagrange type for quantum variational problems involving Hahn derivatives of higher-order.

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A Necessary and Sufficient Condition for Oscillation of Second Order Sublinear Delay Dynamic Equations

Raziye Mert
Çankaya University, Turkey
(Ağacık Zafer)

Time scale calculus approach allows one to treat the continuous, discrete, as well as more general systems simultaneously. In this study we use this access to establish a necessary and sufficient condition for the oscillation of a class of second order sublinear delay dynamic equations on time scales. Some well known results in the literature are improved and extended.

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Subsequential Convergence of Solutions of Dynamic Equations on Time Scales

Ralph Oberste-Vorth
Marshall University, USA

This is joint work with B. Lawrence. We discuss the appropriateness of the Hausdorff-Fell topology on the set of all time scales. Consider sequences of dynamic initial value problems

$$x_n^{\Delta} = f_n(t,x), \quad x_n(t_{0,n}) = x_{0,n}$$

over time scales $\mathbb{T}_n$, respectively, where the time scales $\mathbb{T}_n$, the functions f_n, and the initial conditions $(t_{0,n}, x_{0,n})$, converge to a time scale $\mathbb{T}$, a function f, and (t_0, x_0), respectively. We verify the convergence

of subsequences $x_{n_k}(t)$ of solutions of these initial value problems to a solution of the limit problem

$$x^{\Delta} = f(t, x), \quad x_n(t_0) = x_0.$$

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Oscillation Criteria for Second Order Mixed Nonlinear Differential Equations with Positive and Negative Coefficients

Abdullah Özbekler
Atilim University, Turkey
(A. Zafer)

In this work we obtain some new results on the oscillation of second order mixed nonlinear differential equations with positive and negative coefficients. We give some examples to emphasize the importance of the results.

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Regular Variation on Various Time Scales and Its Application to Dynamic Equations

Pavel Řehák
Academy of Sciences, Czech Republic

This is joint work with my Ph. D. student Jiří Vítovec. We discuss the concept of regular and rapid variation on time scales. We show that parallel theories of regular and rapid variation on individual time scales may differ, and there is a need of certain additional conditions on the graininess μ, which cannot be omitted. In particular, the cases where $\mu(t) = o(t)$ as $t \to \infty$ and $\mu(t) = (q-1)t$ with $q > 1$ (i.e., q-calculus) are examined. The obtained theory is applied to study asymptotic behavior of solutions to second order dynamic equations.

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Quantum Oscillator Equations and Basic Ghost State Functions

Andreas Ruffing
Technische Universität München, Germany
(Moritz Simon)

Some recent developments in the area of quantum oscillator equations are presented. Their filigrane function spaces are looked at in greater detail and particular properties of related complete and incomplete function systems are elucidated.

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Riccati-Asymptotics for Sturm-Liouville Equations Via Ehrling's Lemma

Katja Setzer
University of Ulm, Germany

This is joint work with Werner Kratz. We consider Sturm-Liouville Equations and the corresponding quadratic functional. By using a special case of Ehrling's Lemma from functional analysis we are able to show the positivity of the functional without explicit calculations and in a more general way. This leads easily to a proof of the fact that the positivity of the quadratic functional is equivalent to a certain kind of asymptotic behavior of solutions of the Riccati matrix differential equations associated with this functional.

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Global Stability of Some Symmetric Difference Equations

Stevo Stević
Serbian Academy of Sciences, Yugoslavia

We prove two globally convergence results regarding some general classes of symmetric rational difference equations. One of these results confirm a quite recent conjecture of interest.

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Oscillation of Even Order Nonlinear Delay Dynamic Equations on Time Scales

Ağacık Zafer
Middle East Technical University, Turkey

This is a joint work with Raziye Mert. One of the important methods for studying the oscillation of higher order differential equations is to make a comparison with second order differential equations. The method involves using Taylor series expansion of solutions. In this talk we show how such a method can be used for a class of even order delay dynamic equations on time scales via comparison with second order dynamic inequalities. In particular, it is shown that nonexistence of an eventually positive solution of a certain second order delay dynamic inequality is sufficient for oscillation of even order dynamic equations on time scales. The arguments are based on Taylor polynomials on time scales.

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Krein-Von Neumann and Friedrichs Extensions for Second Order Operators on Time Scales

Petr Zemánek
Masaryk University, Brno, Czech Republic

We consider an operator defined by the second order Sturm–Liouville equation on an unbounded time scale. For such an operator we give characterizations of the domains of its Krein–von Neumann and Friedrichs extensions by using the recessive solution. This generalizes and unifies similar results obtained for operators connected with the second order Sturm–Liouville differential equations by Niessen and Zettl and for operators associated with the second order Sturm–Liouville difference equations by Brown and Christiansen.

[1] B. M. Brown, J. S. Christiansen, On the Krein and Friedrichs extensions of a positive Jacobi operator, *Expo. Math.* 23 (2005), no. 2, 179—186.

[2] H. Freudenthal, Über die Friedrichssche Fortsetzung halbbeschränkter Hermitescher Operatoren (German), *Proc. Akad. Wet. Amsterdam* 39 (1936), 832–833.

[3] H. D. Niessen, A. Zettl, Singular Sturm-Liouville problems: The Friedrichs extension and comparison of eigenvalues, *Proc. London Math. Soc. (3)* 64 (1992), no. 3, 545–578.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Special Session 49: Complexity of Geometry and Analysis of Larger Scale Dynamical Systems

Henk Broer, University of Groningen, The Netherlands
Carles Simó, Universitat de Barcelona, Spain
Renato Vitolo, University of Exeter, UK
Gert Vegter, University of Groningen, The Netherlands

Introduction: The programme 'Complexity of geometry and analysis of larger scale Dynamical Systems' considers the larger scale geometry and dynamics in the product of higher dimensional phase space and parameter space. On the one hand we extend the more familiar theory of lower dimensional dynamics, also applying this by center manifolds, etc. What is lower depends a bit on the precise context, but in general the dimensions for flows do not exceed 2 or 3. Here any form of generalisation can be important. On the other hand we also are interested in genuinely in larger scale phenomena, for instance where multiple resonances play a role. Here one may apply Kolmogorov-Arnold-Moser Theory as well as Singularity Theory, often starting off from normal form approximations and the perturbative point of view, but also (numerical) continuations of these can be great importance. Here computational and algorithmic aspects play an important role. The interest is formed by theoretical results as well as by applications, e.g., to population or climate models. In the theoretical approach coupled cell system may play a role.

On Parametrized KAM Theory

Henk Broer
Johann Bernoulli Inst., Groningen, Netherlands

Parametrized KAM Theory and quasi-periodic bifurcation Theory developed in has been developed from Moser 1966 on by many researchers. It turns out that almost all the known KAM Theorems can be re-discovered in this way. Moreover, novel applications deal with higher dimensional bifurcations where many resonances play a role. The corresponding theory is a marriage of KAM Theory and Singularity Theory in the product of state space and parameter space, where a Cantorisation occurs of the semi-algebraic stratifications known from the latter theory. This research is of importance in larger dimensional modelling.

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A Variational Principle for Model Reduction in Chemical Kinetics: Computing Slow Invariant Manifolds Via Numerical Optimization

Dirk Lebiedz
University of Freiburg, Germany
(Jochen Siehr, Jonas Unger)

A key issue in dimension reduction of large-scale dissipative dynamical systems with multiple time scales and spectral gaps is the identification of slow attracting invariant manifolds. We present theoretical and numerical results for a variational approach to the problem of computing such manifolds for kinetic ODE models using numerical optimization of trajectories. The corresponding objective functional reflects a variational principle that characterizes trajectories on, respectively near, slow invariant manifolds. For a two-dimensional linear system and a common nonlinear test problem we prove that the variational approach asymptotically identifies the exact slow invariant manifold. Numerical results for linear and nonlinear model problems as well as a more realistic higher-dimensional chemical reaction mechanisms are presented.

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High Accuracy Computation of Rotation Numbers and Derivatives with Respect to Parameters

Alejandro Luque
Universitat Politecnica de Catalunya, Spain
(Jordi Villanueva)

We discuss an approach for computing rotation numbers of circle maps and derivatives with respect to parameters. The method is based on suitable averages of iterates of the map together with Richardson extrapolation. We will illustrate the method by studying Arnold Tongues and invariant curves for twist maps. If time permits, we will discuss generalizations to more general contexts.

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Existence of Chaotic Dynamics in Rayleigh-Bénard at Moderate Rayleigh Numbers

Dolors Puigjaner
Universitat Rovira i Virgili, Tarragona, Spain
(Joan Herrero, Carles Simó, Francesc Giralt)

Many challenges in science and engineering are related to the complex dynamics of large scale dynamical systems. The identification of the invariant objects, such as fixed points, periodic orbits, invariant tori and, if they have some hyper-

bolic properties, their invariant manifolds, homoclinic or heteroclinic orbits, and the related creation/destruction of chaoticity are crucial issues in the understanding of the dynamics of any complex system. Recent developed numerical methods allow the efficient computation of invariant manifolds for high-dimensional dissipative dynamical systems. We combine these Newton-Krylov methods with a Galerkin spectral method to compute the bifurcation diagram of steady and periodic solutions for the Rayleigh-Bénard problem inside a cube with perfectly conducting lateral walls for moderate Rayleigh numbers ($\mathrm{Ra} \leq 1.5\times 10^5$). The symmetries of the cubic domain are responsible for the complexity of the bifurcation diagram. Once the velocity and temperature fields are approximated by truncated expansions in terms of a divergence-free set of basis functions we obtain a system of dimension $O(10^4)$ whose unknowns are the coefficients in the expansions.

The bifurcation diagram obtained when the Prandtl number is fixed to $\mathrm{Pr} = 0.71$ and the Rayleigh number is used as a bifurcation parameter shows the presence of regions where different stable solutions coexist. It also shows the existence of several homoclinic solutions. The role of one of these homoclinic solutions in the generation of some chaotic regions for moderate Rayleigh values ($9 \times 10^4 \leq \mathrm{Ra} \leq 10^5$) at Pr slightly above 0.71 will be discussed.

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Computation of Periodic Orbits and Invariant Tori, in Large-Scale Dissipative Systems, by Newton-Krylov Methods

Juan Sanchez Umbria
Universitat Politècnica de Catalunya, Spain
(M. Net and C. Simó)

The computation of invariant objects is an essential tool to study dynamical systems. This kind of computations is more or less routinely performed for low dimensional systems. In the case of PDEs, it is now common to compute fixed points by continuation methods. Other invariant manifolds have only been calculated recently.

A method to compute periodic orbits and invariant tori in high-dimensional systems, obtained as discretizations of systems of parabolic/eliptic PDEs, by continuation and Newton-Krylov methods will be described. They are found as fixed points of a Poincare or generalized Poincare map, so that the dimension of the system of equations to be solved is that of the original system of ODEs. Therefore there is no prohibitive increase in the size of the problem. Due to the dissipative nature of the systems studied, the convergence of the linear solvers is extremely fast. The computation of periodic orbits inside the Arnold's tongues is also considered.

The thermal convection of a binary mixture of fluids, in a rectangular cavity, has been used to test the method. The main branch of tori obtained, starts at a Neimark-Sacker and ends at a pitchfork bifurcation. Later on, the stable branches of invariant tori undergo a cascade of period-doubling bifurcations leading to chaos. In this case, as typically happens, only a finite number of period doublings is found before the chaotic range is reached. Although the method is suitable for stable tori, we have been able to compute an small branch of unstable invariant tori.

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The Dynamics of a Low-Order Model for the Atlantic Multidecadal Oscillation

Alef Sterk
Johann Bernoulli Inst., Groningen, Netherlands
(H. W. Broer, H. A. Dijkstra, C. Simó, R. Vitolo)

Observational and model based studies provide ample evidence for the presence of multidecadal variability in the North Atlantic sea-surface temperature known as the Atlantic Multidecadal Oscillation (AMO). This variability is characterised by a multidecadal time scale, a westward propagation of temperature anomalies, and a phase difference between the anomalous meridional and zonal overturning circulations.

We study the AMO in a low-order model obtained by projecting a PDE for thermally driven ocean flows onto a 27-dimensional function space. We study bifurcations of attractors by varying the equator-to-pole temperature gradient (ΔT) and a damping parameter (γ).

For $\Delta T = 20°\mathrm{C}$ and $\gamma = 0$ the low-order model has a stable equilibrium corresponding to a steady ocean flow. By increasing γ to 1 a supercritical Hopf bifurcation gives birth to a periodic attractor with the spatio-temporal signature of the AMO. Through a period doubling cascade this periodic orbit gives birth to Hénon-like strange attractors. Finally, we study the effects of annual modulation by introducing a time-periodic forcing. Then the AMO appears through a Hopf-Neĭmark-Sacker bifurcation. For $\Delta T = 24°\mathrm{C}$ we detected at least 11 quasi-periodic doublings of the invariant torus. After these doublings we find quasi-periodic Hénon-like strange attractors.

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On Stickiness and Flicker-Noise

Dmitry Turaev
Imperial College, England

We show that for every generic analytic area-preserving map of a two-dimensional disc, given any wild hyperbolic set Λ, the expected value of τ^α is infinite for every $\alpha > 1$, where τ is the passage time across Λ. It follows that given an elliptic island within the chaotic zone, one should not expect the probability density for the time τ spent by a chaotic orbit near the island to decay faster than $\propto \tau^{-2}$. For a constant flow of phase points injected near the island, this estimate gives for the fluctuations of the number of phase points stuck to the island the power spectrum roughly ω^{-1} at small frequencies ω.

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Dynamics in Chaotic Zones of Area Preserving Maps (APMs): Close to Separatrix and Global Instability Zones

Arturo Vieiro
Universitat de Barcelona, Spain
(C. Simó)

In this talk we will focus on the dynamics in a neighbourhood of the separatrices of a resonant zone. The well-known separatrix map, defined on a figure eight when needed, is used to determine the location of rotational invariant curves (r.i.c.) inside and outside the resonance. The interest is on a quantitative description of the dynamics in a neighbourhood of the separatrices: to produce theoretical estimates of the width of the stochastic zone, distance to the r.i.c., existence of tiny islands close to the separatrices, ... In every one of the studied items we will complement the limit analytic study with realistic numerical simulations, describing the analogy when possible.

Then we will focus on the formation of larger domains without r.i.c. (e.g. Birkhoff domains). To this end we introduce the biseparatrix map model. Although this is a qualitative model, the mechanism of destruction of the "last" r.i.c., and hence the process of creation of zones without r.i.c., is clarified by means of this simple model.

The methodology that we will present to study the phase space of an APM consists on gluing different return models. We believe that can be systematically adapted, using suitable numerical/analytical tools, to more general situations.

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Special Session 50: Positive Systems

Tobias Damm, TU Kaiserslautern, Germany
Anke Kalauch, TU Dresden, Germany

Introduction: The session will deal with positivity methods in dynamical systems, differential equations and applications. The discussion will be around aspects of operator theory, Perron-Frobenius type theorems, stability results and matrix analysis.

Monotony of Solutions of Some Difference and Differential Equations

Jozef Bobok
Technical University in Prague, Czech Republic
(Ivo Marek)

In our contribution motivated by some analysis of the first author concerning bounds of topological entropy it is shown that a well known sufficient condition for a difference and differential equation with constant real coefficients to possess strictly monotone solution appears to be also necessary. Transparent proofs of adequate generalizations to Banach space analogs are presented.

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The Problem of Global Identifiability for Systems with Tridiagonal Matrices

Begoña Cantó Colomina
University Polithecnic of Valencia, Spain
(Carmen Coll, Elena Sanchez)

In this work a parametric system with symmetric tridiagonal matrix structure is considered. In particular, parametric systems whose state coefficient matrix has non-zero (positive) entries only on the diagonal, the super-diagonal and the sub-diagonal are analyzed. The structural properties of the model are studied, and some conditions to assure the global identifiability are given. These results guarantee the existence of only one solution for the parameters of the system. In practice, systems with this structure arise, for example, via discretization or finite difference methods for solving boundary and initial value problems involving differential or partial differential

equations.

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On Lyapunov Operators and Their Positivity Properties

Tobias Damm
TU Kaiserslautern, Germany

The Lyapunov operator is a central tool in linear control theory. As a mapping on the space of Hermitian matrices it possesses interesting positivity properties, which are useful in the analysis of dynamical systems. In this talk we give a survey of different generalizations of the Lyapunov operator and show how their monotonicity can be exploited.

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Tropical Aspects of Nonlinear Perron-Frobenius Theory

Stephane Gaubert
INRIA and Ecole Polytechnique, France
(M. Akian, A. Guterman)

Tropical algebra arises when looking at the usual structure or real numbers with logarithmic glasses. This yields a deformation in which order preserving and linear maps degenerate to tropically linear maps (which preserve the maximum, and commute with the addition of a constant). More generally, logarithmic glasses transform the order preserving non-linear maps, which belong to non-linear Perron-Frobenius theory, to dynamic programming operators of zero-sum stochastic games. In this talk, I will survey some results which emerge when seeing non-linear Perron-Frobenius theory from a tropical perspective, and in particular, the representation of the sets of sub-fixed points of an order preserving, homogeneous, map as a tropical convex set. This yields an equivalence between mean payoff games and tropical linear programming (arXiv:0912.2462).

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On the Solution of Nonnegative Discrete-Time Singular Control Systems

Alicia Herrero
Universidad Politecnica de Valencia, Spain
(Francisco J. Ramirez, Nestor Thome)

In the last years, the nonnegativity of linear control systems has received an increasing interest. The characterization of the solution of this kind of systems has been developed by using the nonnegativity of the singular coefficient matrix. Recently, some results have been given, which involve this nonnegativity property and the nonnegativity of the group-projector. In those cases only index 1 matrices have been considered. In order to obtain results for indices greater than one, two different approaches to the nonnegativity of the solution of a singular discrete-time linear control system are presented in this paper. Here, we will consider singular linear systems whose singular coefficient matrix is nonnegative and has nonnegative Drazin inverse. Firstly, we study the nonnegativity of a given system using a representation of this kind of matrices. Next, we will construct a finite number of subsystems whose solutions allow to obtain the general solution of an initial given system. Then, the nonnegativity of the initial system will be studied through its relation with these set of associated subsystems.

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Dynamical Systems Generated by Barriers on Convex Sets

Roland Hildebrand
UJF Grenoble / CNRS, France

A barrier function on an open convex subset of $\mathbf{R}^n$ is a strictly convex function which tends to infinity as the argument tends to the boundary of the set. The Hessian of the barrier confers the set a structure of a Riemannian manifold. This manifold is complete, i.e., the geodesic flow does not leave the set, which allows to use the barrier as a convenient description of the convex set for optimization purposes. In projective space, the notion of convexity of a function, contrary to the notion of convexity of a set, is not well-defined, and the concept of a barrier cannot be directly carried over from the affine case. We present a self-contained theory of barriers in projective space which is build upon the cross-ratio, a fundamental projective invariant, and the interaction of the projective structures on the convex set and its dual. The results provide a new interpretation of the affine theory and have applications in conic optimization.

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Robust Stability of Positive Linear Systems

Anke Kalauch
TU Dresden, Germany

For the investigation of the robustness of a stable linear system under structured perturbations the notions of complex and real stability radius are central tools. Generally it is more difficult to analyse the real stability radius than the complex one. Considering positive systems, both radii coincide. For standard orderings this is a well-known fact, which we extend to more general situations.

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Positive Operators with No Nontrivial Invariant Sublattices

Arkady Kitover
Community College of Philadelphia, USA
(A. W. Wickstead)

Using some results from topological dynamics we construct positive disjointness preserving operators without closed nontrivial invariant sublattices on $L^p(S)$, $1 \leq p \leq \infty$, and on $C(S)$ where S is the unit circle.

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Splittings of Operators and Nonnegative Generalized Inverses

Sivakumar Koratti Chengalrayan
IIT Madras, India

In this talk we present necessary and sufficient conditions for the nonnegativity of specific generalized inverses of operators on (possibly) infinite dimensional spaces. These conditions involve a certain splitting of the operator concerned. The main results, generalize recent results of M. Weber (On Positive Invertibility of Operators and Decompositions, Math. Nach., to appear, 2010).

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Swarms of Birds and Positive Dynamical Systems

Ulrich Krause
Universität Bremen, Germany

For a model of bird flocking due to F. Cucker and S. Smale [1] alternative conditions on the local interaction of birds are provided which lead to the global phenomenon of swarms. The proof employs a result on the collective dynamics within positive dynamical systems in discrete time [2]. F. Cucker and S. Smale, *Emergent behaviour in flocks*, IEEE Trans. on Autom. Control, 52 (2007), 852–862. U. Krause, *Positive particle interaction*, In L. Benvenuti, A. De Santis, L. Farina (Eds.), Positive Systems, Lect. N. in Control and Inf. Sciences, 294 (2003), 199–206.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Stability and Convergence in Discrete Convex Monotone Dynamical Systems

Bas Lemmens
University of Kent, England
(Marianne Akian and Stephane Gaubert)

In this talk I will discuss the behaviour of discrete monotone dynamical systems on Euclidean space, where the map is convex and preserves the partial ordering induced by standard positive cone. Such dynamical systems arise in the analysis of Markov decision processes, game theory, and computer science.

A useful concept in this context is tangential stability of fixed points and periodic points, which is weaker than the usual Lyapunov stability. Among others, we will see that the set of tangentially stable fixed points is isomorphic to a convex inf-semilattice, and a criterion for the existence of a unique tangentially stable fixed point is presented. It will also be shown that the periods of tangentially stable periodic points are orders of permutations on n letters, where n is the dimension of the underlying space. (Unstable periodic points can have arbitrary large periods.) Furthermore a condition is obtained under which there is global convergence to Lyapunov stable periodic orbits.

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Monotonicity and Riccati Differential Equations on Euclidean Jordan Algebras

Oliver Mason
National University of Ireland Maynooth, Ireland

Euclidean Jordan Algebras provide a natural framework for the analysis of symmetric cones such as the cone of positive semi-definite matrices over the real or complex numbers or the second-order cone. In this talk we consider algebraic and differential Riccati equations defined on Euclidean Jordan algebras. Specifically, using general results on monotone dynamics over arbitrary cones, we demonstrate that differential Riccati equations on arbitrary Euclidean Jordan algebras are monotone. This generalizes a well known property of symmetric and hermitian Riccati equations and provides a novel proof of this fact. We also investigate the implications of this result for the existence of solutions to algebraic Riccati equations and inequalities on Euclidean Jordan algebras.

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Positivity, Robust Stability and Comparison of Dynamic Systems

Alexey Mazko
Institute of Mathematics of NAS of Ukraine

We investigate generalized classes of positive and monotone dynamical systems in a partially ordered Banach space. Using some results from nonlinear operators theory, we establish new algebraic conditions for the stability of equilibrium states of the class of monotone type differential and difference systems. Conditions for the positivity and absolute stability of positive differential systems with delay are proposed. Using new technique for construction

of the invariant sets of differential systems in terms of cone inequalities, we generalize known positivity conditions for linear and nonlinear differential systems with respect to typical classes of cones. In addition, we formulate generalized comparison principle for a finite family of differential systems in the form of a cone inequality.

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Integer Interpretations of Some Topological Invariants of Classical Orlicz and Marcinkiewicz Spaces

Alexander Mekler
S.-Petersburg Math. Society, Russia, Germany

In terms of sequences of positive integers the interpretation of some topological invariants of Orlicz and Marcinkiewicz spaces in particular of the coincide of these is represented.

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Output Monotonicity for a Special Class of Linear Differential Systems

Sergio Romero-Vivo
Universidad Politecnica de Valencia, Spain
(Jorge Bondia and Beatriz Ricarte)

The output monotonicity problem of linear differential systems with non-Morishima state matrix is addressed. Suitable state transformations are carried out in order to deduce cone-generated realizations of the corresponding transfer function which guarantee, under certain conditions, monotonicity of the output.

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Analyticity of Weak Solutions to Linear Parbolic Systems and a Model in Finance

Peter Takac
Universität Rostock, Germany

We first motivate our linear parabolic problem by a stochastic volatility model (a linear system of two strongly coupled equations) from mathematical finance. Then we use H^2-type Hardy spaces to establish analyticity of weak solutions to the linear parabolic system. Our proof requires only L^2 approximation of the initial data by H^2 functions combined with a standard a priori L^2 estimate for weak solutions. A characterization of the Fourier transform of a function from H^2 is essential in the proof.

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Periods of Order-Preserving Nonexpansive Maps on Strictly Convex Normed Spaces

Onno van Gaans
Leiden University, Netherlands
(Bas Lemmens)

Let X be a closed convex sublattice of $\mathbb{R}^n$ containing 0 and let $f\colon X \to X$ be an order-preserving map with $f(0) = 0$. If f is nonexpansive with respect to a strictly monotone and strictly convex norm, then there exists an integer p such that the sequence of iterates $(f^{kp}(x))_k$ converges for each $x \in X$ to periodic point of f. The integer p is the order of a permutation on n letters. The analysis relies on an asymptotic decomposition of f into a nonexpansive projection r and an isometry on the range of r.

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Spectraloid Operator Polynomials with Positive Semidefinite Coefficients

Harald Wimmer
University of Wuerzburg, Germany
(Jan Swoboda)

We study operator polynomials of the form $G(z) = Iz^m - \sum_{j=0}^{m-1} C_j z^j$, where the coefficients C_j are positive semidefinite operators on $\mathbb{C}^n$. If $\sum_{j=0}^{m-1} C_j \leq I$ then the spectrum of $G(z)$ lies in the closed unit disc, and $G(z)$ is spectraloid in the sense that the polynomial numerical radius and the spectral radius coincide. The focus is on the spectrum in the boundary of the numerical range. It will be shown that the characteristic values of $G(z)$ on the unit circle are in the normal spectrum of $\sigma(G)$ and that they are semisimple.

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Positive Group Representations in Banach Lattices

Marten Wortel
Leiden University, Netherlands
(Marcel de Jeu)

Inspired by the theory of unitary group representations in Hilbert spaces, we investigate positive group representations in Banach lattices. Using methods based on the atomic structure, we present a characterization of positive strongly continuous representations of compact groups in finite dimensional Banach lattices and certain sequence spaces. We also show that these representations can be decomposed uniquely into band irreducible representations.

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On the Discretisation of Switched Positive Systems

Annalisa Zappavigna
Politecnico di Milano, Italy
(Patrizio Colaneri, Stephen Kirkland, Robert Shorten)

In this paper the discretisation of switched and non-switched linear positive systems using Padé approximations is considered. We show:

1. Diagonal Padé approximations preserve both linear and quadratic co-positive Lyapunov functions.
2. Positivity need not be preserved even for arbitrarily small sampling time for certain Padé approximations.

Sufficient conditions on the Padé approximations are given to preserve positivity of the discrete-time system. Finally, some examples are given to illustrate the efficacy of our results.

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Special Session 51: Holomorphic Dynamics in the Complex Plane and Higher Dimensions

Robert L. Devaney, Boston University, USA
Xavier Jarque, Universitat Rovira i Virgili at Tarragona, Spain
Janina Kotus, Warsaw University of Technology, Poland

Introduction: This special session will feature talks about complex dynamical systems in one dimension — polynomial dynamics, rational dynamics, and transcendental dynamics — as well as higher dimensional systems and holomorphic vector fields.

Mating Cubic Polynomials

Magnus Aspenberg
Sweden
(Pascale Roesch)

In this talk I will present a recent result together with P. Roesch (Toulouse) on the techniques of mating polynomials in degree 3. The technique of mating was invented by A. Douady and J. Hubbard as a way to partially parameterise rational maps (of degree at least 2) by pairs of polynomials (of the same degree). One starts with two polynomials (with the same degree) with locally connected Julia sets. Then the idea is to glue these Julia sets along their boundaries in reverse order so as to obtain a new Julia set of a rational map (of the same degree as the polynomials).

In the quadratic case, a conjecture states that this is possible whenever the (quadratic) polynomials belong to non-conjugate limbs of the Mandelbrot set. This mating-conjecture has been settled in the post-critically finite case by L. Tan, M. Rees, M. Shishikura. In the non-postcritially finite case we have results by S. Zakeri and M. Yampolsky (for Siegel quadratic polynomials) and by M. Yampolsky and M. Aspenberg (for non-renormalisable quadratic polynomials).

In the talk, we aim to show that cubic Newton maps can be viewed as matings of cubic polynomials.

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Hausdorff Dimension of the Sierpiński Julia Sets

Krzysztof Barański
University of Warsaw, Poland
(Michał Wardal)

We consider the family of rational maps on the Riemann sphere given by $F_\lambda(z) = z^n + \lambda/z^n$, where $\lambda \in \mathbb{C}$ and n is a positive integer larger than 1. The topological and combinatorial properties of such maps where studied in several papers by R. Devaney and his collaborators. They showed that if critical points (different from $0, \infty$) tend to infinity under iteration, then the Julia set of F_λ is either a Cantor set, a Cantor set of quasicircles or is homeomorphic to the Sierpiński carpet. We estimate the Hausdorff dimension of the Julia set for such parameters in the case of large n. In particular, we show that under some conditions, the Hausdorff dimension of the Julia set of F_λ is not larger than $1 + c/\log n$.

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A Taste of Transcendental Dynamics

Anna Benini
SUNY (Stony Brook), UB (Barcelona), Spain

Abstract: We will introduce some of the dynamical features of the complex exponential family $e^z + c$, exploring some properties of their Fatou and Julia sets in the special cases in which the map has an attracting cycle or the unique singular value c

is preperiodic. In the end we will give an idea on how this dynamical properties can be translated into analogous properties of the bifurcation locus and the stable locus in parameter plane.

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Checkerboard Julia Sets II

Paul Blanchard
Boston University, USA

(Figen Cilingir, Robert L. Devaney, Daniel M. Look, Elizabeth D. Russell)

Let $F_\lambda(z) = z^n + \lambda/z^d$ with $n \geq 2$, $d \geq 1$, and $\lambda \in \mathbb{C}$. In his talk in this session, Dan Look will describe the topology of the "checkerboard" Julia sets that arise in these families, and he will introduce the parameter spaces. In this talk, we will elaborate on the dynamical invariant that classifies the conjugacy classes of the principal Mandelbrot sets, and we will count of the number of such classes for a fixed n and d.

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The Structure Theorem of Degree d Complex Polynomial Vector Fields in $\mathbb{C}$

Bodil Branner
Technical University of Denmark

(Kealey Dias)

Any polynomial vector field $\xi_P = P(z)\frac{d}{dz}$ in $\mathbb{C}$ is affine conjugate to one (or several) for which the polynomial is monic and centered. Therefore it suffices to study those. We show that one can assign a combinatorial invariant $\mathcal{C}(\xi_\mathcal{P})$ and an analytic invariant $\mathcal{A}(\xi_\mathcal{P})$ to each such vector field. We then define admissible combinatorial and analytic data sets. The Structure Theorem states that a monic and centered polynomial vector field is uniquely determined by the invariants $(\mathcal{C}(\xi_\mathcal{P}), \mathcal{A}(\xi_\mathcal{P}))$ and that for each admissible sets of data $(\mathcal{C}, \mathcal{A})$ there exists a unique monic and centered polynomial vector field realizing those. This result is joint work with Kealey Dias. It generalizes the pioneering work by Douady, Estrada, and Sentenac, who proved the theorem for the structurally stable polynomial vector fields. This talk is about the dynamics of the vector fields, while the talk by Kealey Dias will be about the structure of the parameter space when decomposed into loci of vector fields sharing the same combinatorial data.

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Two Theorems of Herman on Siegel Disks

Arnaud Chéritat
CNRS, Institut de Math. de Toulouse, France

(Pascale Roesch)

We will present two enhancements of two theorems of Herman concerning Siegel disks. The first one is a construction of a Siegel disk whose boundary is non-locally connected, but the dynamical system is holomorphic inside the Siegel disk, and only smooth outside. I will show how to construct an example where the dynamical system is holomorphic beyond the boundary. Second, Herman proved that under some diophantine condition (later called Herman's condition) on a rotation number ϑ, the critical point of a unicritical polynomial with a Siegel disk Δ with rotation number ϑ must belong to the boundary of Δ. In a collaboration with Pascale Roesch, we extend this result to bi-critical polynomials.

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Condense Wandering Triangles

Clinton Curry
Stony Brook University, USA

(Alexander Blokh and Lex Oversteegen)

A *wandering branch point* for a polynomial P with connected Julia set J is a point $z \in J$ such that $J \setminus \{z\}$ has at least three components and z is neither preperiodic or precritical. A cornerstone of Thurston's theory of quadratic laminations is the No Wandering Triangles theorem, which implies that quadratic polynomials have no wandering branch points. In 2004, Alexander Blokh and Lex Oversteegen constructed examples of cubic polynomials with wandering branch points. We introduce a new construction which is more flexible. Thereby we prove that the space of laminations corresponding to wandering branch points is dense in an appropriate parametrization (though it is of first category). Even more, we can choose a dense set of laminations which correspond to polynomials with a wandering branch point whose orbit intersects every proper subcontinuum.

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Julia Sets Converging to Filled Julia Sets of $z^2 + c$

Robert Devaney
Boston University, USA

It is known that, in the family $z^2 + \lambda/z^2$, as $\lambda \to 0$, the Julia set converges to the unit disk, i.e., to the filled Julia set of z^2. In this talk we discuss the more general case of $z^2 + c + \lambda/z^2$ where c is the center of a hyperbolic component in the Mandelbrot set.

We show that these Julia sets converge to the filled Julia set of z^2+c, provided that the second iterates of the critical points reach a limit in a certain region of the filled Julia set of z^2+c. This happens if the filled Julia set of z^2+c is the basilica or the Douady rabbit, for example.

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Parameter Space of Degree d Complex Polynomial Vector Fields in $\mathbb{C}$

Kealey Dias
Christian-Albrechts-Universität zu Kiel, Germany

The space of degree d monic and centered complex polynomial vector fields in $\mathbb{C}$ is parameterized by the $d-1$ complex coefficients $\mathbf{a} = (a_0, \ldots, a_{d-2}) \in \mathbb{C}^{d-1}$ of the associated polynomials $P(z) = z^d + a_{d-2}z^{d-2} + \cdots + a_0$. We decompose this space in a natural dynamical way into loci consisting of vector fields having the same global combinatorial data. We analyze the topological and geometric structure of each locus, and furthermore determine how the different loci fit together, via a description of the possible bifurcations. For a description of the global dynamics of such a vector field, see the talk by Bodil Branner.

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Configurations of Herman Rings for Meromorphic Transcendental Functions

Nuria Fagella
Universitat de Barcelona, Spain
(Jorn Peter)

We study the existence and possible configurations of periodic cycles of Herman rings for transcendental meromorphic functions. By a quasiconformal surgery construction which "transcendentalizes" rational maps, we show that any configuration which is realizable by a rational map, is also realizable by a meromorphic transcendental function.

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Perturbing Baker Domains into Attracting Basins

Tania Garfias-Macedo
Georg-August-Universität Göttingen, Germany

The present work addresses two families of entire transcendental functions with infinitely many critical points in the framework of perturbation preserving the Julia set. First, the family $F_a(z) = z - a + e^z$, with $a \in \mathbb{C}$, has an invariant Baker domain $\mathcal{B}_a$ as unique Fatou component for parameters with $Re(a) > 0$. Its perturbation is $F_{a,\beta}(z) = \beta(z - a + e^z)$, with $\beta \in (0,1)$ and $\beta \to 1$. Second, the functions $G_\lambda(z) = z - 1 + \lambda z e^z$ have an invariant Baker domain $\mathcal{B}_\lambda$ for all $\lambda \in \mathbb{C}^*$. We perturb them as $G_{\lambda,\mu}(z) = (1-\mu)(z - 1 + \lambda z e^z)$, with $\mu \in (0,1)$ and $\mu \to 0$. In both cases, the Fatou sets of the perturbed functions have an attracting basin which becomes the Baker domain in the limit. Furthermore, we obtain convergence in the Hausdorff metric of the Julia sets, even though the Julia set of $G_{\lambda,\mu}$ presents an intricate structure.

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Singular Perturbations in the Quadratic Family with Multiple Poles

Antonio Garijo
Universitat Rovira i Virgili, Spain
(Sebastian Marotta and Elizabeth D. Russell)

We consider the quadratic family of complex maps given by $q_c(z) = z^2 + c$ where c is the center of a hyperbolic component in the Mandelbrot set. Then, we introduce a singular perturbation on the corresponding superattracting cycle by adding one pole to each point in the cycle. When $c = -1$ the Julia set of q_{-1} is the well known basilica and the perturbed map is given by $f_\lambda(z) = z^2 - 1 + \lambda/(z^{d_0}(z+1)^{d_1})$ where $d_0, d_1 \geq 1$ are integers, and λ is a complex parameter such that $|\lambda|$ is very small. We focus on the topological characteristics of the Julia and Fatou sets of f_λ that arise when the parameter λ becomes nonzero.

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Sierpinski Curve Julia Sets in Rational Maps of Degree Two

Xavier Jarque
Universitat Rovira i Virgili at Tarragona, Spain
(Devaney, R. L., Fagella, N., Garijo, A.)

We investigate under which dynamical conditions the Julia set of a rational map of degree two is a Sierpinski curve. Precisely, we force the rational map of degree two to be hyperbolic and satisfying certain dynamic condition (the existence of a period n super–attracting basin), so that there is only one free critical point. Under these assumptions the problem can be discussed in terms of the intersection (or not) of the Fatou domains.

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Thermodynamic Formalism for Some Meromorphic Maps

Boguslawa Karpinska
Warsaw University of Technology, Poland
(Krzysztof Baranski and Anna Zdunik)

In this talk we shall discuss the techniques of thermodynamic formalism for some classes of meromorphic maps and show how they can be applied to study the geometry of Julia sets.

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Generalized Riley Slices

Linda Keen
CUNY – Lehman College and Grad. Center, USA

In this talk we will discuss how to extend the theory of pleating rays for those Kleinian groups in the Riley Slice to groups representing orbifolds. We will show how the dynamics of the boundary of this space can be described by continued fractions.

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Checkerboard Julia Sets

Daniel Look
St. Lawrence University, USA
(Robert L. Devaney, Paul Blanchard, Elizabeth D. Russell, Figen Cilingir)

The parameter plane for the maps $F_\lambda(z) = z^n + \lambda/z^d$ with $n \geq 2$, $d \geq 1$ and $\lambda \in \mathbb{C}$ contains $n-1$ large copies of the Mandelbrot set. We refer to these Mandelbrot sets as the *Principal Mandelbrot sets*. The Julia set for F_λ when λ is drawn from the center of the main cardioid of a principal Mandelbrot set is a *Checkerboard Julia set*. For a given n, d all Checkerboard Julia sets are homeomorphic, yet the dynamics of the respective maps on these Julia sets are not necessarily conjugate. We will investigate a dynamical invariant that explains why some of these maps fail to be conjugate. Further, this invariant allows us to count the exact number of conjugacy classes for a given n, d.

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Buried Points in Julia Sets

John Mayer
University of Alabama at Birmingham, USA
(Clinton P. Curry and E. D. Tymchatyn)

Let $J(f)$ be the Julia set of a rational function f (of degree > 1), and let $J'(f)$ be the subset of points of $J(f)$ not on the boundary of any component of $F(f) = C_\infty \setminus J(f)$, the complement of $J(f)$ in the Riemann sphere, usually called the Fatou set of f. We call $J'(f)$ the *buried points* of the Julia set. (For clarification, note that a buried point is never accessible from a complementary component of $J(f)$, but an inaccessible point is not necessarily buried.) We ask if for a connected Julia set, $J'(f)$ can be totally disconnected without being zero-dimensional? We discuss partial results and examples.

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Linearizers of Entire Functions I

Helena Mihaljevic-Brandt
Christian-Albrechts-Universität zu Kiel, Germany
(Jörn Peter)

For a meromorphic function f with a repelling fixed point z_0, it is well-known that there exists a meromorphic function L which satisfies the Schröder equation $f(L(z)) = L(mz)$ for all z in the complex plane, where m is the multiplier of the point z_0, i.e., $f'(z_0) = m$. The function L is called the Poincaré function or the linearizer of f at z_0 and it is unique with the normalization $L(0) = z_0$ and $L'(0) = 1$. Restricting to the case when f is entire, the resulting linearizer is also entire. There has been great interest in linearizers of polynomials as well as of transcendental entire maps, and for polynomial-linearizers a lot is known. This talk and its second part aim to present some interesting features of linearizers in general as well as to discuss some new results on exceptional and asymptotic values of linearizers of transcendental entire function.

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A Combinatorial Description of Sierpinski Julia Sets

Monica Moreno Rocha
Centro de Investigacion en Matematicas, Mexico

Singular perturbations of z^n give rise to certain one-parameter families of rational maps acting on the Riemann sphere whose Julia sets are homeomorphic to the Sierpinski curve continuum. A common condition among these maps is their escaping critical orbits that eventually enter the immediate basin of infinity after some positive number of iterations.

Let F_A and F_B be two of these maps and assume they are postcritically finite, with critical orbits landing on infinity at the same number of iterations. Then, F_A and F_B are known to be topologically conjugate if and only if their parameters satisfy $A = \omega^{2j} B$, with ω a $(n-1)$th primitive root of unity (Devaney & Pilgrim, 2009).

In this talk we address the question of characterizing conjugacy classes in a combinatorial way. To do so, we present a combinatorial model of Sierpinski curve Julia sets for postcritically finite rational

maps. In particular, the combinatorial invariant obtained from the model can be used to characterizes hyperbolic components in the connectedness locus of these rational families.

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Linearizers of Entire Functions II

Jörn Peter
University of Kiel, Germany
(Helena Mihaljević-Brandt)

For a meromorphic function f with a repelling fixed point z_0, it is well-known that there exists a meromorphic function L which satisfies the Schröder equation $f(L(z)) = L(mz)$ for all z in the complex plane, where m is the multiplier of the point z_0, i.e., $f'(z_0) = m$. The function L is called the Poincaré function or the linearizer of f at z_0 and it is unique with the normalization $L(0) = z_0$ and $L'(0) = 1$. Restricting to the case when f is entire, the resulting linearizer is also entire. There has been great interest in linearizers of polynomials as well as of transcendental entire maps, and for polynomial-linearizers a lot is known. This talk will present more interesting results on linearizers of transcendental entire functions.

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Density of Hyperbolicity in Spaces of Real Transcendental Entire Functions

Lasse Rempe
University of Liverpool, England
(Sebastian van Strien)

We prove density of hyperbolicity in some spaces of real transcendental entire functions with finitely many singular values.

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Periodic Slices in Parameter Space, Focus on Hyperbolic Components

Pascale Roesch
UPS, IMT, Toulouse, France

Following J. Milnor, we look in the space of rational maps at the slices where the rational maps have some fixed critical relations. For instance periodic slices are the “slices” where some critical points stay with some given periods. We will give several examples in which the intersection of the hyperbolic components with the periodic slices are “well understood”.

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Complex Dynamics and Symbolic Dynamics

Elizabeth Russell
US Military Academy (West Point), USA
(Paul Blanchard, Robert Devaney, Antonio Garijo)

This talk will focus on the families of maps of the form z_n+c+A/z_n where the parameter A is a center of a hyperbolic component of the Multibrot set and $n > 2$. We show that, for A sufficiently small the Julia set of this map contains a countable collection of simple closed curves (some of which are “decorated”) together with an uncountable collection of point components. The presence of these point components will be shown to exist via symbolic dynamics. This generalizes a result of McMullen which shows that, when $A = 0$, the Julia set is a single Cantor set of simple closed curves.

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Special Session 52: Singular Perturbations

Freddy Dumortier, Hasselt University, Belgium
Peter De Maesschalck, Hasselt University, Belgium
Martin Wechselberger, University of Sydney, Australia

Introduction: The aim of this special session is to get informed about recent results on singular perturbations, both from a pure, applied and numerical point of view. Besides scheduling talks from established mathematicians, we will give opportunity to junior researchers to present their work. Topics include (non-exhaustive): (geometric) singular perturbation theory, mixed-mode oscillations, canards, singularly perturbed PDE, delay in bifurcations.

Bifurcation Delay – the Case of the Sequence: Stable Focus – Unstable Focus – Unstable Node

Eric Benoît
Universié de La Rochelle, France

We consider a two dimensional family of real vector fields. We suppose that there exists a stationary point where the linearized vector field has successively a stable focus, an unstable focus and an unstable node. It is known that when the parameter

moves slowly, a bifurcation delay appears due to the Hopf bifurcation. The main question investigated is the continuation of the delay in the region of the unstable node. For a typical example, it is proved that the delay is pushed after the focus node transition.

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Detectable Canard Cycles with Singular Slow Dynamics of Any Order at the Turning Point

Peter De Maesschalck
Hasselt University, Belgium
(F. Dumortier)

In this talk, we study limit cycles that appear in a class of planar slow-fast systems, near a canard limit periodic set of FSTS-type. Limit periodic sets of FSTS-type are closed orbits, composed of a Fast branch, an attracting Slow branch, a Turning point, and a repelling Slow branch. Techniques to bound the number of limit cycles near a FSTS-limit periodic set are based on the study of the so-called slow divergence integral, calculated along the slow branches. Typically, these techniques are based on the presence of a nonzero speed in the slow dynamics. We extend the technique to the case where the slow dynamics has singularities of any (finite) order that accumulate to the turning point, and in which case the slow divergence integral becomes unbounded. Bounds on the number of limit cycles near the FSTS-limit periodic set are derived by examining appropriate derivatives of the slow divergence integral.

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Mixed-Mode Dynamics in the Olsen Model

Mathieu Desroches
University of Bristol (UK), England
(Bernd Krauskopf and Hinke M Osinga)

We consider the four-dimensional Olsen model for the Peroxidase-Oxidase reaction — a system that naturally displays complex oscillatory solutions known as Mixed-Mode Oscillations (MMOs). One difficulty compared to other models showing MMOs is that the Olsen model does not have explicit timescale separation. We show how computing two-dimensional (locally) invariant manifolds, which act as extended slow manifolds, can give useful insight into the overall dynamics of MMOs. More specifically, we use numerical continuation of parametrised families of two-point boundary value problems to find these manifolds and their possible transversal intersections, which play the role of separating canard-like orbits.

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Slow-Fast Bogdanov-Takens Bifurcations

Freddy Dumortier
Hasselt University, Belgium
(Peter De Maesschalck)

The talk deals with perturbations from planar vector fields having a line of zeros and representing a singular limit of Bogdanov-Takens (BT) bifurcations. We introduce, among other precise definitions, the notion of slow-fast BT-bifurcation and we provide a complete study of the bifurcation diagram and the related phase portraits. Based on geometric singular perturbation theory, including blow-up, we get results that are valid on a uniform neighbourhood both in parameter space and in the phase plane.

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Delay Induced Canards

Thomas Erneux
Universite Libre de Bruxelles, Belgium
(Sue Ann Campbell and Emily Stone)

We consider a model for regenerative chatter in a drilling process. The model is a nonlinear delay differential equation where the delay arises from the fact that the cutting tool passes over the metal surface repeatedly. A Hopf bifurcation is at the origin of the chatter vibration. We show that for zero delay, the Hopf bifurcation can be degenerate and that for small delay this leads to a canard explosion. That is, as the chip width is increased beyond the Hopf bifurcation value, there is a rapid transition from a small amplitude limit-cycle to a large relaxation cycle. Our analysis relies on singular perturbation techniques allowing us to transform the DDE perturbation problem as an ODE problem.

S. A. Campbell, E. Stone, and T. Erneux, Delay induced canards in a model of high speed machining, Dynamical Systems 24, 373 (2009)

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Global Recurrences of Fast-Slow Systems

Jean-pierre Francoise
Université P.-M. Curie, Paris 6, France

Fast-slow systems may display delay to bifurcation. This fine effects can be analysed with different techniques such as asymptotics developments matching or geometrical methods. These properties provide important tools to explain via dynamical systems mixed-mode oscillations observed ubiquitously in physiology. We focus on this talk on global recurrence which can occur for such systems like weakly and strongly coupled oscillators. We also discuss some related synchronization mechanisms.

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Slow-Fast Dynamics in a Combustion Model

Anna Ghazaryan
University of Kansas, USA
(Chris Jones)

We consider a system related to the model which describes combustion processes in inert porous media proposed by Sivashinsky in 2002. We study existence and stability properties of the wavefronts in this system using geometric singular perturbation theory.

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Singular Hopf Bifurcation

John Guckenheimer
Cornell University, USA

When an equilibrium crosses a fold curve of a slow-fast system, a Hopf bifurcation occurs robustly nearby. In systems with one slow and one fast variable, this produces a canard explosion of the periodic orbits emerging from the Hopf bifurcation. In systems with two slow variables, Szmolyan et al. labelled the intersection of the equilibrium with the fold curve a folded saddle-node, type II. This lecture will discuss normal forms for this problem, analyze the dynamics of these normal forms and describe how singular Hopf bifurcations together with a "global return mechanism" generate mixed mode oscillations.

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Limit Cycles of Singularly Perturbed Quadratic Vector Fields

Yulij Ilyashenko
Moscow Universities, Cornell University, Russia

Restricted Hilbert 16th problem for quadratic vector fields requires an estimate of the number of limit cycles of these fields provided that limit cycles and vector fields considered are subjects to parameter depending restrictions. The estimate is given by a function that has poles on a set of equations having either a center or a line of singular points. This estimate was obtained by J. Llibre and the author. In the talk this result will be improved. Namely, a function is given that majorizes the number of so called δ-tame limit cycles and has no poles on the set of equations with the line of singular points. To do that, we investigate the generation of limit cycles of quadratic vector fields from the so called *van der Pol type* limit cycles. Methods of the theory of relaxation oscillations are intensively used.

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The Bifurcation of Slow-Fast Separatrix Loop in Singular Systems Family

Pavel Kaleda
Lomonosov Moscow State University, Russia

A singular system on a plane is a 1-parametric vector fields family that has a curve of singular points when parameter is equal to zero. This is a natural generalization of the slow-fast system notion. Some new global phenomena arise in singular systems due to the fact that the phase space is not a product of slow and fast variables anymore.

In the report the new slow-fast separatrix loop bifurcation phenomenon is discussed. The bifurcation takes place in a singular system with an additional parameter. When both parametes are equal to zero, the system has a so-called singular separatrix loop that is a closed curve composed of a slow curve arc and a fast system trajectory arc. There is a separatrix loop family (it's called the slow-fast separatrix loop) that corresponds to a curve γ in parameter plane.

The work was supported by part by the grant RFBR 07-01-00017-a.

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The Role of Multiple Timescales in Models of Intracellular Calcium Dynamics

Vivien Kirk
University of Auckland, New Zealand
(Emily Harvey, James Sneyd, Martin Wechselberger, Wenjun Zhang)

Calcium is crucially important in cells, regulating many aspects of cell physiology. Experiments show that intracellular calcium dynamics typically evolves over two or more time-scales, and mathematical models of calcium dynamics are constructed to reflect this. This talk will describe recent progress in the analysis of a range of models of intracellular calcium, focussing on the use of appropriate singular limits for making predictions about dynamics and on the ways in which singular limits in calcium models differ from singular limits in other excitable systems.

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A New Type of Relaxation Oscillations in a Model of the Mitotic Oscillator

Ilona Kosiuk
Max Planck Inst. for Math. in the Sci., Germany
(Peter Szmolyan)

We present a geometric analysis of a new type of relaxation oscillations in a minimal model for the

mitotic oscillator. The model developed by A. Goldbeter describes the mitosis part of the cell division in eukaryotes. We rewrite the model as a three dimensional singularly perturbed system in the variables (X, M, C). The equations are however not in standard form, i.e. away from the critical manifold all variables are fast. The critical manifold consists of four planes $M = 0$, $X = 0$, $M = 1$, and $X = 1$ which intersect along four non-hyperbolic lines where an exchange of stability occurs. In addition, each of these planes changes its stability at another non-hyperbolic line given by $C = C_{crit}$, $M = M_{crit}$, $C = C_{crit}$, and $M = M_{crit}$, respectively. For certain parameter values the model exhibits an interesting type of relaxation oscillations with the following dynamics: slow motion in the attracting part of the plane $M = 0$ towards the (non-hyperbolic) edge $(X, M) = (0, 0)$, very slow drift along the edge $(X, M) = (0, 0)$, slow motion in the attracting part of the plane $X = 0$ to a point p on the non-hyperbolic line $M = M_{crit}$, and a fast jump from p to the attracting part of the plane $M = 1$. The second half of the cycle is generated in a similar manner. This novel type of relaxation oscillations is studied by means of several blow-up transformations.

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A Boundary Value Approach to Computing Slow Manifolds and Canard Orbits

Bernd Krauskopf
University of Bristol, England
(Mathieu Desroches and Hinke M Osinga)

We present a general technique for the computation of two-dimensional slow manifolds in systems with one fast and two slow variables. It is based on the continuation of a one-parameter family of orbit segments, given as solutions of a suitably-defined boundary value problem. In this way, we are able to deal effectively with the numerical challenges of strong attraction to and strong repulsion from the slow manifolds. Visualization of the computed surfaces gives unprecedented insight into the geometry of the system. In particular, our technique allows us to find and then continue canard solutions as the intersection curves of attracting and repelling slow manifolds. The method is illustrated with the self-coupled FitzHugh-Nagumo system, where mixed-mode oscillations arise due to a folded singularity.

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Mixed-Mode Oscillations in the Koper Model: Analytical and Numerical Methods

Christian Kuehn
Cornell University, USA
(M. Desroches, J. Guckenheimer, B. Krauskopf, H. Osinga, M. Wechselberger)

The Koper model describes a prototypical chemical reaction. We are going analyze mixed-mode oscillations (MMOs) in the Koper model from the viewpoint of multiple time scale dynamics. Due to the structure of the equations several analytical calculations are possible. Furthermore we also employ numerical methods and illustrate that the combination of both approaches improves the original analysis by Koper considerably. In our analysis we shall also describe the singular bifurcation diagram of the Koper model and demonstrate the relation to the normal form for singular Hopf bifurcation. We are going to emphasize typical MMO transition sequences. Furthermore an overview and an outlook on the classification of mixed-mode oscillations will be given.

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Spherically Symmetric Standing Waves for a Liquid/Vapor Phase Transition Model

Xiao-Biao Lin
North Carolina State University, USA
(Dr. Haitao Fan, Georgetown University, USA)

We study fluid flow involving liquid/vapor phase transition in a cone shaped section, simulating the flow in fuel injection nozzles. Assuming that the flow is spherically symmetric, and the fluid has high specific heat, we look for standing wave solutions inside the nozzle. The model is a system of viscous conservation laws coupled with a reaction-diffusion equation. We look for two types of standing waves-Explosion and Evaporation waves. If the diffusion coefficient, viscosity and typical reaction time are small, the system is singularly perturbed. Transition from liquid mixture to vapor occurs in an internal layer inside the nozzle. First, matched formal asymptotic solutions are obtained. Internal layer solutions are obtained by the shooting method. Then we look for a real solution near the approximation.

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Mixed-Mode Oscillations and Canard Orbits in a Reduced Hodgkin-Huxley Model

Hinke Osinga
University of Bristol, England
(Mathieu Desroches and Bernd Krauskopf)

We study a three-dimensional reduced Hodgkin-Huxley model with one fast and two slow variables.

We concentrate on mixed-mode periodic orbits that are organized by canard orbits, which arise geometrically as intersection curves of two-dimensional attracting and repelling slow manifolds near a folded-node singularity. Using the continuation of orbit segments, we compute the slow manifolds and associated canard orbits. We show how the subsequent continuation of canard orbits allows us to find and investigate new types of dynamics, such as the interaction between canard orbits and a saddle periodic orbit that is generated in a singular Hopf bifurcation.

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Canard Dynamic Structures and Their Roles in Generating Abrupt Transitions between Firing Frequency Regimes in Neural Models: The Stellate Cell Case

Horacio Rotstein
New Jersey Institute of Technology, USA
(Tilman Kispersky, John A. White, Horacio G. Rotstein)

In two- and three-dimensional systems, the term canard structure referes to the combination of nonlinearities (nullclines and nullsurfaces) and time scale separation that have the potential to produce the canard phenomenon. In the context on neural dynamics, canard structures have been shown to underlie the mechanism of generation of subthreshold and mixed-mode oscillations (subthreshold oscillations interspersed with spikes) in biophysical (conductance-based) models of single cells. A prototypical example is a model of stellate cells (SCs) in layer II of the medial entorhinal cortex of the brain. These cells are known for their ability to display robust rhythmic subthreshold spiking activity in the theta frequency range (4 - 10 Hz) that persist in both excitatory and inhibitory newtorks including SCs.

Recent experimental studies have shown that SCs become hyper-excitable in animal models of temporal lobe epilepsy. These studies have also demonstrated the existence of recurrent connections among SCs (excitatory), reduced levels of recurrent inhibition in epileptic networks as compared to control ones, and comparable levels of recurrent excitation among SCs in both network types. In this work, we show that minimal, recurrently connected networks of SCs and interneurons (inhibitory cells) exhibit an abrupt, threshold-like transition from theta to hyper-excitable spiking (about 60 Hz) as the result of small increases in the amount of recurrent excitation. These abrupt transitions are observed in the absence of inhibition and in single, self-coupled SCs, which represent a network of coupled synchronous SCs, but not in synaptically isolated SCs. Experimental results confirm our theoretical predictions. We use dynamical systems tools to explain how synaptic excitation interacts with the canard structure present in the SC model to generate these abrupt transitions between firing frequency regimes.

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Liénard Equations and Slow-Fast Systems

Robert Roussarie
University of Burgondy, France

Let $\mathcal{L}^n$ be the $(n-1)$-dimensional space of classical Liénard differential equations :

$$\mathcal{L}^n \ : \dot{x} = y - F_a^n(x), \quad \dot{y} = -x.$$

Here F_a^n is a polynomial of degree n, $F_a^n(x) = \sum_{i=1}^{n-1} a_i x^i + x^n$, where $a = (a_1, \cdots, a_{n-1}) \in \mathbb{R}^{n-1}$. An easy remark is that the boundary of this space is made by the layer equations (obtained for $\varepsilon = 0$) of the Liénard slow-fast system family

$$\mathcal{SL}^n_{\bar{a},\varepsilon} \ : \dot{x} = y - F_{\bar{a}}^n(x), \quad \dot{y} = -\varepsilon x,$$

where $\bar{a} = (\bar{a}_1, \cdots, \bar{a}_{n-1}) \in S^{n-2}$ (the $(n-2)$-sphere) and $\varepsilon \sim 0 \in \mathbb{R}^+$. As a consequence of this remark, the problem of Smale about existence of a uniform finite bound for the number of limit cycles in $\mathcal{L}^n$, reduces to the following problem :

Is it true that each canard cycle in the family $\mathcal{SL}^n_{\bar{a},\varepsilon}$ has a finite cyclicity?.

A lot of positive partial results have recently been obtained for this last question, in joint works by Freddy Dumortier and myself. In this talk I want to review some of them and also to point out some remaining open questions.

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Heteroclinic Orbits in Slow-Fast Hamiltonian Systems with Slow Manifold Bifurcations

Stephen Schecter
North Carolina State University, USA
(Christos Sourdis)

Motivated by a problem in which a heteroclinic orbit represents a moving interface between ordered and disordered crystalline states, we consider a class of slow-fast Hamiltonian systems in which the slow manifold loses normal hyperbolicity due to a transcritical or pitchfork bifurcation as a slow variable changes. We show that under assumptions appropriate to the motivating problem, a singular heteroclinic solution gives rise to a true heteroclinic solution. In contrast to previous approaches to such problems, our approach uses blow-up of the bifurcation manifold, which allows geometric matching of inner and outer solutions.

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Canard Cycles in Generic Slow-Fast Systems on the Two-Torus

Ilya Schurov
Moscow State University, Russia

We show that there exist generic slow-fast systems with only one (time-scaling) parameter on the two-torus, which have attracting canard cycles for arbitrary small values of this parameter. This is in drastic contrast with the planar case, where canards usually occur in two-parametric families. The number of canard cycles is no more than the number of fold points of the slow curve. This estimate is exact for every system from some open set. The work is supported in part by RFBR 07-01-00017-a.

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Chaotic Dynamics in a Fast-Slow Model of Bursting Neurons

Jianzhong Su
University of Texas at Arlington, USA
(Feng Zhang, Qishao Lu)

In this talk, we use mathematical analysis to study the transition of dynamic behavior in a system of two synaptically coupled neurons (Hindmarsh-Rose model), based on flow-induced Poincare map. The individual HR neuron has chaotic behavior, but they become regularized when coupled. Through geometric analysis we first investigate the bifurcation structure of its fast subsystem to show that the emerging of regular patterns of neurons are due to topological structure changes of its underlying bifurcations. Then we focus on the transition phase of coupling strength where the bursting solutions need to pass near two homoclinic bifurcation points located on a branch of saddle points and study the flow-induced Poincare maps. We observe that as the gap between the homoclinic points narrows, the chaotic behavior begin to vanish. That, along the Lyaponov exponent calculation, show the fine structure of the pathway to chaotic bursting behavior and regular bursting of HR neurons when the synaptic coupling of neurons gets stronger.

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Blow-Up Analysis of Glycolytic Relaxation Oscillations

Peter Szmolyan
Vienna University of Technology, Austria
(Ilona Gucwa)

In this talk a singularly perturbed planar system modeling oscillatory processes in glycolysis is studied. In suitably scaled variabes the governing equations depend singularly on two small parameters ε and δ. In previous work by L. Segel and A. Goldbeter it was argued that a limit cycle of relaxation type exists for $\varepsilon \ll \delta \ll 1$. The existence of this limit cycle is proven by analyzing the problem in the spirit of geometric singular perturbation theory. The degeneracies of the limiting problem corresponding to $(\varepsilon, \delta) \to (0, 0)$ are resolved by repeatedly applying the blow-up method. It is shown that the blow-up method leads to a clear geometric picture of this fairly complicated two parameter multiscale problem.

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Shooting at the Slow Manifold

Ferdinand Verhulst
University of Utrecht, Netherlands

In many cases, two-point nonlinear boundary value problems can be analysed (if a solution exists) by boundary layer methods and asymptotic matching techniques. It happens also that a slow manifold exists which enables us to using a shooting method; this provides us at the same time with asymptotic validity estimates. In some cases there exist both stable and unstable slow manifolds and the shooting may then involve a canard. A few examples will illustrate the phenomena.

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Folds and Canards in Advection-Reaction-Diffusion Models

Martin Wechselberger
University of Sydney, Australia

We explore a class of nonlinear advection-reaction-diffusion (ARD) models where the diffusion is considered small. In biochemical reactions, this assumption is reflected by a large Peclét number and by a large Damköhler number (of the second kind). Our aim is to explain the genesis of travelling wave patterns in such ARD models. We provide a geometric point of view, complementary to classical PDE analysis, and derive criteria for the existence of smooth and sharp interfaces in the wave form based on the underlying geometry of these ARD models. In particular, we show that folded invariant manifolds and canards play an essential role in the creation and form of the wave profiles.

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Special Session 53: Global or/and Blow-Up Solutions for Nonlinear Evolution Equations and Their Applications

Shaohua George Chen, Cape Breton University, Canada
Ming Mei, Champlain College and McGill University, Canada

Introduction: This special session is devoted to the recent developments in global or/and blowup solutions for nonlinear evolution equations and their applications, include delay, localized, nonlocal, degenerate evolution equations, steady states and their properties.

A Priori Estimates for Solutions of Differential Inclusions

Ovidiu Carja
Al. I. Cuza University of Iasi, Romania

We get a priori estimates for solutions of differential inclusion $y' \in Ay + F(y)$, where A generates a C_0-semigroup in a Banach space and F is a multifunction. The existence of a global solution is also considered.

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Boundedness and Blowup Solutions for Quasilinear Parabolic Systems with Lower Order Terms

Shaohua Chen
Cape Breton University, Canada

We will present the bounded and blowup solutions of the quasilinear parabolic system $u_t = u^p(Du+av)+f(u,v,Du,x)$ and $v_t = v^q(Dv+bu)+g(u,v,Dv,x)$ with homogeneous Dirichlet boundary condition. Under suitable conditions on the lower order terms f and g, all solutions are bounded if $(1+c_1)\sqrt{ab} < \lambda$ and blow up in a finite time if $(1+c_1)\sqrt{ab} > \lambda$, where lambda is the first eigenvalue of $-D$ with Dirichlet data and $c_1 > -1$ related to f and g.

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Quasineutral Limits for Navier Stokes Poisson System

Donatella Donatelli
University of L'Aquila, Italy

We are concerned with the analysis of a vanishing Debye length type limit for a coupled Navier Stokes Poisson in $3-D$. As is well known, the Navier Stokes Poisson system is a simplified model to describe the dynamics of a plasma where the compressible electron fluid interacts with its own electric field against a constant charged ion background. While studying the quasineutral limit for this system, the incompressible limit regime yields to introduce a suitable time scaling, which introduces a singularity by the coupling term with the electric field and it leads to the formation of acoustic waves.

Our approach is based on the idea of estimating the behaviour of the acoustic waves as the parameter Debye lenght goes to zero. We exploit the dispersive properties of the equations satisfied by the fluctuation density.

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Global Existence for the Semilinear Gellerstedt-Type Equation

Anahit Galstyan
University of Texas-Pan American, USA

We discuss the issue of global existence of the solutions of the Cauchy problem for the one-dimensional semilinear weakly hyperbolic equations, appearing in the boundary value problems of gas dynamics. We give some sufficient conditions for the existence of the global weak solutions. Our approach is based on the fundamental solution of the operator and the L_p-L_q estimates for the linear Gellerstedt-type equations.

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Gradient Blowup Rate for Nonlinear Parabolic Equations

Bei Hu
University of Notre Dame, Indiana, USA
(J-S. Guo, Z-C. Zhang)

In this talk, we shall discuss our recent results on the gradient blowup rate of parabolic equations with nonlinear gradient terms.

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On the Uniform Decay in Viscoelastic Cauchy Problems with Singular Kernels

Mohammad Kafini
KFUPM, Saudi Arabia

In this paper we consider a linear Cauchy viscoelastic problem. We show that, for compactly supported initial data and singular kernel, the decay of the first energy of solutions is polynomial. Indeed, we are

concerned with the following Cauchy problem

$$\begin{cases} u_{tt} - \Delta u + \int_0^t g(t-s)\Delta u(x,s)ds = 0, \\ \qquad\qquad\qquad\qquad x \in \mathbb{R}^n,\ t > 0 \\ u(x,0) = u_0(x),\ u_t(x,0) = u_1(x), \quad x \in \mathbb{R}^n \end{cases}$$

where u_0, u_1 are two compactly supported functions and g is a positive nonincreasing function defined on $\mathbb{R}^+$. This problem in bounded domains has been extensively studied by many authors to cite but a few. For the unbounded case, unfortunately we could not find much. Very few papers appeared in the literature.

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Global Nonexistence Results for a Class of Hyperbolic Systems

Mokhtar Kirane
University of La Rochelle, France
(Belkacem Said-Houari)

Blow-up results to a system of non-autonomous nonlinear wave equations are presented. Our results extend known results in the literature like those of DelSanto, Georgiev and Mitidieri among others.

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Global Existence for Higher Dimension Strongly Coupled Parabolic Systems

Dung Le
University of Texas at San Antonio, USA

We introduce the nonlinear heat approximation method and establish everywhere regularity of weak solutions to a general strongly coupled parabolic systems consisting of more than two equations given on arbitrary dimention domains. The global existence results will also be discussed.

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Entire Solutions in Nonlocal Dispersal Equations

Wan-Tong Li
Lanzhou University, Peoples Rep. of China
(Yu-Juan Sun and Zhi-Cheng Wang)

We consider entire solutions of nonlocal dispersal equations with Fisher-KPP type and bistable nonlinearities in one-dimensional spatial domain, i.e., $u_t = J * u - u + f(u)$. Here the entire solutions are defined in the whole space and for all time $t \in \mathbb{R}$. For the Fisher-KPP type nonlinearity, we establish the existence of entire solutions by combining two traveling wave solutions with different speeds and coming from both ends of the real axis and some spatially independent solutions, here a comparison principle is employed. For the bistable nonlinearity, a two-dimensional manifold of entire solutions which behave as two traveling wave solutions coming from both directions is established by an increasing traveling wave front with nonzero wave speed. Furthermore, we show that such an entire solution is unique up to space-time translations and Liapunov stable. A key idea is to characterize the asymptotic behaviors of the solutions as $t \to -\infty$ in terms of appropriate subsolutions and supersolutions.

We have to emphasize that a lack of regularizing effect occurs. This is probably the first time the existence of entire solutions of reaction equations with nonlocal dispersal has been studied.

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Hyperbolic Damped p-System and Diffusion Phenomena

Ming Mei
Champlain College and McGill University, Canada

In this talk, we study the 2×2 hyperbolic p-system with damping. The damping effort makes such a system to behave as a diffusion equation. The focus in this talk is to show how to find the best asymptotic profile for the damped p-system, and what are the optimal convergent rates. The most new results and progress will be reported.

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Steady States for a Heterogeneous Quantum-Classical 1-D Hydrodynamic Model for Semiconductor Devices

Bruno Rubino
University of L'Aquila, Italy
(Federica Di Michele, Pierangelo Marcati)

It is well known that, from the numerical point of view, the Schrödinger equation is very complicated and the drift diffusion equations do not provide good results in modeling modern semiconductor device. In this talk we presented the results obtained with Di Michele and Marcati (University of L'Aquila, Italy). In particular, we will study a hybrid model linking the classical hydrodynamic model and the quantum hydrodynamic one. For this, we consider a device domain divided into two different zones. In the first one (classical region) we work by using the classical hydrodynamic equation; in the second one (quantum region) we will use the quantum equation. A key point to get over is the following: information is not available at the interface point, because it is impossible to take experimental measure there. In this work we will prove the existence of a unique solution of the hybrid model in the one dimensional steady state case.

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Rates of Convergence to Steady States for a Semilinear Parabolic Equation

Christian Stinner
Universität Duisburg-Essen, Germany

We study the asymptotic behavior of nonnegative solutions to the Cauchy problem for the semilinear parabolic equation $u_t = \Delta u + u^p$ where the nonlinearity is supercritical or critical in the sense of Joseph and Lundgren. In the supercritical case, we prove that the convergence to zero or to nontrivial regular steady states can take place with very slow rates which are arbitrarily slow in some sense and are slower than any algebraic rate. In particular, any rate resulting from iterated logarithms occurs if the initial data are chosen properly. In the critical case, we outline that the rates of convergence to regular or singular steady states contain an additional logarithmic factor as compared to the supercritical case. The results are obtained by comparison with suitably constructed sub- and supersolutions.

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Nonexistence for a Viscoelastic Problem in the Whole Space

Nasser-Eddine Tatar
King Fahd University, Saudi Arabia

We establish a nonexistence result for a viscoelastic problem with a nonlinear source in the whole space. The equation is of hyperbolic type and involves a memory term. This memory term is a time convolution of a certain kernel (relaxation function) and the Laplacian of the solution. It expresses the fact that the stress at any instant depends on the entire past history of strains the material has undergone. Our result is obtained without imposing compactness of support for the initial data and without using the argument on the finite propagation speed property of the wave. The case of a system of two interacting viscoelastic equations will also be discussed.

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Co-Compact Imbeddings and PDE with Self-Similar Nonlinearities

Kyril Tintarev
Uppsala University, Sweden

A continuous imbedding of Banach spaces $X \subset Y$ is called co-compact (relative to a group D of bounded linear operators on X) if every sequence $u_k \in X$ – such that $g_k u_k$, with any $g_k \in D$, converges in X weakly to zero – converges strongly in Y. In particular, Sobolev imbeddings over $\mathbb{R}^N$ are co-compact. Other examples of co-compact imbeddings are imbeddnings of Besov spaces, of subelliptic Sobolev spaces over manifolds and Strichartz imbeddings. In applications, necessary compactness properties often can be derived from co-compactness combined with specific properties of the problem. In this sense the co-compactness approach is a functional-analytic formalization of the classical concentration-compactness principle. In this talk we survey known cocompact imbeddings and their applications with emphasis on settings not covered by the original concentration compactness of P.-L. Lions.

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Highly Time-Oscillating Solutions for Very Fast Diffusion Equations

Michael Winkler
Universität Duisburg-Essen, Germany
(Juan Luis Vazquez)

We consider the Dirichlet problem for $u_t = \nabla \cdot (u^{m-1}\nabla u)$ on a bounded domain $\Omega \subset \mathbb{R}^n$ and positive $m < (n-2)/n$. The initial data are supposed to exhibit a singularity of inverse power type at some interior point of Ω, and the evolution of such a singularity under this nonlinear diffusion process is discussed.
We first review known results which link the strength of the initially present singularity to the respective possibilities of immediate, or delayed, or absent regularization to occur. Next, we outline how by skilfully choosing the behavior of u_0 near the singular point it is possible to construct global in time solutions $u(x,t)$ which have the strange property that they oscillate as $t \to \infty$ between divergence to infinity at times $t_{2i} \to \infty$ and convergence to a constant at times $t_{2i-1} \to \infty$.

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Analytical Blowup Solutions of the Euler-Poisson/Navier-Stokes-Poisson Equations in R^N

Manwai Yuen
The Hong Kong Polytechnic University, Hong Kong

In this talk, we first provide a literature review to the analytical solutions of the compressible Euler-Poisson equations in R^N. New analytical solutions of the Navier-Stokes-Poisson equations with density-dependent viscosity in R^N, have been constructed. After that, analytically periodic solutions for the Euler-Poisson equations in R^N with a negative cosmological constant are also presented, based on known solutions. Moreover, non-radially (line) symmetric blowup solutions for the Euler-Poisson equations in R^N will be proposed.

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Special Session 54: Applied Hyperbolic and Elliptic Dynamics

Marian Gidea, Northeastern Illinois University, USA
Ilie Ugarcovici, DePaul University, USA

Introduction: The focus of this special session is on current research on the theory and applications of hyperbolic and elliptic dynamics to billiards, celestial mechanics, mathematical biology, chaotic phenomena in science, spectral problems, statistical limit laws, etc.

Computer Assisted Proof for Normally Hyperbolic Invariant Manifolds

Maciej Capinski
University of Sci. and Tech., Krakow, Poland
(Carles Simó)

We present a proof of existence of normally hyperbolic invariant manifolds for maps. The proof is based on local estimates on derivatives of maps and allows for rigourous-computer-assisted implementation. We give an example of a driven logistic map in which standard (non-rigorous) computer simulation gives evidence of a chaotic attractor. With our method we prove that the simulation is false and that the map possesses a global attractor which is an attracting invariant curve. The reason behind false numerical results lies in the fact that even though the curve is globally attracting, in parts it is strongly expanding. In the expansion regions small roundoff errors lead to false simulation of chaos. On this example we demonstrate that our method can be used to obtain rigorous results even for maps for which standard numerical simulations brake down. We also demonstrate that standard computations of Lyapunov exponents can give misleading results.

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On Topological Entropy of Billiard Tables with Small Inner Scatterers

Yi-Chiuan Chen
Academia Sinica, Taiwan

We present an approach to studying the topological entropy of a class of billiard systems. In this class, any billiard table consists of strictly convex domain in the plane and strictly convex inner scatterers. We show that a billiard system in this class generically admits a set of non-degenerate anti-integrable orbits which corresponds bijectively to a topological Markov chain of arbitrarily large topological entropy. The anti-integrable limit is the singular limit when scatterers shrink to points. On auxiliary circles encircling these scatterers we define a length functional whose critical points are well-defined at the anti-integrable limit and give rise to billiard orbits when the scatterers are not points. Consequently, we prove the topological entropy of the first return map to the scatterers can be made arbitrarily large provided the inner scatterers are sufficiently small.

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Filtering for Chaotic Maps

Gianluigi Del Magno
CEMAPRE – Univ. Tecnica de Lisbona, Portugal
(Jochen Bröcker)

The 'Filtering problem' consists in estimating the current state of a dynamical system from a record of noisy measurements. More precisely, consider two stochastic processes $\{X_n\}$ and $\{Y_n\}$ with $\{Y_n\}$ being a random perturbation of $\{X_n\}$. We consider $\{X_n\}$ as an unobservable process and $\{Y_n\}$ as a measurement process. Then, the 'Filtering problem' consists in computing the conditional expectation of X_n given the observations $Y_1, \ldots, Y_n$. The asymptotic properties of the filtering process $\{Z_n\}$ are of great interest both from a theoretical perspective as well as in applications. In this talk, I will report on a result (joint work with Jochen Bröcker) concerning the asymptotic properties of the filtering process when $\{X_n\}$ is generated by the iterations of an expanding map.

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Shadowing and Diffusion in Hamiltonian Systems

Marian Gidea
Northeastern Illinois University, USA
(Clark Robinson)

We will discuss some topological methods to prove the existence of shadowing orbits, i.e. orbits with prescribed itineraries, in monotone twist mappings of the annulus. We will also apply topological methods to show the existence of diffusing orbits, i.e., orbits that travel 'far' and 'chaotically' with respect to the action variable, in certain Hamiltonian systems close to integrable.

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Infinite Genus Surfaces Admitting Two Affine Multi-Twists

William Hooper
City College of New York, USA

Thurston constructed the first examples of pseudo-Anosov automorphisms of surfaces using a combinatorial construction starting with a finite graph representing the intersections of a pair of multi-curves. The surface constructed has the natural structure of a translation surface (or a Riemann surface paired with a holomorphic one-form).

We will begin by noting that Thurston's construction works equally well for infinite graphs, and yields a infinite genus surface admitting pseudo-Anosov automorphisms. I will consider the straight-line flows on these infinite genus surfaces, and state a theorem which describes the locally finite ergodic invariant measures for many of these flows. (In finite genus, these types of questions are answered by work of Thurston, Masur and Veech.)

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Topological Classification of Heteroclinic Intersections

Hector Lomeli
ITAM, Mexico

We study heteroclinic intersections between two normally hyperbolic invariant manifolds. We show that, in some situations, these intersections can be classified using the fundamental group of a reduced manifold. We show that, in the case of intersections that come from a perturbation of a degenerate saddle connection, it is also possible to use the Euler class on a normal bundle. We show some examples, including some numerical results on a quadratic volume preserving map, that we computed using the *parametrization method.*

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Study of Elliptic Tori Via KAM Methods without Action-Angle Variables

Alejandro Luque
Universitat Politecnica de Catalunya, Spain
(Jordi Villanueva)

We discuss a KAM theorem for elliptic lower dimensional tori of Hamiltonian systems via parameterizations. The method is based in solving iteratively the functional equations that stand for invariance and reducibility. In contrast with classical methods, we do not assume that the system is close to integrable nor that it is written in action-angle variables. We want to highlight that the approach presents many advantages compared with methods which are built in terms of canonical transformations, e.g., it produces simpler and more constructive proofs that lead to more efficient numerical algorithms for the computations of these objects.

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Transport in Transient Dynamical Systems

James Meiss
University of Colorado, USA
(Brock Mosovsky)

A transitory system is autonomous except on a compact interval, undergoing a transition between two steady states. They provide perhaps the simplest example of nonautonomous and aperiodic dynamical systems. Nonautonomous dynamical systems have received much study recently using the concepts of "Lagrangian coherent structures" (LCS) and finite time Lyapunov exponents (FTLE) especially for application to fluid mixing. For the transitory case, the coherent structures are precisely defined for the past and future systems, and the natural question is "what is the transport between these structures?" This is a natural testbed for understanding LCS.

A simple example corresponds to a 2D fluid flow with a pair of gyres that transition to a new pair. Another corresponds to the acceleration of a particle trapped in a moving potential well. The interesting quantity to compute is the flux from the past to the future coherent structures. We show how these fluxes can be finding particular heteroclinic orbits and computing integrals along these orbits to determine their actions. These results are compare to those obtained by the FTLE technique.

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Connecting Dynamics and Parameterization of Invariant Manifolds

Jason Mireles James
Rutgers University, USA
(Rafael de la Llave, Jean-Philippe Lessard, Hector Lomelí, Konstantin Mischaikow)

I will discuss several problems, coming from both discrete and continuous time dynamical systems, where the parameterization method for stable and unstable manifolds can be used to compute connecting dynamics.

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Blowing-Up of Deterministic Fixed Points in Stochastic Population Dynamics

Mario Natiello
Lund University, Sweden

(Hernán G. Solari)

We discuss the stochastic dynamics of biological (and other) populations presenting a limit behaviour for large environments (called *deterministic limit*) and its relation with the dynamics in the limit. The discussion is circumscribed to hyperbolic, linearly stable fixed points of the deterministic dynamics, and it is shown that the cases of extinction and non-extinction, equilibriums present different features. Mainly, non-extinction equilibria have associated a region of stochastic instability surrounded by a region of stochastic stability. The instability region does not exist in the case of extinction fixed points, and a linear Lyapunov function can be associated with them. Stochastically sustained oscillations of two subpopulations are also discussed in the case of complex eigenvalues of the stability matrix of the deterministic system.

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From Limit Cycles to Strange Attractors

William Ott
University of Houston, USA

(Mikko Stenlund)

We define a quantitative notion of shear for limit cycles of flows. We show that strange attractors and SRB measures emerge when systems with limit cycles exhibiting sufficient shear are subjected to periodic pulsatile drives. The forcing does not overwhelm the intrinsic dynamics. Rather, it acts as an amplifier, amplifying the effects of the intrinsic shear. The strange attractors possess a number of precisely-defined dynamical properties that together imply chaos that is both sustained in time and physically observable. We base the analysis on the recent theory of rank 1 maps developed by Wang and Young.

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Combination of Scaling Exponents and Properties of Renormalization Operators

Nikola Petrov
University of Oklahoma, USA

(Rafael de la Llave, Arturo Olvera)

We report some numerical computations of scaling exponents corresponding to different "routes to chaos", i.e., different kneading sequences in the case of unimodal maps, and different rotation numbers in the case of maps of the circle and in the case of dynamics on the boundary of Siegel disks in the complex plane. We observe that there exist certain approximate relations between the exponents corresponding to different routes.

We formulate a new principle, called the Principle of Approximate Combination of Scaling Exponents (PACSE) which consists of several statements that relate scaling exponents for complicated transitions with the ones for simpler transitions. For example, the scaling exponents for cubic critical circle maps with a rotation number with continued fraction expansion $\langle (1^k 2)^\infty \rangle$ with the exponents corresponding to rotation number the golden mean $\langle 1^\infty \rangle$ and rotation number the silver mean $\langle 2^\infty \rangle$.

We propose a conjectural explanation of PACSE based on the geometry of the function spaces and the properties of appropriately defined renormalization operators.

This is a joint work with Rafael de la Llave (University of Texas, Austin, USA) and Arturo Olvera (UNAM, Mexico City, Mexico).

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Arnold's Mechanism of Diffusion in the Spatial Circular Restricted Three Body Problem

Pablo Roldán
Universitat Politècnica de Catalunya, Spain

(Amadeu Delshams and Marian Gidea)

We show the existence of Arnold's mechanism of diffusion in the spatial circular Restricted Three Body Problem using a semi-numerical argument.

Specifically, we consider the center manifold associated to the equilibrium point L_1 in the Sun-Earth system. The existence of the center manifold for a practical range of energy values h has been proved recently using computer-assisted techniques by M. Capinski and the author and will be presented in a separate talk.

For small values h of the energy, the center manifold restricted to the energy level $H = h$ is a normally hyperbolic invariant manifold Λ. In the normal form approximation, Λ consists of a (1 parameter) family of invariant tori.

We find a transition chain of tori in Λ such that the unstable whisker of a torus intersects transversally the stable whisker of another neighboring torus. Moreover, we show how to construct an orbit that shadows the transition chain.

The argument is semi-numerical, consisting of a combination of geometrical methods (NHIM, scattering maps), topological methods (correctly aligned windows) and numerical methods (normal form, numerical integration of trajectories, numerical intersection of st/unst manifolds).

We emphasize that the argument is completely

constructive, so we can construct different transition chains (thus shadowing orbits) that exhibit interesting dynamics. Therefore the results can be applied to e.g. space mission design.

We remark also that the methods apply to the more general setting of a normally hyperbolic manifold whose stable and unstable manifolds intersect transversally, so the argument could be adapted to some other models different from the spatial RTBP.

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Dirac Physical Measures for Transitive Flows

Radu Saghin
CRM Barcelona, Spain
(Edson Vargas)

I will present some examples of transitive flows which have a unique physical measure supported on hyperbolic fixed points. The construction uses the method of periodic approximations in dimension two, and an example by Hu and Young in dimension three.

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Hyperbolic and Elliptic Behavior in Population Dynamics

Ilie Ugarcovici
DePaul University, USA
(M. Gidea, J. Meiss, H. Weiss)

I will describe two classes of population models that exhibit some of the paradigms of hyperbolic and/or elliptic dynamics. Rigorous and numerical results will be presented.

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Special Session 55: Perturbed Differential and Difference Equations

Conall Kelly, University of the West Indies, Jamaica
Alexandra Rodkina, University of the West Indies, Jamaica

Introduction: The scope of this session includes

1. qualitative analysis of systems evolving in discrete or continuous time under the influence of stochastic or deterministic perturbations;
2. analysis of numerical methods for perturbed differential equations;
3. applications of perturbed differential and difference equations.

Characterisation of Mean Square Convergence Rates in Solutions of Stochastic Volterra Equations with Applications to Finance

John Appleby
Dublin City University, Ireland
(Xuerong Mao and Markus Riedle)

In this talk we characterise the asymptotic behaviour in a mean square sense of scalar bilinear stochastic differential equations of Volterra type. In particular, we study equations in which the kernel of the Volterra operator exhibits slow convergence to zero. Necessary and sufficient conditions for mean square stability are obtained in terms of an underlying deterministic Volterra differential equation. In a similar manner, necessary and sufficient conditions for the mean square to decay at a certain rate in a weighted space of integrable functions are also found. The results can be applied to ARCH-like models of financial markets to describe the presence of slowly decaying autocorrelation (or long memory) in the asset returns.

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Characterisation of Global Stability of Differential Equations with Respect to a Fading Stochastic Perturbation

Jian Cheng
Dublin City University, Ireland
(John Appleby and Alexandra Rodkina)

In this talk we study the asymptotic behaviour of a nonlinear stochastic differential equation of the form

$$dX(t) = -f(X(t))\,dt + \sigma(t)\,dB(t), \quad t \geq 0$$

where B is standard Brownian motion. The equation is a perturbed version of a globally stable autonomous equation with a unique equilibrium, where the intensity of the stochastic perturbation fades over time. We consider necessary and sufficient conditions for the solution of the equation to be globally asymptotically stable almost surely.

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Long Memory and Asymptotic Behaviour in an Affine Stochastic Differential Equation with an Average Functional

John Daniels
Dublin City University, Ireland
(John Appleby)

We consider the stochastic differential equation with an average functional

$$dX(t) = \left(aX(t) + b\tfrac{1}{1+t}\int_{-1}^{t} X(s)\,ds\right) dt + \sigma\,dB(t), \quad t \geq 0.$$

We classify the asymptotic behaviour of the solution of this equation into recurrent behaviour with large fluctuations and transient behaviour. We use results from admissibility theory to characterise conditions for this process to exhibit long-memory, i.e. that the autocorrelations decay at a non-integrable polynomial rate. It transpires that this long-memory process can be made asymptotically close to a short-memory process. We consider our results using admissibility theory in light of similar results derived using special functions.

We complement the analysis in continuous time by considering the stochastic difference equation with an average component

$$X(n+1) = \alpha X(n) + \frac{\beta}{n+1}\sum_{j=0}^{n} X(j) + \sigma\xi(n+1), \quad n \in \{0, 1, \ldots\}.$$

We look at conditions under which the solution of this difference equation will exhibit similar asymptotic behaviour to that of the continuous equation.

This is a joint work between John Appleby and John Daniels.

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Ważewski's Method for Dynamic Equations

Josef Diblík
Brno University of Technology, Czech Republic
(M. Ružičková, Z. Šmarda)

The Ważewski's method, which is well-known for ordinary differential equations, is developed for a system of dynamic equations on an arbitrary time scale. Sufficient conditions guaranteeing the existence of at least one solution with graph staying in previously defined open set are derived. This result, generalizing some previous results concerning the asymptotic behavior of solutions of discrete equations, is suitable for investigating of asymptotic behavior of solutions of dynamical systems.

The investigation was supported by the Grant 201/10/1032 of Czech Grant Agency (Prague), by the Council of Czech Government MSM 00216 30503, MSM 00216 30519 and MSM 00216 30529, by the Grant 1/0090/09 of the Grant Agency of Slovak Republic (VEGA) and project APVV-0700-07 of Slovak Research and Development Agency.

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Structural Instability in Biological Networks

Michael Grinfeld
University of Strathclyde, Glasgow, Scotland
(Fuaada Mohd-Siam and Steven D. Webb)

Bistability, or more generally multistability, is often a desirable property of molecular biological networks, as it allows the network to act as switch with memory. Recently we discovered that many models of networks that exhibit bistability (for example, the autophosphorylating kinase model of Lisman and cell-cycle models of Sontag et al.) are structurally unstable in the following sense: while the system is bistable when closed to inflows or outflows of material, it can lose the property of bistability under arbitrarily small rates of synthesis and degradation of species. We will explain the mechanism of this instability and will discuss its implications.

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Reduction of the Order of Kinetic Equations in the Problem of the Attainment of Bounds by Random Processes

Vladimir Kazakov
National Polytechnic Institute, Mexico

The problem of the attainment of bounds by random processes has usually solved by Fokker-Plank-Kolmogorov (FPK) equation. Any multidimensional continuous Markov process is completely described by the system of stochastic equations of the first order with white noise as influence. On the basis of this system the kinetic coefficients can be obtained. The complexity of a numerical solution of boundary problems increases as the dimensions of the Markov process increases due to the increasing dimension of the FPK equation. In order to overcome these difficulties it is necessary to use the generalized FPK equation which is valid for non Markov non differential processes. In this case the kinetic coefficients must be calculated as derivatives with respect of time from corresponding conditional cumulant functions. We consider the Gaussian process of a speech message. In the Markov interpretation this process is a component of two dimensional Markov process and its behavior is described by the two dimensional FPK equation in the transition regime. The application of the generalized FPK equation provides a possibility to reduce the equation dimension from two to one. We discuss: the method of the

kinetic coefficients derivation, the boundary condition, results of numerical calculations of the attainment probability.

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On the Implications of Feedback Geometry for Stochastic Numerical Methods

Conall Kelly
University of the West Indies, Jamaica
(Evelyn Buckwar)

When a numerical method is applied to a differential equation, the result is a difference equation. Ideally the dynamics of the difference equation should reflect those of the original as closely as possible, but in general this can be difficult to check. It is therefore useful to perform a linear stability analysis: applying the method of interest to a linear test equation possessed of an equilibrium solution with known stability properties, and determining the asymptotic properties of the resultant difference equation for comparison.

We examine the issues that arise for this kind of analysis in the context of stochastic differential equations, and review the relevant literature. These issues have yet to be adequately addressed and, seeking to improve matters, we propose a new approach and demonstrate its usage for the class of Theta-Maruyama methods with constant step-size.

This is joint work with Evelyn Buckwar, of Heriot-Watt University, and is part of an ongoing research project partly funded by a University of the West Indies New Initiative Grant.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Denoising in Diffusively Coupled Dynamical Systems

Georgi Medvedev
Drexel University, USA

Dynamics of systems with multiple stable or metastable states can be very sensitive to random perturbations. We show that by combining randomly perturbed dynamical systems in a coupled network, one can preserve the attractors of the underlying local deterministic systems, while drastically reducing the effects of noise on the local dynamics. In large networks, the effects of noise can be effectively controlled by varying the strength of coupling, which provides a powerful mechanism of denoising. The mechanism of denoising is closely related to that of synchronization. We discuss both effects as well as several applications in computational neuroscience.

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Fuzzy Stochastic Integral Equations

Mariusz Michta
University of Zielona Gora, Poland

In general, investigating dynamic systems by deterministic ordinary differential equations we cannot usually be sure that the model perfectly describes the system because our knowledge of dynamic system is often incomplete. Thus, the theory of ordinary differential equations has been extensively developed in connection with fuzzy-valued analysis. To consider a stochastic case we introduce the notion of a fuzzy trajectory stochastic integral which represents a family of fuzzy sets on the space of square integrable random vectors. Next we present an existence of solutions of a fuzzy integral stochastic equation driven by Brownian motion under weaker conditions than Lipschitz continuity imposed on the right-hand side. Similarly as in a deterministic case, in our approach we interpret the fuzzy stochastic equation as a family of stochastic differential inclusions. The idea used next is to solve those inclusions via martingale problem approach and then apply Negoita-Ralescu type theorem.

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Differential and Stochastic Inclusions in Banach Lattices

Jerzy Motyl
University of Zielona Gora, Poland
(M. Michta)

Differential inclusions form a useful tool for solving some control problems. In general, for deterministic or stochastic differential inclusions an appropriate kind of regularity of their multivalued structure is required. In particular, the properties such as the Lipschitz continuity, lower, upper semicontinuity or monotonicity of set-valued mappings have most often been considered. Such regularities imposed on set-valued operators allow us to use results on the existence of exact or at least approximate selections having an appropriate kind of regularity, and therefore to reduce the set-valued problems to single-valued ones. Hence regular selections have attracted considerable interest for proving the existence of solutions of set-valued problems.

Let X be a Banach space while $(Y, \preceq)$ a Banach lattice. In the talk we consider a new class of set-valued functions (called "upper separated") with values in Banach lattices. We will show that such multifunctions admits convex and locally Lipschitz selections. However, they need not satisfy any of the classical regularity properties mentioned above.

In the second part of the talk we will discuss some applications of the selection properties obtained above to get new existence results for deter-

ministic and stochastic differential inclusions. Some examples will be also presented.

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Discretized Itô Formula and Stability of the Systems of Linear Stochastic Difference Equations

Alexandra Rodkina
UWI, Mona, Jamaica

We consider the system of two linear stochastic difference equations perturbed by the mutually independent random variables. The tails of the dstributions of the random variables decay more quickly than some polynomials. Applying the Discretized Itô formula and the martingale convergence techniques we derive conditions which guarantee that the solutions of the system converge to zero almost surely for all initial values. We also derive conditions which guarantee that the solution of the system do not converge to zero almost surely. Each set of the conditions is presented in the form of a system of 3 inequalities.

For some partial cases of the coefficients of the system these two sets of conditions resemble the necessary and sufficient conditions.

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Stability in Systems Perturbed Internally by Quasiperiodic Terms

Thomas Waters
NUI Galway, Ireland

Systems of linear ODEs with quasiperiodic coefficients provide an interesting extension to the archetypal Mathieu equation, and an important issue to examine is the stability of the solutions. By taking the amplitude as a small parameter, we can use various perturbation techniques to derive analytic expressions for certain resonance curves in the frequency space which grow with the perturbation into bands of instability. This brings out the complex resonant properties of systems of this type. We find the boundary between stable and unstable zones in the frequency space is quite intricate. We will also show how systems of this type arise naturally in applications.

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Special Session 56: Spectral, Linear and Nonlinear Stability of Coherent Structures

Milena Stanislavova, University of Kansas, USA
Jared Bronski, University of Illinois, USA

Introduction: The goal of this session is to get together researchers that study stability of special solutions of nonlinear PDEs from diverse points of view. Spectral and linear stability will be investigated for periodic waves and ground state solutions of a variety of equations. Several nonlinear stability results will be presented, including the nonlinear stability of semi-discrete shocks and the interaction manifolds for the planar reaction diffusion systems. Using an array of different methods and techniques to predict the long time behavior of special solutions of equations of mathematical physics will be the unifying idea.

An Index Theorem Governing the Stability of Waves of KdV Type

Jared Bronski
University of Illinois, USA
(Mat Johnson; Todd Kapitula)

We present a theorem giving the stability of periodic KdV waves in terms of some geometric information on the classical mechanics of the traveling wave ordinary differential equation. This can be considered a rigorous Whitham theory type calculation.

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Conditional Stability Theorem for the One Dimensional Klein-Gordon Equation

Aslihan Demirkaya
University of Kansas, USA

We explicitly construct the center-stable manifold for the steady state solutions of the one-dimensional Klein-Gordon equation. The main difficulty in the one-dimensional case is that the required decay of the Klein-Gordon semigroup does not follow from Strichartz estimate alone. In this talk, I will explain how to resolve this issue by proving an additional weighted decay estimate, which will allow us to close the argument.

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Stability of Traveling Waves for a Class of Reaction-Diffusion Systems

Anna Ghazaryan
University of Kansas, USA
(Yuri Latushkin, Stephen Schecter)

We consider solutions of front and pulse type in a certain class of reaction-diffusion systems which represent a generalization of systems that arise in chemical reaction models. Under certain conditions imposed on the nonlinearity, pulses and fronts are not stable on the spectral level. We obtain results that show that in this case the instability is of a convective character.

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Transverse Stability of Periodic Waves

Mariana Haragus
Universite de Franche-Comte, France

We discuss the question of spectral stability of one-dimensional periodic traveling waves of the Kadomtsev-Petviashvili equations, KP-I and KP-II, with respect to two-dimensional perturbations. We restrict to periodic waves with small amplitude. Using Floquet decomposition in the direction of propagation, and Fourier decomposition in the perpendicular direction, the spectral stability problem is formulated in terms of the spectra of a two-parameter family of linear operators. These operators have point spectrum, only, and we show how to locate this spectrum, and in particular how to detect instabilities.

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Weak Interaction of Coherent Structures in Lattice Differential Equations

Aaron Hoffman
Boston University, USA
(J. D. Wright)

We study spatially discrete evolution equations on a lattice. Assuming that the lattice admits asymptotically stable traveling waves, we show that the manifold of linear superpositions of two well-separated traveling waves that are moving apart from each other is a local attractor for the dynamics.

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Linear Stability for Solutions to the Vortex Filament Equation

Stephane Lafortune
College of Charleston, USA

In its simplest form, the self-induced dynamics of a vortex filament in a perfect fluid is governed by the Vortex Filament Equation (VFE), a nonlinear partial differential equation that is related to the cubic, focussing Nonlinear Schrödinger (NLS) equation via the well-known Hasimoto map. The NLS equation is a fundamental example of completely integrable partial differential equation, exhibiting many of the properties of completely integrable Hamiltonian systems (constants of motions, Poisson structures, commuting flows, action-angle variables, etc.), and arising as the compatibility condition of a pair of linear equations (the AKNS system): an eigenvalue problem and an evolution equation for an auxiliary eigenfunction.

The squared eigenfunctions of the AKNS system play a central role in linear stability studies of solutions of the NLS equation, as they provide a large (and often complete) set of solutions of the linearization of the NLS equation about a given solution. Using the squared eigenfunctions of the AKNS system and the relation between the VFE and NLS equations, we construct solutions of the linearized VFE equation and relate the stability properties of vortex filaments to those of the associated NLS potentials.

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Reverse Draining of a Magnetic Soap Film

Joceline Lega
University of Arizona, USA
(Derek Moulton)

This talk will be concerned with the dynamics of the one-dimensional thin-film equation with no-flux boundary conditions and in the presence of a spatially dependent external forcing. I will discuss regular and singular equilibrium solutions, explain how their nature depends on the properties of the external forcing, and apply these considerations to the reverse draining of a one-dimensional magnetic soap film subject to an external nonuniform magnetic field.

The results [1] will be summarized in a bifurcation diagram that reveals a rich structure and demonstrates the complexity hidden in a relatively simple-looking evolution equation. I will also show numerical simulations that illustrate the convergence of the solutions of the thin-film equation toward singular equilibrium configurations and describe how numerical solutions traverse the bifurcation diagram, as the amplitude of the forcing is slowly increased and then decreased.

Time permitting, I will discuss how the forced thin film equation should be regularized to avoid the presence of singular solutions and say a few words about the dynamics of the regularized equation.

[1] D. E. Moulton & J. Lega, *Reverse draining of a*

magnetic soap film - Analysis and simulation of a thin film equation with non-uniform forcing, Physica D 238, 2153-2165 (2009).

$\longrightarrow \infty \diamond \infty \longleftarrow$

Existence and Stability of Multi-Hump Pulses in Systems with Reflection and Phase Invariance

Vahagn Manukian
University of Kansas, USA
(Björn Sandstede)

We investigate the existence and stability of standing and travelling multi-hump waves in partial differential equations with reflection and phase symmetries. We focus on 2- and 3-pulse solutions that arise near bi-foci and apply our results to the complex cubic-quintic Ginzburg-Landau equation.

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Stability and Instability of Kink Wave Solutions to the Sine-Gordon Equation

Robert Marangell
University of North Carolina, USA
(C. K. R. T. Jones)

We give a proof for the conditions of stability and of instability of certain traveling wave solutions to the sine-Gordon equation. For a traveling kink wave solution of speed c, the kink wave is stable if and only if $c^2 - 1 \ldots$

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Nonlinear Stability of Semidiscrete Shocks

Bjorn Sandstede
Brown University, USA
(Margaret Beck, Hermen Jan Hupkes, and Kevin Zumbrun)

The nonlinear stability of travelling Lax shocks in semidiscrete conservation laws involving general spatial forward-backward discretization schemes is considered. It is shown that spectrally stable semidiscrete Lax shocks are nonlinearly stable. In addition, it is proved that weak semidiscrete Lax profiles satisfy the spectral stability hypotheses made here and are therefore nonlinearly stable. The nonlinear stability results are proved by constructing the resolvent kernel using exponential dichotomies, which have recently been developed in this setting, and then using the contour integral representation for the associated Green's function to derive pointwise bounds that are sufficient for proving nonlinear stability.

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Interaction Manifolds in Planar Reaction Diffusion Systems

J. Douglas Wright
Drexel University, USA

We consider a general planar reaction diffusion equation which we hypothesize has a localized traveling wave solution. Under assumptions which are no stronger than those needed to prove the stability of a single pulse, we prove that the PDE has solutions which are roughly the linear superposition of two pulses, so long as they move along trajectories which are not parallel. In particular we prove that if the initial data for the equation is close to the sum of two separated pulses, then the solution converges exponentially fast to such a superposition so long as the distance between the two pulses remains sufficiently large.

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Special Session 57: Hyperbolic Dynamics and Smooth Ergodic Theory

Eugen Mihailescu, Romanian Academy, Romania
Bernd Stratmann, University of Bremen, Germany

Introduction: The goal of the Session is to bring together specialists working in Hyperbolic Dynamics and Smooth Ergodic Theory, as well as in applications and/or related subjects. We hope that this will facilitate a productive exchange of ideas and continue, or initiate discussions in these fields. Some of the topics of the Session are: uniform and non-uniform hyperbolic dynamics, fractal dimension theory, dynamics of non-invertible maps, thermodynamic formalism, SRB measures, Bernoullicity of maps, aspects from complex dynamics, attractors/repellers, etc.

Attractors for Unimodal Quasiperiodically Forced Maps

Lluis Alseda
Universitat Autonoma de Barcelona, Spain

We consider unimodal quasiperidically forced maps, that is, skew products with irrational rotations of the circle in the base and unimodal interval maps in the fibres. The map in the fibre over x is a unimodal map f of the interval $[0,1]$ onto itself multiplied by $g(x)$, where g is a continuous function from the circle to $[0,1]$. The case when g does not take the value 0 has been extensively studied by various authors. Here we consider a more difficult case, the "pinched" one, when g attains value 0. This case is similar to the one considered by Gerhard Keller, except that the function f in his case is increasing. Since in our case f is unimodal, the basic tools from the Keller's paper do not work.

We prove that under some additional assumptions on the system there exists a strange non chaotic attractor. It is a graph of a non-trivial function from the circle to $[0,1]$, which attracts almost all trajectories. Both Lyapunov exponents on this attractor are non positive. There are also cases when the dynamics is completely different, because one can apply the results of Jerome Buzzi implying the existence of an invariant measure absolutely continuous with respect to the Lebesgue measure. Finally, there are cases when we can only guess what the behaviour is by performing computer experiments.

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From Mixing Rates to Recurrence Times

José Alves
University of Porto, Portugal
(J. M. Freitas, S. Luzzatto, S. Vaienti)

One of the most efficient tools for studying the mixing rates of certain classes of dynamical systems is through Young towers: if a given system admits an inducing scheme whose tail of recurrence times decays at a given speed, then that system admits an SRB measure with mixing rate of the same order. In this talk we shall consider the inverse problem: assume that a given dynamical system has an SRB measure with a certain mixing rate; under which conditions does that measure come from an inducing scheme with the tail of recurrence times decaying at the same speed? We have optimal results for the polynomial case. The exponential case raises interesting questions on the regularity of the observables.

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Quasi-Invariant Measures and Escape Rates

Wael Bahsoun
Loughborough University, England
(Christopher Bose)

A result by Keller and Liverani on the stability of non-essential spectrum of transfer operators can be used to study existence of absolutely continuous conditionally invariant measures (accim) for interval maps with holes. In particular its says that if a mixing Lasota-Yorke map is perturbed by introducing a 'sufficiently small' hole in the phase space, then the resulting open dynamical system admits an accim. In this talk we show how the condition 'sufficiently small' can be verified rigorously on a computer. In particular, for a given Lasota-Yorke map, we use Ulam's method on the closed dynamical system T to give a computable size of the hole H for which the open dynamical system T_H must admit an accim.

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The One-Sided Bernoulli Property for One-Dimensional Dynamical Systems

Henk Bruin
University of Surrey, England
(Jane Hawkins)

A measure preserving dynamical system is called one-sided Bernoulli if it isomorphic to a one-sided full shift equipped with some stationary product (Bernoulli) measure. This notion is of interest for non-invertible systems, and much more rare and in a way more subtle than the property of having a two-sided Bernoulli natural extension. Measure-theoretic entropy, for example, is not a complete invariant for one-sided Bernoulli systems. In this talk I want to discuss some necessary conditions for the one-sided Bernoulli properties to hold. Examples are taken from one-dimensional dynamics (both real and complex). The talk is based on a joint paper with Jane Hawkins (Chapel Hill, NC).

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Variational Principles for Dynamical Systems with Holes

Mark Demers
Fairfield University, USA

We introduce a hole into a general dynamical system and study the relation between the escape rate and the pressure on the survivor set, the set of points which never enters the hole. We derive a variational principle for a broad class of systems with holes that requires only weak assumptions on the size and boundary of the hole. When the underlying dynamical system is smooth (before the intro-

duction of the hole) the variational principle allows us to determine how the escape rate changes as we vary the size and position of the hole. This is joint work with Paul Wright and Lai-Sang Young.

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Kleinian Groups with Dimension Gap

Kurt Falk
Universität Bremen, Germany

The dynamics of geometrically finite hyperbolic manifolds is well understood by means of Patterson-Sullivan theory. For geometrically infinite manifolds, or manifolds given by infinitely generated Kleinian groups, nonrecurrent dynamics becomes the "thick part" of dynamics, not only in the sense of measure but also Hausdorff dimension. The associated Kleinian groups display a gap between the dimension of their conical limit sets and the total dimension. This can also be interpreted as a dimension gap between recurrent and nonrecurrent dynamics in the manifold. I will present some classical results on this topic, alongside with newer research I was involved in.

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A Semi-Invertible Oseledets Theorem with Applications to Transfer Operator Cocycles

Gary Froyland
University of New South Wales, Sydney, Australia
(Simon Lloyd, Anthony Quas)

Oseledets' celebrated Multiplicative Ergodic Theorem (MET) is concerned with the exponential growth rates of vectors under the action of a linear cocycle on R^d. When the linear actions are invertible, the MET guarantees an almost-everywhere pointwise splitting of R^d into subspaces of distinct exponential growth rates (called Lyapunov exponents). When the linear actions are non-invertible, Oseledets' MET only yields the existence of a filtration of subspaces, the elements of which contain all vectors that grow no faster than exponential rates given by the Lyapunov exponents. The authors recently demonstrated [G. Froyland, S. Lloyd, and A. Quas, Coherent structures and exceptional spectrum for Perron–Frobenius cocycles, Ergodic Theory and Dynam. Systems (to appear).] that a *splitting* over R^d is guaranteed *without* the invertibility assumption on the linear actions. Motivated by applications of the MET to cocycles of (non-invertible) transfer operators arising from random dynamical systems, we demonstrate the existence of an Oseledets splitting for cocycles of quasi-compact non-invertible linear operators on Banach spaces.

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Return Times Distribution for α-Mixing Dynamical Systems

Nicolai Haydn
University of Southern California, USA
(Y Psiloyenis)

We prove that for a dynamical system which has an invariant measure that is α-mixing with respect to a given partition the limiting dstribution of return times is Poisson almost everywhere. This is shown by using the Chen-Stein method which also yields rates of convergence. Our theorem improves on previous results by allowing for infinite partitions and dropping the requirement that the invariant measure have finite entropy with respect to the given partition. The points where these estimates apply are those that don't have period type behaviour. We also prove that Lai-Sang Young's Markov Towers are α-mixing and thus have Poisson distributed limit dstribution of return times. In this case the rate of convergence is determined by the decay rate of the 'tail dstribution' of the tower.

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Remarks on Hausdorff-Dimensions of Certain Subsets of Limit Sets for Infinitely Generated Kleinian Groups

Martial Hille
Humboldt University Berlin, Germany

In the Theory of Kleinian groups several subsets of the limit set have been introduced. We will mainly focus on the 'uniformly radial limit set' and the 'Jørgensen limit set'. For infinitely generated Kleinian Groups these are of enourmous interest. In this talk I will give a short overview of the main results on the Hausdorff dimensions of these sets and their relation. We will also present our most recent results on this question. Extensions to the theory of Iterated Function Systems and (pseudo) Graph Directed Markov systems may also be discussed.

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Induced Topological Pressure and Regularity

Marc Kesseböhmer
University Bremen, Germany
(Johannes Jaerisch and Sanaz Lamei)

We introduce the notion of induced topological pressure and discuss some of its basic properties. In particular, in the context of countable state Markov shifts, this new approach allows us to connect the classical notion of topological pressure with the notion of Gurevič pressure. In this way we obtain new

insides into regularity properties for the thermodynamical and multifractal formalism.

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Inverse SRB Measures, Dimensions, and 1-Sided Bernoullicity for Endomorphisms

Eugen Mihailescu
Romanian Academy, Romania

The dynamics of non-invertible hyperbolic smooth maps (hyperbolic endomorphisms) presents many differences from the dynamics of hyperbolic diffeomorphisms or that of expanding maps. In this talk I will present new results about non-reversible non-expanding dynamical systems and asymptotic dstributions of preimages for them. We introduce an inverse SRB measure in the non-invertible case. We present also results about the fractal dimensions (stable dimension, Hausdorff dimension of the global unstable set, pointwise dimensions of measures) for hyperbolic endomorphisms on locally maximal sets with overlaps. Finally we give new results about the 1-sided Bernoullicity (or lack of it) for certain equilibrium measures of endomorphisms on folded basic sets.

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Equilibrium States and Phase Transition for Partially Hyperbolic Diffeomorphisms

Isabel Rios
Universidade Federal Fluminense, Brazil
(R. Leplaideur and K Oliveira)

We prove the existence of equilibrium states associated to continuous potentials, for a family of partially hyperbolic horseshoes. We show that, for a certain smooth potential, there are at least two different equilibrium measures.

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On Random Topological Markov Chains with Big Images and Preimages

Manuel Stadlbauer
University of Porto, Portugal

In this talk, the notion of big images and preimages (or ‘finite primitivity’ in the terminology of Mauldin & Urbanski) is extended to random topological Markov chains. It then turns out that this combinatorical condition is sufficient for the existence of a random conformal measure, a random eigenfunction of the Ruelle operator and, moreover, implies that the system is relatively exact. In particular, this suggests that these systems are an analogue of deterministic systems with the Gibbs-Markov property.

Moreover, these results extend the work of Bogenschütz, Gundlach and Kifer to random shift spaces with countably many states, and as an application one obtains a partial solution to a question of Orey on the convergence to the stationary distribution for Markov chains in random environment.

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Strong Renewal Theorems and Lyapunov Spectra for α-Farey-Lueroth and α-Lueroth Systems

Bernd Stratmann
University of Bremen, Germany
(M. Kesseboehmer, S. Munday)

In this talk we introduce and study α-Farey-Lueroth maps and their corresponding jump transformations, α-Lueroth maps, for arbitrary partitions of the unit interval. These maps represent linearized generalisations of the Farey map and the Gauss map from elementary number theory. First, we give some of their topological and ergodic-theoretic properties. Then, the first main result is to establish weak and strong renewal laws for what we call α-sum-level sets for the α-Lueroth map. The second main result is to obtain an almost complete description of the Lyapunov spectra of the α-Farey-Lueroth maps and the α-Lueroth map in terms of the thermodynamical formalism. Various examples are given, demonstrating the diversity of different behaviours of these spectra in dependence on the chosen partition α.

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Myrberg Points and Ergodicity of Geodesic Flow

Pekka Tukia
University of Helsinki, Finland

Let G be a discrete Moebius acting on the $(n+1)$-dimensional ball and on its boundary the n-sphere. We assume that a conformal measure m is given in the limit set; sometimes such measures are called Patterson measures. It is possible to consider in this situation the geodesic flow in the hyperbolic convex hull of the limit set of G. It is well known that the geodesic flow is ergodic if and only if the conical limit points have full measure. Myrberg points are a subclass of conical limit points. Myrberg points have full measure if and only if conical limit points have full measure. We show that if Myrberg points have full measure, then the geodesic flow is ergodic. This should give a new way to approach the ergodicity of the geodesic flow.

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Local and Global Curvatures of Self-Similar Random Fractals

Martina Zaehle
University of Jena, Germany

An approach to local and global Lipschitz-Killing curvatures of self-similar (random) sets via ergodic theorems for associated dynamical systems is presented. Extensions to more general fractals are indicated.

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Return- and Hitting Time Statistics for Null-Recurrent Dynamical Sytems

Roland Zweimueller
University of Vienna, Austria
(Francoise Pene, Benoit Saussol))

The asymptotic behaviour of return- and hitting time dstributions of small sets has been studied for a large variety of (finite) measure preserving dynamical systems, and it is well known that the emergence of an exponential limit law (or the convergence to a Poisson process) is a very common and robust phenomenon. However, very little is known about null-recurrent (infinite measure preserving) systems. I will survey some recent joint work with Francoise Pene and Benoit Saussol (Univ Brest, F) which clarifies the (different!) asymptotic behaviour in a number of interesting and relevant situations including classical random walks, certain skew products, and intermittent maps. I may also mention related results for the Poisson suspensions of (non-interacting infinite particle systems driven by) these systems.

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Special Session 58: Hamiltonian and Reversible Systems

Amadeu Delshams, Universitat Politècnica de Catalunya, Spain
Oksana Koltsova, Imperial College London, United Kingdom

Introduction: Main tentative topics: Applications to celestial mechanics, space science, plasma physics, accelerators, etc; Structure of the phase space of Hamiltonian and Reversible Systems; Common features of Hamiltonian and Reversible Systems; Differents features of Hamiltonian and Reversible Systems; KAM theory (invariant tori, invariant manifolds); Splitting of separatrices; Normal forms and bifurcations; Numerical and symbolic tools for Hamiltonian and Reversible Systems; Variational methods; Detection and measure of the non-integrability; Passage through resonance; Arnold diffusion (geometry and estimates); Stability; Transport in conservative and reversible systems

Hill's Determinant of a Periodic Orbit

Sergey Bolotin
Steklov Mathematical Institute, Moscow, Russia

Hill discovered that the multipliers ρ, ρ^{-1} of time periodic Hill's equation $\ddot{x} = a(t)x$ satisfy

$$\rho + \rho^{-1} - 2 = \beta \det H,$$

where $\det H$ is a properly regularized determinant of the matrix of Hill's operator and $\beta > 0$ a factor depending on the regularization. Hill's formula admits several generalizations.

Consider a τ-periodic Lagrangian system with the Lagrangian $L(x, v, t)$ strictly convex in the velocity. Periodic orbits are extremals of

$$A(\gamma) = \int_0^\tau L(\gamma(t), \dot{\gamma}(t), t)\, dt$$

on the space of τ-periodic curves. The second variation of A at γ is a bilinear form on the set X of τ-periodic $W^{1,2}$ vector fields along γ:

$$h(\xi, \eta) = \int_0^\tau \left((D\xi(t), D\eta(t)) + (U(t)\xi(t), \eta(t)) \right) dt.$$

Here $(\,\cdot\,,\,\cdot\,)$ is a positive definite scalar product defined by the Hessian of the Lagrangian with respect to velocity, and D is a covariant derivative:

$$\frac{d}{dt}(\xi, \eta) = (D\xi, \eta) + (\xi, D\eta).$$

Define a scalar product on X by

$$\langle \xi, \eta \rangle = \int_0^\tau \left((D\xi, D\eta) + (\xi, \eta) \right) dt.$$

Then $h(\xi, \bar{\eta}) = \langle H\xi, \bar{\eta} \rangle$ where $H : X \to X$ is the Hessian operator.

Let P be the linear Poincaré map of γ, and Q the operator of parallel transport around γ.

Theorem: *Hill's determinant* $\det H$ *converges absolutely and*

$$\det H = (-1)^m \det Q \frac{e^{m\tau} \det(I - P)}{\det^2(e^\tau I - Q)}.$$

For systems with first integrals, Hill's formula degenerates and we obtain a suitable reduced version. We also discuss applications of this formula to stability problems.

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On Parametrized KAM Theory

Henk Broer
Johann Bernoulli Inst., Groningen, Netherlands

Parametrized KAM Theory and quasi-periodic bifurcation Theory developed in has been developed from Moser 1966 on by many researchers. It turns out that almost all the known KAM Theorems can be re-discovered in this way. Moreover, novel applications deal with higher dimensional bifurcations where many resonances play a role. The corresponding theory is a marriage of KAM Theory and Singularity Theory in the product of state space and parameter space, where a Cantorisation occurs of the semi-algebraic stratifications known from the latter theory. One nice application of the theory is a description of the destruction of resonant Lagrangean tori in Hamiltonian systems.

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Finding Centre Manifolds in the Restricted Three Body Problem

Maciej Capinski
University of Sci. and Tech., Krakow, Poland
(Pablo Roldan)

We present a topological tool for finding centre manifolds for ODEs. The proof of their existence is constructive and performed in the state space of the system. Assumptions of our theorem are formulated in a way which allows for implementation in computer assisted proofs. We apply our method to obtain a centre manifold of Lyapunov orbits around L_1 in the restricted circular three body problem. Our method allows us to obtain explicit bounds on a section of the manifold and also to prove its uniqueness within an investigated region.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Rigorous Verification of the Geometric Mechanism of Diffusion for a General Class of a Priori Unstable Hamiltonian Systems

Amadeu Delshams
Universitat Politecnica de Catalunya, Spain
(Gemma Huguet, Rafael de la Llave, Tere M. Seara)

In this talk we introduce a version of the geometric mechanism for diffusion with a more applied approach since the hypotheses are more concise, more easily verifiable and they still guarantee the existence of diffusion in a wide general case of a priori unstable Hamiltonian system with $2 + 1/2$ degrees of freedom.

The simplification of the hypothesis also allows us to present in an easily understandable way the mechanism of diffusion, and particularly the construction of the scattering map and the combination of two types of dynamics on a manifold.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Exponentially Small Splitting of Separatrices for the Pendulum with a Fast Periodic Meromorphic Perturbation

Marcel Guardia
Universitat Politecnica de Catalunya, Spain
(Tere M. Seara)

The problem of the exponentially small splitting of separatrices for one degree of freedom Hamiltonian System perturbed with a fast non-autonomous periodic perturbation has been widely studied in the past years. However, since the size of the splitting is very sensitive on the analytic properties of both the unperturbed system and the perturbation, in most of the results achieved up to now, some restrictive hypotheses have been imposed. For instance, the perturbations are are usually taken just $\mathcal{C}^1$ in time but the unperturbed system is taken analytic entire in the other variables and the perturbation is restricted to be a polynomial or a trigonometric polynomial.

Nevertheless, in many applications (for instance, in celestial mechanics) these hypotheses do not hold since both the unperturbed system and the perturbation are not entire but have singularities in the complex domain. The work explained in this talk is a very first step towards understanding the exponentially small splitting of separatrices for non-entire hamiltonian systems.

We consider a toy model in which the unperturbed system is the classical pendulum (and thus still entire) but the perturbation is meromorphic since has polar singularities in the complex domain. We will show how the exponentially small distance between the perturbed invariant manifolds depends strongly on the width of the strip of analyticity, even becoming non-exponentially small if the analyticity strip is too narrow.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Multiple Homoclinic Orbits and Homoclinic Tangles in Reversible Systems

Ale Jan Homburg
University of Amsterdam, Netherlands

I will discuss homoclinic tangles, in particular their existence near multiple homoclinic orbits, in reversible differential equations. Differences with conservative systems will be highlighted.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Multi-Round Periodic Orbits Near a Homoclinic Orbit to a Hamiltonian Saddle-Center

Oksana Koltsova
Imperial College London, England
(Ruslan Biryukov)

We present multi-round periodic dynamics in a 2 d.o.f. Hamiltonian system with a homoclinic orbit to a saddle-center. To study the global behaviour near the homoclinic orbit we use asymptotic symmetries of the return map as a consequence of self-similarities. Moreover, we have studied bifurcations of the periodic orbits for small periods.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Quasiperiodic Excitation in Hamiltonian Systems

Tamás Kovács
MPI PKS, Germany

The Sitnikov problem is a special case of the three-body problem in celestial mechanics. It is known that the dynamics can be described as a one dimensional explicitely time-dependent motion. Therefore, all the main features of chaotic behavior in conservative systems can appear in it. In recent years large number of numerical and analytical studies dealt with this problem, consequently, the phase space structure and the dynamical evolution of the problem is well-mapped. In this talk I will focus on a little perturbation of the Sitnikov problem, namely the periastron shift in two-body problem, that comes from general relativity and alter the external impulsive force quasiperiodic. The main question is how the structure of the phase space does change under this new condition.

$\longrightarrow \infty \diamond \infty \longleftarrow$

One-Round Dynamics Near a Homoclinic Orbit to a Reversible Saddle-Center

Jeroen Lamb
Imperial College London, England
(Oksana Koltsova)

We study one-round dynamics in the neighbourhood of a homoclinic orbit to a saddle-center equilibrium in reversible and/or Hamiltonian vector fields in $\mathbb{R}^4$. We employ return maps to study periodic solutions and use a singularity theoretical approach to study the intersection of the center-stable and center-unstable manifolds at the point homoclinic connection and its generic unfolding. We emphasize similarities and differences between the reversible, Hamiltonian and reversible-Hamiltonian cases.

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Abundance of Attracting, Repelling and Elliptic Periodic Orbits in Two-Dimensional Reversible Maps

J. Tomás Lázaro
Universitat Politècnica de Catalunya, Spain
(Delshams, A., Gonchenko S. V., Gonchenko V. S., Sten'kin, O.)

We study dynamics and bifurcations of two-dimensional reversible maps having non-transversal heteroclinic cycles containing symmetric saddle periodic points. For a one-parameter family of reversible maps unfolding the initial heteroclinic tangency it is proved the existence of infinitely many sequences (cascades) of bifurcations as well as birth of asymptotically stable, unstable and elliptic periodic orbits. This result is a counterpart of the one obtained by Lamb and Stenkin (2004), where two initial non-symmetric saddle points were considered.

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On Bifurcations in Hamiltonian Systems

Lev Lerman
The University of Nizhny Novgorod, Russia

My goal here is to present several new results about bifurcations in Hamiltonian systems. The first one is about the structure of a 3 d.o.f. Hamiltonian systems near a homoclinic orbit to a periodic orbit of the 1-elliptic type (two real nonzero multipliers and two others on the unit circle without strong resonances). We present conditions when such system has transverse homoclinic orbits to 2-dimensional KAM tori on the center manifold of the periodic orbit, hyperbolic subsets. Also one-parameter unfoldings of such the system and bifurcations will be discussed. Another bifurcation scenario concerns of the passage through a Hamiltonian system with homoclinic orbits to a degenerate equilibrium with zero eigenvalues, reconstructions of hyperbolic sets, bifurcations of periodic and homoclinic orbits. If time remains, I would like to discuss a general problem about the structure of a Hamiltonian systems near a transverse homoclinic orbit to a hyperbolic invariant torus.

Author acknowledges a support from the Russian Foundation of Basic Research.

$\longrightarrow \infty \diamond \infty \longleftarrow$

On the Similarity of Hamiltonian and Reversible Vector Fields in 6D

Ricardo Martins
Unicamp, Brazil
(Marco Antonio Teixeira)

In this work we study the problem of the formal conjugacy between reversible and hamiltonian vector fields in 6D. We show that, under certain conditions, reversible vector fields with a non resonant equilibria are formally conjugated to (integrable) hamiltonian vector fields around the equilibria. We also show that hamiltonian vector fields with a non resonant equilibria are formally conjugated to reversible vector fields around the equilibria. Moreover, if one begin with a completely integrable hamiltonian vector field, then the conjugacy can be made analytic.

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Dynamical Properties of Certain Families of Symmetric Periodic Orbits in N-Body Problems

Daniel Offin
Queen's University, Canada
(Mark Lewis, Abdalla Mansur)

Global variational methods have produced a fascinating variety of symmetric periodic solutions in the classical N-body problem. We discuss a method based on the variational structure of these solutions which allows in certain cases the determination of their stability type. The main tool used in this is the Maslov index of curves of Lagrangian planes which contain the variational structure. We will apply this technique to several examples, including the minimizing equal mass Hip-Hop family of the $2N$-body problem, and the rhomboid solutions of the planar four body problem. The idea for this determination of stability type can be seen clearly in the case of the Conley-Zehnder index for curves in the symplectic group Sp(2) where the stability is exactly determined by the parity of a certain intersection number.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Oscillator and Thermostat

Dmitry Treschev
Steklov Mathematical Institute, Russia

We study the problem of a potential interaction of a finite-dimensional Lagrangian system (an oscillator) with a linear infinite-dimensional one (a thermostat). In spite of the energy preservation and the Lagrangian (Hamiltonian) nature of the total system, under some natural assumptions the final dynamics of the finite-dimensional component turns out to be simple while the thermostat produces an effective dissipation.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Some Remarks on 3:1 Resonance of Area Preserving Maps

Piotr Zgliczyński
Jagiellonian University, Poland
(Carles Simo, Tomasz Kapela)

We investigate the following area preserving Henon map

$$h(x,y) = R_\alpha(x, y - x^2), \qquad \alpha = 2\pi/3 \qquad (1)$$

where R_α is a rotation by α. We focus on $\alpha = 2\pi/3$ case, the 3:1 resonance.

I will discuss a computer assisted proof of the existence of the symbolic dynamics and the existence of the 'hyperbolic' set with interesting properties: some points have zero Lapunov exponent, some nonzero and for others points the limit defining Lapunov exponent oscilates between zero and some nonzero value.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Special Session 59: Topological Invariants in Dynamical Systems

Grzegorz Graff, Gdansk University of Technology, Poland
Waclaw Marzantowicz, Adam Mickiewicz University in Poznan, Poland

Introduction: The session will be focused on topological invariants such as Nielsen and Lefschetz numbers, zeta-type functions, fixed point index and fixed point indices of iterations, Conley index; and their applications in low dimensional dynamics, periodic point theory and differential equations.

On the Determination of the Topology of Vector Fields Singularities by the Newton Polyhedron

Clementa Alonso
University of Alicante, Spain

In this talk we will determine a class of polyhedra and a generic set, in the set of three dimensional real vector fields with an isolated singular point P, for which the topology around P is given by discrete elements associated to the Principal Part.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Wecken Theorems for N-Valued Maps

Joel Better
none at present, Spain

We discuss Wecken Theorems for n-valued maps. We present H. Schirmer's results on comapct triangulable manifolds of dimension greater than three, the author's reults for equivariant n-valued maps on suitable finite simplicial G-complexes as well as his results (in collaboration with R. Brown) for n-valued maps on the 2-sphere.

$\longrightarrow \infty \diamond \infty \longleftarrow$

A Topological Version of the Poincaré-Birkhoff Theorem

Marc Bonino
University Paris 13, France

The classical Poincaré-Birkhoff theorem asserts that an area-preserving homeomorphism of the annulus which "turns the two boundary components in opposite directions" possesses at least two fixed points. We will present a topological version of this theorem and will discuss of the relationship with previous close results.

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Averaging Method and Periodic Solutions of Evolution Systems

Aleksander Cwiszewski
Nicolaus Copernicus University, Torun, Poland

We shall consider evolution systems of the form

$$(P)_\omega : \qquad \dot{u}(t) = A(\omega t)u(t) + F(\omega t, u(t))$$

where $\{A(t)\}_{t\geq 0}$ is a family of linear operators generating evolution system on a Banach space E, $F : [0, +\infty) \times E \to E$ is a continuous perturbation and $\omega > 0$ is a parameter. We shall discuss a general averaging principle saying that, under the time almost periodicity of $\{A(t)\}_{t\geq 0}$ and F, if $\omega \to +\infty$, then solutions of $(P)_\omega$ converge to solutions of the averaged autonomous equation $\dot{u} = -\widehat{A}u + \widehat{F}(u)$. Next, assuming some compactness and T-periodicity, we use Krasnosel'skii type formulae, relating the fixed point index of the translation along trajectories operator of the averaged equation with the topological degree of its right hand side, to derive efficient criteria for the existence of T-periodic solutions for $\omega = 1$. The results apply to parabolic and hyperbolic partial differential equations.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Viable Trajectories in State-Dependent Impulsive Systems

Grzegorz Gabor
Nicolaus Copernicus University, Toruń, Poland

The talk is devoted to the problem of the existence of solutions to the following single-valued or multivalued differential problem

$$\begin{cases} \dot{x}(t) = f(x(t)) \text{ [resp. } \in F(x(t))] \quad \text{for a.e. } t \geq 0 \\ x(t^+) := \lim_{s\to t^+} x(s) = I(x(t)) \text{ [resp. } \in I(x(t))] \\ \qquad \text{for } x(t) \in M \subset \partial K \\ x(t) \in K \quad \text{for every } t \geq 0, \end{cases}$$

where $K \subset \mathbb{R}^n$ is a closed set. In other words, for a given closed subset K of a state space, we look for *viable* trajectories in K.

If we do not assume usual tangency conditions on the whole set K, we allow trajectories for a non-impulsive problem, even all of them, to leave K. They escape from K through a so-called *exit set*. To prevent this, we place a barrier M being an essential subset of the exit set, and define an impulse map I moving trajectories back to the set K.

We look for sufficient conditions for a non-impulsive differential problem, a set K, the exit set, the barrier M and the impulsive map to obtain viable trajectories for an impulsive system. Some suitable topological tools detecting such trajectories are provided. We show a connection between piecewise continuous dynamics of an impulsive system with topological invariants for flows, and a discrete dynamical (multivalued) system on the exit set with an appropriate index theory for multivalued maps. Topological properties of the exit set are important to solve the problem. It is strictly related to the celebrated Ważewski retract method and the fixed point theory.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Local Fixed Point Indices of Iterations of Planar Maps

Grzegorz Graff
Gdansk University of Technology, Poland
(Piotr Nowak-Przygodzki and Francisco R. Ruiz del Portal)

Let f be a continuous planar map. We consider a fixed point p of f which is neither a sink nor a source and such that $\{p\}$ is an isolated invariant set. Under these assumption we prove, using Conley index methods and Nielsen theory, that the sequence of fixed point indices of iterations $\{ind(f^n, p)\}_{n=1}^{\infty}$ is periodic, bounded by 1, and has infinitely many non-positive terms, which is a generalization of Le Calvez and Yoccoz theorem [Annals of Math., 146 (1997), 241-293] onto the class of non-injective maps. We apply our result to study the dynamics of continuous maps on 2-dimensional sphere.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Fixed Point Indices of Iterated Smooth Maps in Arbitrary Dimension

Jerzy Jezierski
Warsaw University of Life Sciences, Poland
(Grzegorz Graff, Piotr Nowak-Przygodzki)

Let us consider the sequence of fixed point indices $i_k = \text{ind}(f^k, 0)$, where $f : \mathbb{R}^n \to \mathbb{R}^n$ is a continuous self-map and 0 is an isolated fixed point for each iteration.

Albrecht Dold noticed that the sequence $\{i_k\}_k$ must satisfy *Dold congruences.* Later, Babenko and Bogatyi showed that this condition is also sufficient: each sequence of integers satisfying Dold congruences can be realized as a sequence of indices for a self-map of $\mathbb{R}^3$.

On the other hand, it turned out that if f is smooth then there are much more restrictions on the sequence $i_k = \text{ind}(f^k; 0)$ [Chow, Mallet-Paret and Yorke]. These restrictions depend on the dimension of $\mathbb{R}^n$ and of course imply Dold congruences. The following question appears. Let us fix the dimension n and a sequence of integers satisfying conditions of Chow, Mallet-Paret and Yorke for dimension n. Can the sequence be realized by a smooth self-map of $\mathbb{R}^n$? This was answered pos-

itively for $n = 2$ (Babenko-Bogatyi) and for $n = 3$ (Graff, Nowak-Przygodzki).

In this lecture we show that the answer is positive in all remaining dimensions. As a consequence, we obtain the complete list of fixed point indices for smooth maps. This result allows also to extend Wecken Theory on periodic points of smooth maps.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Equivariant Degree Theory and Its Applications, Part 2

Wieslaw Krawcewicz
University of Texas at Dallas, USA

(Zalman Balanov, Slawomir Rybicki, Heinrich Steinlein)

Following our recent survey paper "A short treatise on the equivariant degree theory and its applications, J. Fixed Point Theory Appl. (to appear)", we give an introduction to three variants of the equivariant degree with emphasizing the aspect that, due to nice properties and computational routines, one can easily apply it without going into the deep and technical topological background. The first part will concentrate on these degrees, while a second part, by Wieslaw Krawcewicz, will describe several original applications of the equivariant degree without free parameter, the twisted equivariant degree and the gradient equivariant degree.

$\longrightarrow \infty \diamond \infty \longleftarrow$

On a Generalized Poicare-Hopf Formula in Infinite Dimension

Wojciech Kryszewski
Nicholas Copernicus University, Torun, Poland

(Aleksander Cwiszewski)

We prove a formula relating the fixed point index of rest points of a completely continuous semiflow defined on a (not necessarily locally compact) metric space in the interior of an isolating block B to the Euler characteristic of the pair (B, B^-), where B^- is the exit set. The proof relies on a general concept of an approximate neighborhood extension space and a full fixed point index theory for self maps of such spaces. As a consequence, a generalized Poincare-Hopf type formula for differential equations determined by a perturbation of the generator of a compact C_0-semigroup is obtained.

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A Symmetry of Map and Periodic Points

Wacław Marzantowicz
UAM Poznań, Poland

(J. Jezierski)

We assume that X is a compact connected polyhedron, G is a finite group acting freely on X, and $f : X \to X$ an G-equivariant map. We find a formulae for the least number of $n-$periodic points in the equivariant homotopy class of f i.e. $\inf_h \#\mathrm{Fix}(h^n)$ (where h is G-homotopic to f). As an application we prove that the set of periodic points of an equivariant map is infinite provided the action on the rational homology of X is trivial and the Lefschetz number $L(f^n)$ does not vanish for infinitely many indices n commeasurable with the order $|G|$ of G. Moreover, then we have at least linear growth, in n, of the number of points of period n.

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Otopy Classes of Local Maps

Piotr Nowak-Przygodzki
Res. Group on Nonlinear Anal. at UAM, Poland

(Piotr Bartłomiejczyk)

Let U be an open subset of R^n. A map $f : U \to R^n$ is called a local map if $f^{-1}(0)$ is compact. One can assume in addition that f is gradient and/or proper. In this way one obtains four different sets consisting of some local maps: 1) all of local maps; 2) gradient; 3) proper; 4) gradient and proper. For each of these sets we consider a notion of otopy (gradient and/or proper respectively), which is a generalization of the notion of homotopy. We prove in each of above four cases that a function assigning topological degree to every otopy class is a bijection between otopy classes and integers.

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Automatic Computation of the Conley Index. Algorithms and Applications

Paweł Pilarczyk
University of Minho, Braga, Portugal

In this talk, an algorithmic approach to the computation of the Conley index for semidynamical systems with discrete time will be introduced. This index is a topological invariant developed in the 1970s by C. Conley as a generalization of the Morse index of an isolated fixed point in a gradient flow. In addition to algorithms for constructing index pairs, in this talk a method and software for the computation of the homological version of the Conley index will be discussed. The usefulness of this theory and the related software will be illustrated by an advanced application to classifying global dynamics in

multi-parameter systems via Morse decomposition, in which the recurrent dynamics is restricted to isolated invariant subsets, called Morse sets, and the dynamics outside these sets is gradient-like. Computation of the Conley indices of the constructed Morse sets plays a crucial role in this application.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Location of Fixed Points in the Presence of Two Cycles

Alfonso Ruiz
University of Granada, Spain

Any orientation-preserving homeomorphism of the plane having a two cycles has also a fixed point. This well known result does not provide any hint on how to locate the fixed point, in principle it can be anywhere. Campos and Ortega considered the class of Lipschitz-continuous maps and locate the fixed point in the region determinded by the ellipse with foci at the two cycle and eccentricity the inverse of the Lipschitz-constant. It will be shown that this region is not optimal and a sub-domain can be removed from the interior. A curious fact is that the ellipse mentioned above is relevant for the optimal location of the fixed point in a neighbourhood of the minor axis but it is of no relevance around the major axis. An essential ingredient in the proof is M. Brown's result in "Fixed points for orientation preserving homeomorphisms of the plane which interchange two points".

$\longrightarrow \infty \diamond \infty \longleftarrow$

Rotation Numbers of Attractors

Francisco R. Ruiz del Portal
Universidad Complutense de Madrid, Spain
(Luis H. Corbato and Rafael Ortega)

A rotation number r can be assigned to an attractor of a planar homeomorphism h. We analyze some properties of attractors depending of its rotation number. If $r \in \mathbb{Q}$, h is dissipative and U is unbounded then there exists a periodic point lying in $\mathbb{R}^2 \setminus U$. On the other hand, if $r \notin \mathbb{Q}$ then h induces a Denjoy homeomorphism in the circle of prime ends of $\partial(U)$.

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A Poincaré Formula for the Fixed Point Indices of the Iterates of Planar Homeomorphisms

José Salazar
Universidad de Alcalá, Spain
(Francisco R. Ruiz del Portal)

Let $U \subset \mathbb{R}^2$ be an open subset and let $f : U \to \mathbb{R}^2$ be an arbitrary local homeomorphism with $Fix(f) = \{p\}$. We compute the fixed point indices of the iterates of f at p, $i_{\mathbb{R}^2}(f^k, p)$, and identify these indices in dynamical terms. Therefore we obtain a sort of Poincaré index formula without differentiability assumptions. Our techniques apply equally to both, orientation preserving and orientation reversing homeomorphisms.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Firing Maps for Differential Equations with Almost Periodic Coefficients

Justyna Signerska
Polish Academy of Sciences, Poland
(Wacław Marzantowicz)

Integrate-and-fire models are the systems given by differential equations of the type $\dot{x} = f(t, x)$, where "threshold-reset" mechanism is incorporated into the dynamics. It is well known that if the function f is periodic in time, such a system induces a firing map which is a lift of a degree-one circle map. Thus the standard tools of rotation theory, e.g. the rotation number, prove to be useful in investigating properties of firing maps. However, since it is also possible to define rotation numbers and rotation sets for functions of the real line with almost periodic displacement, we analyze firing maps and firing rate for the class of integrate-and-fire models with almost periodic stimulus (either uniformly almost periodic or in the sense of Besicovitch). In particular, we give rigorous proofs that the firing rate for the Perfect Integrator Model can be obtained as the limit of firing rates for periodic systems and show when the firing map has almost periodic displacement. These results are illustrated by suitable examples.

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The Fuller Index and Bifurcation Points

Robert Skiba
Nicolaus Copernicus University, Poland
(Wojciech Kryszewski)

In this talk, we present the theory of a cohomological index of Fuller type detecting the periodic orbits of a multivalued dynamical system generated by a differential inclusion. The presented theory is applied to establish a general result on bifurcation of periodic orbits from an equilibrium point of a differential inclusion. This is a joint work with Wojciech Kryszewski.

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A Set of Axioms for the Degree of a Tangent Vector Field on Differentiable Manifolds

Marco Spadini
Università di Firenze, Italy
(Massimo Furi and Maria Patrizia Pera)

Given a tangent vector field on a finite dimensional real smooth manifold, its degree (also known as *characteristic* or *rotation*) is, in some sense, an algebraic count of its zeros and gives useful information for its associated ordinary differential equation. When, in particular, the ambient manifold is an open subset U of $\mathbb{R}^m$, a tangent vector field v on U can be identified with a map $\vec{v}\colon U \to \mathbb{R}^m$, and its degree, when defined, coincides with the Brouwer degree with respect to zero of the corresponding map $\vec{v}$.

As is well known, the Brouwer degree in $\mathbb{R}^m$ is uniquely determined by three axioms, called *Normalization*, *Additivity* and *Homotopy Invariance.* Here we shall provide a simple proof that in the context of differentiable manifolds the degree of a tangent vector field is uniquely determined by suitably adapted versions of the above three axioms.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Nielsen Coincidence Theory of Iterates

P. Christopher Staecker
Fairfield University, USA
(Philip Heath)

We will discuss a new Nielsen-type theory of coincidences of iterates of self-maps. This theory is an attempt to combine techniques of Nielsen theories of coincidence and periodic points. We define the number $NP_{n,m}(f,g)$, which is a lower bound for the minimum number of coincidence points of f^n and g^m which are not also coincidence points of lower iterates. This number is invariant under homotopies of f and g.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Equivariant Degree Theory and Its Applications, Part 1

Heinrich Steinlein
Universität München, Germany
(Zalman Balanov, Wiesław Krawcewicz, Sławomir Rybicki)

Following our recent survey paper "A short treatise on the equivariant degree theory and its applications, J. Fixed Point Theory Appl. (to appear)", we give an introduction to three variants of the equivariant degree with emphasizing the aspect that, due to nice properties and computational routines, one can easily apply it without going into the deep and technical topological background. The first part will concentrate on these degrees, while a second part, by Wiesław Krawcewicz, will describe several original applications of the equivariant degree without free parameter, the twisted equivariant degree and the gradient equivariant degree.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Cohomological M-Index Over the Phase Space for Discrete Semidynamical Systems

Jacek Szybowski
AGH University of Science and Technology, Poland
(Kinga (Stolot) McInerney)

We define a cohomological index of Mrozek-Reineck-Srzednicki type for discrete dynamical systems. This notion is an extension of the index over the phase space for a semidynamical system defined by Szybowski and the index over the base morphism defined by Stolot for multivalued dynamical systems. As it is an algebraic invariant, computation of the index presented in this paper can be algorithmized and the important additivity property is more likely to hold. This is a joint work with Kinga (Stolot) McInerney.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Some Results on Multidimensional Perturbations of 1-Dim Maps

Piotr Zgliczyński
Jagiellonian University, Poland

We consider continuous maps $F : \mathbb{R} \times \mathbb{R}^k \to \mathbb{R} \times \mathbb{R}^k$ which are close to $F_0(x,y) = (f(x),0)$, where $f : \mathbb{R} \to \mathbb{R}$ is continuous.

We address the following question, assume that f has an interesting dynamical property, will it continue to F if $|F(z) - F_0(z)|$ is small for z in some suitable compact sets. In this context as the 'interesting dynamical properties' we will consider the set of periods and the topological entropy. We will present some positive answers in this direction.

[1] P. Zgliczyński, *Sharkovskii's Theorem for multidimensional perturbations of one–dimensional maps.* Ergodic Theory and Dynamical Systems 19 (1999), 1655–1684.
[2] M. Misiurewicz and P. Zgliczyński, *Topological entropy for multidimensional perturbations of one dimensional maps.* Int. J. of Bifurcation and Chaos 11 (2001), 1443–1446.
[3] Ming-Chia Li and P. Zgliczyński, *On stability of forcing relations for multidimensional perturbations of interval maps*, preprint, http://www.ii.uj.edu.pl/~zgliczyn/papers.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Special Session 60: Deterministic and Stochastic Dynamical Systems and Applications

Tomás Caraballo, Universidad de Sevilla, Spain
José Valero, Universidad Miguel Hernández, Spain

Introduction: The aim of this session is to offer an overview on recent results concerning the asymptotic behaviour of solutions of stochastic and deterministic partial and ordinary differential equations. The main topics of the session are: existence and properties of pullback attractors for stochastic and non-autonomous equations, stability, stabilization, attractors for equations without uniqueness, dynamics of equations with delay and finite-dimensional dynamics for dynamical systems.

Short Time Dynamics Random Perturbations of Hyperbolic ODE

Sergio Almada
Georgia Tech, USA

(Yuri Bakhtin)

We consider the stochastic differential equation $dx_\varepsilon = b(x_\varepsilon)dt + \varepsilon dW$ and study the dynamics of this system as $\varepsilon \to 0$. We assume that the unperturbed ODE $\frac{dx}{dt} = b(x)$ has a unique hyperbolic fixed point. We outline how the application of normal form theory combined with stopping time arguments for nonlinear stochastic differential equations allows us to compute some short time assymptotics for this process. Among this assymptotic results, we show how an unpredicted (according to Large deviations theory) assymetry is created. Then we moved forward to explain what the consequences of this result are for the case in which the unperturbed system admits an heteroclinic network.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Stability of Gradient Semigroups under Perturbations

Alexandre Carvalho
Universidade de São Paulo, Brazil

(E. R. Aragão Costa, T. Caraballo and J. A. Langa)

This work is dedicated to give conditions under which gradient semigroups in a general metric space are stable under perturbation. This is done proving that gradient-like semigroups are gradient semigroups (posses a Lyapunov function) and using the perturbation results proved in A. N. Carvalho and J. A. Langa, J. Differential Equations 246 (7) 2646-2668 (2009).

$\longrightarrow \infty \diamond \infty \longleftarrow$

Stochastic Embedding of Lagrangian Systems and Applications

Jacky Cresson
University of Pau, France

Most physical systems are modelled by an ordinary or a partial differential equation, like the n-body problem in celestial mechanics. In some cases, for example when studying the long term behaviour of the solar system or for complex systems, there exist elements which can influence the dynamics of the system which are not well modelled or even known. One way to take these problems into account consists of looking at the dynamics of the system on a larger class of objects, that are eventually stochastic. We develop a theory for the stochastic embedding of ordinary differential equations. We apply this method to Lagrangian systems. In this particular case, we extend many results of classical mechanics namely, the least action principle, the Euler-Lagrange equations, and Noether's theorem. We also obtain a Hamiltonian formulation for our stochastic Lagrangian systems. Many applications are discussed in particular for variational formulation of classical PDEs like the Schrödinger equation and celestial mechanics.

$\longrightarrow \infty \diamond \infty \longleftarrow$

On Random Dynamical Systems and Fractional Brownian Motions

Maria Garrido-Atienza
University of Sevilla, Spain

In this talk we consider stochastic differential equations driven by a fractional Brownian motion process. Our aim is to show under which sufficient conditions these equations with non–trivial stochastic integrals generate random dynamical systems.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Birth-and-Growth Processes for Nonconvex Random Closed Sets

Thomas Lorenz
Goethe University, Frankfurt am Main, Germany

The main goal of this talk is an approach to stochastic shape evolution - without a priori assumptions about the regularity of shape boundaries. From a conceptual point of view, it is a stochastic counterpart of Aubin's morphological equations, which

deal with closed-loop evolutions of nonempty compact subsets of the Euclidean space.

Here shapes are described as random closed subsets (not necessarily bounded, but just with a square integrable selection) and, they evolve “along” stochastic differential inclusions. A quite broad class of growth processes is covered if the multivalued right-hand side is prescribed appropriately as a function of time and the current set. Then the resulting time-dependent random closed sets coincide with reachable sets of a nonautonomous stochastic differential inclusion. In particular, this approach to growth processes is not restricted to convex sets. First, we focus on sufficient conditions on the existence of unique solutions in finite time intervals. The aspect of birth is taken into consideration in form of a given set-valued map of time which specifies where an additional growth process is initiated at a possibly later point of time. Secondly, we specify sufficient conditions on this nucleation map for solving the related birth-and-growth problem approximatively.

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On Pullback Attractors in H_0^1 for Nonautonomous Nonlinear Reaction-Diffusion Equations

Grzegorz Łukaszewicz
University of Warsaw, Poland

Applying a method based on the notion of the Kuratowski measure of noncompactness together with a new way of dealing with well known estimates of solutions we would like to present an elegant proof of the existence of a unique minimal pullback attractor for the evolutionary process associated with a nonautonomous nonlinear reaction-diffusion equation

$$\frac{\partial u}{\partial t} - \triangle u + f(u) = g(t), \quad u_{|\partial\Omega} = 0, \quad u(\tau) = u_0$$

in the phase space $H_0^1(\Omega)$, (Ω a bounded and smooth domain in R^n) in which the nonlinear term satisfies the usual conditions

$$\begin{aligned} &f \in C^1(R, R), \\ &C_1|u|^p - C_2 \le f(u)u \le C_3|u|^p + C_4, \\ &f_u(u) \ge -C_5 \end{aligned}$$

and the right hand side satisfies the integrability condition

$$\int_{-\infty}^{t} \exp\{\lambda s\} ||g(s)||^2_{L^2(\Omega)} ds < \infty \qquad \text{for all } t \in R, (\lambda > 0)$$

− thus generalizing a number of similar recent results. To get the needed estimates we introduce a general inequality of Gronwall's type which also proves helpful in other contexts.

$\longrightarrow \infty \diamond \infty \longleftarrow$

The Kneser Property for the Discretization of Reaction-Diffusion Systems

Francisco Morillas Jurado
Universitat de Valencia, Spain
(José Valero Cuadra)

In this work we study the Kneser property for a system of reaction-diffusion equations on an unbounded domain. In particular, we study a reaction-diffusion system in which the non-linear term satisfies some dissipative and growth conditions which guaranty the existence but not the uniqueness of solutions of the Cauchy problem. Also, we use a finite difference method to discretize this system, obtaining in this way a system of infinite ordinary differential equations which approximates the initial system. Namely, we obtain a lattice dynamical system (LDS). For this LDS, in a previous work, we obtained the existence of a global compact attractor for the corresponding multivalued semiflow. Now we prove that the Kneser property holds and use it to obtain that the global attractor is connected.

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Exponential Ordering for Nonautonomous Neutral Functional Differential Equations with Applications to Compartmental Systems

Sylvia Novo
Universidad de Valladolid, Spain
(Rafael Obaya, Victor M. Villarragut)

This talk deals with the study of monotone skew-product semiflows generated by families of nonautonomous neutral functional differential equations with infinite delay and stable D-operator, when the exponential ordering is considered. Under adequate hypotheses of stability for the order on bounded sets, it is shown that the omega-limit sets are copies of the base to explain the long-term behavior of the trajectories. The application to the study of the amount of material within the compartments of a neutral compartmental system with infinite delay shows the improvement with respect to the standard ordering.

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Pullback Attractors and PDE's

Luis Felipe Rivero Garvía
Universidad de Sevilla, Spain
(Tomás Caraballo, Alexandre N. Carvalho and José A. Langa)

In the framework of autonomous dynamical systems, the concept of global attractor plays an important role in the study of their asymptotic behaviour. However in the non-autonomous context, it is necessary to extend this theory for evolution processes and pullback attractors. In this talk we will show how the theory of pullback attractors can be applied to a concrete PDE, given some results about its structure and forward attraction under some hypotheses.

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Finite-Dimensional Dynamics and Slices of the Unit Cube in $\mathbb{R}^n$

James Robinson
University of Warwick, England

Can one construct a finite set of ordinary differential equations that reproduce the dynamics on a finite-dimensional attractor? The question can be reduced to two sub-problems: (i) the smoothness of the vector field on the attractor, and (ii) the smoothness of embeddings of finite-dimensional sets into Euclidean space. In this talk I will concentrate on (ii), and show that it is related to the problem of the finding the maximal $n-1$ volume of slices through the unit cube in $\mathbb{R}^n$ (the answer – independent of n – is $\sqrt{2}$, a result due to Keith Ball).

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Concave or Sublinear Monotone Non-Autonomous Infinite Delay FDEs

Ana Sanz
University of Valladolid, Spain
(C. Núñez, R. Obaya)

A large number of mathematical models describing different phenomena in engineering, biology and other applied sciences present some monotonicity properties with respect to the state argument, which permits to apply the theory of monotone dynamical systems to their analysis. Often some additional physical conditions occur so that the model exhibits concave (or convex) nonlinearities. There are also well-known phenomena in applied sciences for which only positive state arguments make sense, and for which the dynamics can be essentially described by a sublinear vector field.

With this motivation, we study the dynamical behavior of the trajectories defined by a recurrent family of monotone functional differential equations with infinite delay and concave or sublinear nonlinearities. We analyze different sceneries which require the existence of a lower solution and of a bounded trajectory ordered in an appropriate way, for which we prove the existence of a globally asymptotically stable minimal set given by a non-autonomous equilibrium.

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Master-Slave Dynamics for Random Dynamical Systems

Björn Schmalfuss
University of Paderborn, Germany
(I. D. Chueshov)

We construct the coupling of a stochastic parabolic and a stochastic hyperbolic differential equation describing the synchronization of two systems. The solution of these equations is given in mild form. In particular these systems are driven by white or fractional or Levy noise. Then the dynamics of the parabolic equation (slave) can be determined by a random inertial manifold for these both systems given by the graph of a Lipschitz function defined on the domain of the master (hyperbolic equation) system. To obtain this manifold the Lyapunov-Perron method is used.

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Special Session 61: Topological Methods for the Qualitative Analysis of Differential Equations

Marco Spadini, Università di Firenze, Italy
Pierluigi Benevieri, University of Sao Paulo, Brasil

Introduction: The main topic of the session will be topological methods such as degree theory, fixed point index theory, Morse theory, Maslov index, spectral flow, and their applications to various problems in ordinary, functional and partial differential equations. Particular emphasis will be given to existence, multiplicity and bifurcation of solutions. Contribution to the more theoretical aspect of the above mentioned methods will be welcome as well.

Existence Results for Quasilinear Variational Inequalities Involving Multivalued Maps

Irene Benedetti
University of Florence, Italy
(F. Mugelli and P. Zecca)

Let $\Omega \subset \mathbb{R}^N$ be a bounded domain with Lipschitz boundary, we use topological methods to establish existence results for a class of nonlinear variational inequalities on convex closed sets. The inequalities considered involve a quasilinear operator of class S_+ and the nonlinear part is given by the sum of a Carathéodory map and a multimap. We take into account the cases of elliptic variational inequalities and parabolic variational inequalities and, in both cases, we study an upper semicontinuous and a lower semicontinuous general type of multivalued nonlinearity.

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Retarded Functional Differential Equations on Manifolds

Pierluigi Benevieri
University of Sao Paulo, Brazil
(Alessandro Calamai, Massimo Furi, Maria Patrizia Pera)

We study existence and bifurcation of periodic solutions for T-periodic retarded functional differential equations of the type $x'(t) = f(t, x_t)$, where f is a T-periodic functional vector field on a smooth manifold. The approach followed is topological, based on the fixed point index theory.

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An Eigenvalue Problem for Planar Dirac Systems

Walter Dambrosio
Universitá di Torino, Italy
(Anna Capietto, Duccio Papini)

In this talk we will present some bifurcation results for nonlinear planar Dirac systems. A crucial step in the proof is the study of the spectral properties of the linear part of the system.

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Multiplicity Results for Asymptotically Linear Elliptic Problems

Francisco Odair de Paiva
IMECC/UNICAMP, Brazil

Our aim is to present some results on multiplicity of solutions for the semilinear problem

$$\begin{array}{lll} -\Delta u = g(x,u) & \text{in} & \Omega \\ u = 0 & \text{on} & \partial\Omega, \end{array}$$

where $\Omega \subset \mathbb{R}^N$ is a bounded domain with smooth boundary $\partial\Omega$, $g : \Omega \times \mathbb{R} \to \mathbb{R}$ is a function of class C^1 which is asymptotically linear at infinity and $g(0) = 0$.

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Second Order Problems with Non-Linear Boundary Conditions

Andrea Gavioli
Universitá di Modena e Reggio Emilia, Italy

We consider, on the interval $I = [0, T]$, the second order equation $\ddot{x} = a(t)V'(x)$, where V is a non-negative double well potential, and $a(t) > 0$ is a bounded, measurable function. We are looking for solutions $x(t)$ of the given equation which fulfil boundary conditions of the kind $\dot{x}(0) = \sqrt{2lV(x(0))}$, $\dot{x}(T) = \sqrt{2lV(x(T))}$, where $l > 0$ is given. This problem arises from the search of heteroclinic solutions to the equation $\ddot{x} = a_l(t)V'(x)$, where $a_l = a$ on I, $a_l = l$ outside I. The technique we are going to explain relies on the well-known property of preservation of area which is satisfied by the associated flow on the phase plane.

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Chaotic Dymamics in Periodic Isolating Segments

Anna Gierzkiewicz
Jagiellonian University, Kraków, Poland

I am going to present my joint work with Klaudiusz Wójcik, where we study arithmetical properties of Lefschetz sequence $\{L_k\}_{k\in\mathbb{N}}$ of periodic isolating segment. At the beginning I will shortly present the method of isolating segments, which is a powerful tool of studying differential equations periodic in time.

The Lefschetz numbers of monodromy map allow to study chaotic dynamics of Poincaré map P. Chaos is here defined by semi–conjugacy of the map P on some compact invariant set with the left–shift map σ on the set $\Sigma_2 = \{0,1\}^{\mathbb{Z}}$ of bi–infinite sequences of two symbols and existence of infinitely many periodic points of P.

I am going to focus on periodic sequences $\{L_k\}$. It appears that using basic theorems of number theory, such as Dirichlet theorem, one can obtain nontrivial criteria of chaotic behaviour and existence of periodic points for P. Moreover, interesting facts concerning the cardinality of the set of periodic points of P may be deduced from the boundedness of $\{L_k\}$.

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On Detecting Bounded and Periodic Orbits: Guiding Functions and Isolating Segments

Grzegorz Kosiorowski
Jagiellonian University, Krakow, Poland

I focus on the relationship between two topological techniques for detecting periodic solutions of non-autonomous periodic ODEs. The first one is the well-known Krasnosielski's method of guiding functions. The second one is the method of Srzednicki's periodic isolating segments. The aim of this talk is to show that the existence of the complete set of guiding functions guarantees the existence of periodic isolating segment that carries the same information concerning periodic solutions of the considered system. Moreover, a generalized definition of Krasnosielski's guiding functions can be used to prove a criterion for the compactness of the set of bounded orbits for a differential equation.

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Generalized Synchronization in Linearly Coupled Time-Periodic Systems

Alessandro Margheri
Universidade de Lisboa, Portugal
(Rogerio Martins)

We show that some results by R. Smith can be used in the framework of synchonization theory providing also some estimates for the parameters of the system for which the synchronization occurs.

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Multiple Nontrivial Solutions of a Prescribed Mean Curvature Problem

Franco Obersnel
Università di Trieste, Italy

We study the existence of multiple nontrivial solutions of the prescribed mean curvature problem

$$-\mathrm{div}\Big(\nabla u/\sqrt{1+|\nabla u|^2}\Big) = f(x,u) \quad \text{in } \Omega, \qquad u = 0 \quad \text{on } \partial\Omega,$$

under different assumptions on the behaviour of the function f near the origin and at infinity.

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Coexistence and Optimal Control Problems for a Degenerate Predator-Prey Model

Duccio Papini
University of Siena, Italy
(W. Allegretto, G. Fragnelli, P. Nistri)

We present a predator-prey mathematical model for two biological populations which dislike crowding. The model consists of a system of two degenerate parabolic equations with nonlocal terms and drifts. We provide conditions on the system ensuring the coexistence, namely the existence of two non-trivial non-negative periodic solutions representing the densities of the two populations. Since the presence of predators above a certain threshold may cause the disappearance of the prey, we keep the number of predators under this threshold by means of an external control action consisting of harvesting the exceeding predators. A minimization problem for a cost functional associated with this control action and with some other significant parameters of the model is also considered.

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Atiyah-Singer Family Index Theorem and Bifurcation of Solutions of Nonlinear Elliptic Boundary Value Problems

Jacobo Pejsachowicz
Politecnico di Torino, Italy

Topological invariants derived from index bundle of the linearization of a family of Fredholm maps at points of the trivial branch provide an algebraic measure of the number of bifurcation points from the branch. On the other hand, the index bundle depends only on coefficients of leading terms of the linearization. From this, using the Agranovich reduction and Fedosov's formulation of the Atiyah-Singer index theorem for families of pseudo-differential operators we obtain sufficient conditions for bifurcation of solutions of nonlinear elliptic systems with Shapiro-Lopatinskij boundary conditions which are

of different nature from the ones that can be obtained by means of the classical Liapunov-Schmidt approach.

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A Solution Set Analysis of a Nonlinear Operator Equation Within the Leray-Schauder Type Fixed Point Approach

Anatoliy Prykarpatsky
State Pedag. University, Drohobych, Ukraine
(Denis L. Blackmore)

Here we study the solution set of a nonlinear operator equation in a Banach subspace $\mathcal{L}^n \subset C(X)$ of the finite codimension $n \in \mathbb{Z}_+$ with X being an infinite compact Hausdorf space, and defined by conditions $\alpha_i^*(f) := \int_X f(x)d\mu_i(x) = 0, f \in C(X)$, with norms $||\mu_i|| = 1, i = 1, \ldots, n$, by means of reducing it to the Leray-Schauder type fixed point problem.
AMS Classification: Primary 65L07, 65N12; Secondary 65M12.

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Persistence in Seasonally Forced Epidemiological Systems

Carlota Rebelo
Universidade de Lisboa, Portugal
(N. Bacaer, A. Margheri)

We consider a general class of time periodic epidemiological systems. Using topological methods, we give sufficient conditions for the persistence of the disease in the population.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Lower and Upper Solutions in Periodic Second Order Equations Always Lead to Instability

Antonio Ureña
University of Granada, Spain

We consider periodic problems associated to second-order equations having an ordered pair $\alpha \leq \beta$ of lower and upper solutions and study the dynamics in the vicinity of the periodic solutions which lie between them.

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Reaction-Diffusion Systems with Unilateral Obstacles

Martin Väth
Academy of Sciences of the Czech Rep.
(J. Eisner, M. Kučera)

A reaction-diffusion system of activator-inhibitor or substrate-depletion type is considered which is subject to diffusion-driven instability. Nevertheless there is a large parameter domain in which the system is exponentially stable. It is explained in the talk why even in this parameter domain the system is subject to a global bifurcation if some obstacle (for instance, a unilateral membrane) are involved. Moreover, it is discussed how this is obtained by using degree theory for multivalued maps, even if for the obstacle itself certain physical limitations are assumed.

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A New Spectrum of Nonlinear Operators

Alfonso Vignoli
Dipartimento di Matematica, Italy
(Alessandro Calamai, Massimo Furi)

This is a joint work with Massimo Furi (University of Florence) and Alessandro Calamai (University of Ancona). We introduce a new concept of spectrum (at a point) for nonlinear operators which is particularly suited for C^1-Frechet differentiable maps. In this case the spectrum at a given point coincides with the spectrum of the Frechet differential at that point. Therefore, in the linear case, our spectrum coincides with the usual spectrum of a linear operator. If time permits, we shall give an indication of work in progress with Jorge Ize of the Universidad Nacional de Mexico related to equivariant nonlinear spectral theory.

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Visibility of Critical Points of Functionals from Finite-Dimensional Approximations

Massimo Villarini
Universita di Modena e Reggio Emilia, Italy
(Federica Sani, Universitá di Milano)

We consider a C^2 functional $F : E \to \mathbb{R}$ defined in a separable Hilbert space E, and look to their Raylegh-Ritz (or Galerkin) approximations, (RRG)-approximations for short:

$$F_N : E_N \mapsto \mathbb{R}$$

where E_N is generated by finitely many elements of an Hilbert bases of E, $\overline{\cup E_N}^E = E$ and $F_N = F_{|E_N}$.

A critical point z of F is *visible* from (RRG)-approximations if there exists $\overline{N}$ such that for any $N > \overline{N}$ there exists a critical point z_N of F_N and such critical points converge to z in E.

While in general a critical point, even a nondegenerate one, of a generic smooth functional may be not visible from their finite-dimensional approximations, we single out some classes of functionals,

of relevance for applications, whose critical points are visible from their (RRG)-approximations.

We give applications to functionals with symmetries, and in particular to the problem of existence of periodic orbits of an Hamiltonian system.

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Special Session 62: Topological and Variational Methods in Differential and Difference Equations

Feliz Minhos, University of Evora, Portugal
Alberto Cabada, University of Santiago de Compostela, Spain

Introduction: Boundary value problems (BVPs), composed by ordinary differential equations (ODEs) or difference equations (DEs) and some conditions on the boundary (BC), have emerged naturally from various fields of science, and they have received great attention of the international mathematical community.

By the diversity of applications and the variety of problems (nonlinear, nonlocal, functional, ...) there is a wide range of methods and techniques available. The aim of this Special Session is to present and discuss new trends in related fields such as variational methods (critical point theory, linking theorems, ...), and topological methods (fixed points theorems, lower and upper solutions, topological degree, ...) applied to ODEs, DEs, time-scales, ...

The expected results may also cover various forms of qualitative data of solutions, existence, uniqueness, multiplicity, ..., in a theoretical or applied point of view.

Anti-Maximum Principle for One Dimensional p-Laplacian Operator with Sign Changing Potential and Periodic Boundary Value Conditions

Alberto Cabada
University of Santiago de Compostela, Spain
(Jose A. Cid; Milan Tvrdy)

It is well known that the second order equation $u''+M\,u$ coupled with periodic boundary value conditions satisfies a maximum principle for all negative M and an anti-maximum principle for all positive M less that the first eigenvalue of the corresponding Dirichlet problem. Moreover both results are optimal. The introduction of a nonnegative but non constant potential $M(t)$ in this equation makes the problem more difficult to deal with. Recently, Torres & Zhang presented sharp conditions on $\|M\|_q$ ensuring the validity of the anti-maximum principle. Such results have been improved by Cabada & Cid for $M(t)$ a sign changing potential. On the other hand, Cabada, Lomtatidze & Tvrdý dealing with quasilinear operators, have shown that if $M \geq 0$ satisfies some suitable a priori bound in L_∞, then an anti–maximum principle holds for the operator $(\varphi_p(u'))'+M(t)\,\varphi_p(u)$ together with periodic boundary value conditions. In this talk we fill the gap between the linear problem with $1 \leq q \leq \infty$, for $M(t)$ changing sign, and the p-Laplacian equation with $q = \infty$, for $M(t) \geq 0$. We will provide sharp L_q-estimates on the potential $M(t)$ in order to ensure the validity of the anti-maximum principle even in the case that $M(t)$ changes sign.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Maximun Principle to Both Sides of an Eigenvalue

Juan Campos
Universidad Granada, Spain

We are going to consider $L : \mathrm{Dom}(L) \subset L^1(\Omega) \to L^1(\Omega)$ a linear operators of elliptic type. In this case there is many examples where the positivity of the operator $R_\lambda := (L + \lambda I)^{-1}$ is connected with $\lambda < \lambda_0$. This situation is this is usually referred as maximum principle. When $\lambda > \lambda_0$, sometimes the resolvent operator R_λ became negative and this is usually referred as antimaximum principle. Let observe that this fact is also a maximum principle but the operator $-L$.

We will give conditions on the operator in order to have both maximum and antimaximun principle near a principal eigenvalue. We will give also examples where this property holds.

$\longrightarrow \infty \diamond \infty \longleftarrow$

A Semilinear Vector Version of Krasnoselskii's Cone Compression-Expansion Theorem

Casey Cremins
University of Maryland, USA

A fixed point index for semilinear operators is used to prove a cone compression-expansion type existence theorem applicable to systems of semilinear equations.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Investigation of Asymptotic Behavior of Solutions of Systems of Discrete Equations

Josef Diblík
Brno University of Technology, Czech Republic
(I. Hlavičková, M. Ružičková, Z. Šmarda)

We investigate the asymptotic behavior for $k \to \infty$ of the solutions of the system of n difference equations

$$\Delta u(k) = F(k, u(k)).$$

We show that under appropriate conditions there exists at least one solution of the system considered the graph of which stays in a prescribed domain. To prove the main result, we present a method which connects two techniques – Liapunov type and retract type approach.

The investigation was supported by the Grant 201/10/1032 of Czech Grant Agency (Prague), by the Council of Czech Government MSM 00216 30503, MSM 00216 30519 and MSM 00216 30529, by the Grant 1/0090/09 of the Grant Agency of Slovak Republic (VEGA) and project APVV-0700-07 of Slovak Research and Development Agency.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Maximum Principles for One Component of Solution Vector for Systems of Functional Differential Equations

Alexander Domoshnitsky
Ariel University Center, Israel

In known assertions on differential inequalities for linear systems of ordinary as well as functional differential equations, very hard conditions on the sign of coefficients were assumed. One of the reasons of these limitations was the wish of authors to get assertions about all component of solution vector. In many practical problems it is not necessary since only several components of solution vector are important. The method to study only one component of solution vector is proposed. The method is based on the construction of a functional differential equation for one of the component of solution vector. The results about existence and uniqueness of solutions, about differential inequalities, positivity of elements in a corresponding lines of Green's matrix and stability are proposed.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Heteroclinics for Some Equations Involving the p-Laplacian

Ricardo Enguiça
ISEL, Portugal
(Luis Sanchez)

Consider the differential equation

$$\left(|u'|^{p-2}\, u'\right)' + cu' + g(u). = 0, \tag{1}$$

where $p > 1$ and $g(u)$ is a type A function in $[0, 1]$, that is, continuous, $g(0) = g(1) = 0$ and g is positive in $(0, 1)$. The solutions of this equation can be seen as travelling wave solutions of the partial differential equation

$$u_t = \frac{\partial}{\partial x}\left(\left|\frac{\partial u}{\partial x}\right|^{p-2} \frac{\partial u}{\partial x}\right) + g(u).$$

Our objective is to prove the existence of an heteroclinic solution of (1) with $u(-\infty) = 1$ and $u(+\infty) = 0$.

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Well Ordered (Weakly Coupled) Lower and Upper Solutions for Differential Equations with Piecewise Constant Arguments

Juan Bosco Ferreiro
University of Santiago de Compostela, Spain
(Alberto Cabada)

We use the method of weakly coupled lower and upper solutions to derive the existence of extremal coupled quasi-solutions and extremal solutions of differential equations with piecewise constant arguments and nonlinear boundary conditions. By assuming different monotonicity assumptions on the boundary value conditions, we also deduce existence results of extremal solutions lying between a pair of well ordered lower and upper solutions. Furthermore, some results about the uniqueness of solution are given.

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Non Ordered Lower and Upper Solutions to Fourth Order Functional Boundary Value Problems

Joao Fialho
CIMA-UE, Portugal
(A. Cabada, F. Minhos)

In this paper, given $f : [a, b] \times (C\,[a, b])^3 \times \mathbb{R} \to \mathbb{R}$ a L^1-Carathéodory function, it is considered the functional fourth order equation

$$u^{(iv)}(x) = f\left(x, u, u', u'', u'''(x)\right)$$

together with the nonlinear functional boundary conditions

$$\begin{aligned} L_0\left(u, u', u'', u(a)\right) &= 0, \\ L_1\left(u, u', u'', u'(a)\right) &= 0, \\ L_2\left(u, u', u'', u''(a), u'''(a)\right) &= 0, \\ L_3\left(u, u', u'', u''(b), u'''(b)\right) &= 0. \end{aligned}$$

Here L_i, $i = 0, 1, 2, 3$, are continuous functions satisfying some adequate monotonicity assumptions.

It will be proved an existence and location result in presence of non ordered lower and upper solutions and without monotone assumptions on the right hand side of the equation.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Coupled Fixed Points of Multivalued Operators and First-Order ODEs with State-Dependent Deviated Arguments

Rubén Figueroa
University of Santiago de Compostela, Spain
(Rodrigo L. Pouso)

We establish a coupled fixed points theorem for a meaningful class of mixed monotone multivalued operators and then we use it to derive some results on existence of quasisolutions and unique solutions to first-order functional differential equations with state-dependent deviating arguments. Our results are so general that they apply for functional equations featuring discontinuities with respect to all of their arguments, but we emphasize that they are new even for differential equations with continuously state-dependent delays.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Existence of Solutions to Boundary Value Problems for Systems of Second Order Equations on Times Scales

Marlene Frigon
University of Montreal, Canada

The notion of solution-tube of systems of second order equations on times scales is introduced. Using this notion, we present existence results to boundary value problems for systems of second order equations on times scales. We first consider the case where the right member of the equation does not depend on x^{Δ}. Then we present a result for the more general case where the right member of the equation is $f(t, x(\sigma(t)), x^{\Delta}(t))$.

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$\longrightarrow \infty \diamond \infty \longleftarrow$

Radial Solutions for Boundary Value Problems Involving the Mean Curvature Operator in Minkowski Space

Petru Jebelean
West University of Timisoara, Romania
(C. Bereanu and J. Mawhin)

We discuss some existence and multiplicity results of radial solutions to homogeneous Dirichlet and Neumann boundary value problems for equations of the type

$$\operatorname{div}\left(\frac{\nabla v}{\sqrt{1-|\nabla v|^2}}\right) = f(|x|, v, \frac{dv}{dr}) \quad \text{in } \Omega \subset \mathbb{R}^N,$$

where Ω is a ball or an annular domain.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Continuous Dependence in Front Propagation of Convective Reaction-Diffusion Equations

Cristina Marcelli
Technical University of Marche, Italy
(Luisa Malaguti, Serena Matucci)

The talk concerns the study of the continuous dependence of the threshold wave speed and of the travelling wave profiles for reaction-diffusion-convection equations

$$u_t + h(u)u_x = \Big(d(u)u_x\Big)_x + f(u)$$

with respect to the diffusion, reaction and convection terms.

The diffusion term is allowed to assume also negative values, so that the model includes aggregation-diffusion phenomena too.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Sufficient Conditions for the Solvability of Higher Order Functional Boundary Value Problems

Feliz Minhós
University of Évora, Portugal

This work considers a n^{th} order fully nonlinear differential equation, $n \geq 2$, which includes the $\varphi-$Laplacian, with functional boundary conditions.

Applying lower and upper solutions, α and β, respectively, it is obtained an existence and location result that allows us to conclude that the order relation between α and β, and the assumptions of the

monotone type on the nonlinearity, depend on three data: the relation between $\alpha^{(n-2)}$ and $\beta^{(n-2)}$, the boundary ends and if n is even or odd.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Existence Result for a Stationary Problem Arising in Option Pricing with Transaction Costs

Eva Morais
CEMAPRE, Portugal
(M. R. Grossinho)

We establish an existence and uniqueness result concerning stationary convex solutions for a nonlinear Black-Scholes equation type related to option pricing models with transaction costs. We use the method of upper and lower solutions, together with some Nagumo condition, which also yields information on the localization of the solution. This is a joint work with M. R. Grossinho (CEMAPRE / ISEG-UTL).

$\longrightarrow \infty \diamond \infty \longleftarrow$

Solvability of a Prescribed Mean Curvature Equation: A Lower and Upper Solutions Approach

Pierpaolo Omari
University of Trieste, Italy

We study existence and multiplicity of solutions of the prescribed mean curvature problem

$$-\mathrm{div}\Big(\nabla u/\sqrt{1+|\nabla u|^2}\Big) = f(x,u) \quad \text{in } \Omega, \qquad u = 0 \quad \text{on } \partial\Omega.$$

Our approach is based on variational techniques and a lower and upper solutions method specially developed for this problem in a BV setting.

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Heteroclinic Solutions of Boundary Value Problems on the Real Line Involving General Nonlinear Differential Operators

Francesca Papalini
Technical University of Marche, Italy
(Giovanni Cupini, Cristina Marcelli)

We discuss the existence of heteroclinic solutions defined on the whole real line, for non-autonomous equations governed by strongly nonlinear differential operators, including also the classical p-Laplacian. In this context we provide some sufficient conditions which result to be optimal for a large class of problems.

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A Functional-Analytical Approach to the Asymptotics of Recursions

Christian Poetzsche
Munich University of Technology, Germany

We propose a functional-analytical method to investigate the long-term behavior of recursions (difference equations). It is based on a formulation of given (implicit) recursions as abstract operator equations in sequence spaces. This enables us to tackle the problem of asymptotic behavior using appropriate tools from nonlinear analysis in form of fixed-point and coincidence theorems. We obtain quantitative convergence results and can verify summable or subexponential decay.

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Asymptotic Properties of Singular Second-Order Differential Equations

Irena Rachunkova
Palacky University, Olomouc, Czech Rep.

Singular differential equations of the form

$$(p(t)u'(t))' = p(t)f(u(t)), \tag{1}$$

are investigated on the half-line $[0,\infty)$. The function $p : [0,\infty) \to [0,\infty)$ has a continuous first derivative which is positive on $(0,\infty)$ and $p(0) = 0$. Therefore equation (1) has a singularity at $t = 0$. The function f is supposed to be locally Lipschitz continuous on the real line and it has at least three zeros $L_0 < 0 < L$. Positivity or negativity of f between its zeros characterize two different cases of asymptotic behaviour of solutions of (1).

We provide conditions which guarantee the existence of solutions converging to L or to 0 provided $xf(x) < 0$ or $xf(x) > 0$ between zeros L_0, 0 and L. A global behaviour of solutions on $[0,\infty)$ (positivity, monotonicity, number of zeros, ...) is characterized as well.

Equations of this type arise in the study of phase transitions of van der Waals fluids, in population genetics, where they serve as models for the spatial distribution of the genetic composition of a population, in the homogenenous nucleation theory, in the nonlinear field theory when describing bubbles generated by scalar fields of the Higgs type in the Minkowski spaces or in relativistic cosmology.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Nonlinear Sturm-Liouville Problems Subject to Global Boundary Conditions

Jesus Rodriguez
North Carolina State University, USA
(Zachary Abernathy)

We consider Sturm-Liouville problems subject to nonlinear perturbations in both the dynamics and the boundary conditions. For such boundary value problems we provide easily verifiable sufficient conditions for the existence of solutions.The criteria we present allows for global boundary conditions and sheds light on the interplay between properties of the nonlinearities and the spectrum of the associated linear Sturm-Liouville problem.

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Comparison of Liénard Type Equations

Felix Sadyrbaev
LU MII and Daugavpils University, Latvia
(S. Atslega)

We compare behavior of solutions of two differential equations, the first one is the generalized Liénard equation

$$x'' + f(x)x' + g(x) = 0 \qquad (1)$$

and the second one is a Liénard type equation

$$x'' + f(x)x'^2 + g(x) = 0, \qquad (2)$$

reducible to a conservative system.

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Heteroclinic Orbits for $u''' = f(u)$

Luís Sanchez
Universidade de Lisboa, Portugal
(Denis Bonheure, José Ángel Cid and Colette De Coster)

Consider the problem

$$u''' = f(u), \quad u(-\infty) = -1, \quad u(+\infty) = 1, \qquad (1)$$

where $f : \mathbb{R} \to \mathbb{R}$ satisfies the following assumptions:
($h1$) f is continuous and even.
($h2$) $f(u)(u-1) > 0$ for all $u \geq 0$, $u \neq 1$.
($h3$) There exists $N_0 > 0$ such that

$$F(s) \geq 0 \quad \text{for all } s \geq N_0,$$

where $F(s) = \int_0^s f(r)dr$.

We prove that under these conditions (1) has an odd solution. This is a heteroclinic connecting the equilibria ∓ 1.

We use approximated problems in finite intervals and the Leray-Schauder continuation principle.

If moreover f is nondecreasing on $(0,\infty)$ and locally Lipschitz, the solution is unique.

We also consider related non-symmetric problems, thus dropping ($h1$). Namely, under certain growth conditions on f near $+\infty$, there exist solutions of

$$u''' = f(u), \quad u \leq 0 \quad \text{for } t \in]-\infty, 0], \\ u(-\infty) = -1, \; u(0) = 0$$

and the continuation of such a solution is either a heteroclinic connecting ∓ 1 or explodes in finite time.

Results for this type of problems were studied among other authors by McCord (J. Math. Anal. Appl. 1986), Mock (J. Differential Equations 1979) and Toland (Proc. Royal Soc. Edinburgh 1988).

$\longrightarrow \infty \diamond \infty \longleftarrow$

Positive Solutions for Dirichlet Problems of Singular Nonlinear Fractional Differential Equations

Svatoslav Stanek
Palacky University, Olomouc, Czech Rep.
(Ravi P. Agarwal; Donal O'Regan)

We investigate the existence of positive solutions for singular fractional boundary value problem: $D^\alpha u(t) + f(t, u(t), D^\mu u(t)) = 0$, $u(0) = u(1) = 0$. Here $1 < \alpha < 2$, $0 < \mu \leq \alpha - 1$, D^α is the standard Riemann-Liouville fractional derivative, f is a positive Carathéodory function on $[0,1] \times (0,\infty) \times (-\infty,\infty)$ and $f(t,x,y)$ is singular at $x = 0$. The proofs are based on regularization and sequential techniques. The solvability of regular problems is reduced to the existence of a fixed point of an operator S_n. The existence of a fixed point of S_n is proved by a fixed point theorem of cone compression type due to Krasnosel'skii.

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Multiplicity of Solutions for Discrete Problems with Double-Well Potentials

Petr Stehlik
University of West Bohemia, Czech Republic
(Josef Otta)

In this talk we present some basic multiplicity results for a general class of nonlinear discrete problems with double-well potentials. Variational techniques are used to obtain the existence of saddle-point type critical points. Partial difference equations as well as problems involving discrete p-Laplacian are considered. Finally, we discuss the boundedness of solutions and the applicability of these results.

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Asymptotically Autonomous Second Order Differential Equations

Jan Tomeček
Palacky University in Olomouc, Czech Republic

Some models in the nonlinear field theory, hydrodynamics and other areas of mathematical physics lead to a singular boundary value problem for second order differential equation on the half–line $(0, \infty)$ in the form

$$(p(t)u')' + p(t)f(u) = 0,$$
$$u'(0) = 0, \quad u(\infty) = 0,$$

where $p > 0$ on $(0, \infty)$, $p(0) = 0$, $f : \mathbb{R} \to \mathbb{R}$. The existence and uniqueness of several types of solutions to these problems were investigated intensively in the past. Differential equations of such problems can be characterized by autonomous equations of the form

$$u'' + f(u) = 0.$$

Depending on the behaviour of the function f, there are several qualitatively different phase portraits of this autonomous equation. They correspond to different nonautonomous BVPs requiring special approaches. In the talk, a survey of some earlier and contemporary results will be presented.

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Periodic Solutions of Second-Order Differential Equations with Attractive-Repulsive Singularities

Pedro Torres
University of Granada, Spain
(Robert Hakl)

New sufficient conditions for the existence of a solution to the problem

$$u''(t) = \frac{g(t)}{u^{\mu}(t)} - \frac{h(t)}{u^{\lambda}(t)} + f(t) \qquad \text{for a.e. } t,$$

$$u(0) = u(\omega), \qquad u'(0) = u'(\omega)$$

are established. The main feature is that our conditions do not require that the coefficients g, h, f are uniformly bounded by positive constants, in fact they may vanish in some compact interval. In this sense, our theorems complement classical results by Lazer-Solimini ('87), Rachukova-Tvrdy-Vrkoc ('01) and others.

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On Periodic Singular Problems

Milan Tvrdý
Academy of Sciences, Prague, Czech Republic

In the contribution we will present a survey of the recent development in the theory of singular periodic problems for equations with a quasilinear differential operator.

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Periodic Solutions of Planar Nonautonomous Polynomial ODEs

Pawel Wilczynski
Jagiellonian University, Cracow, Poland

Our goal is to discuss the existence of periodic solutions of ODEs (in complex number notation) $\dot{z} = v(t, z) = \sum_{j=0}^{n} a_j(t)z^j$ where $n \geq 3$ and all a_j are continuous and T-periodic. We are also show that some of the periodic solutions are asymptotically stable or unstable. We are interested especially in periodic solutions which are close to the branches of zeros of the vector field v. We focus on the mechanism of generating periodic solutions in context of multiplicity of the branches. We also detect heteroclinic solutions to the periodic ones.

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Special Session 63: Nonlinear Evolution Equations and Related Topics

Mitsuharu Otani, Waseda University, Japan
Tohru Ozawa, Waseda University, Japan

Introduction: This session will focus on the recent developments in the theory of Nonlinear Evolution Equations and Related Topics including the theory of abstract evolution equations in Banach spaces as well as the studies (the existence, regularity and asymptotic behaviour of solutions) of various types of Nonlinear Partial Differential Equations.

Non-Uniqueness of Solutions for Some Doubly Nonlinear Parabolic Equations

Goro Akagi
Shibaura Institute of Technology, Japan

We exhibit the non-uniqueness of solutions $u = u(x,t)$ for some class of doubly nonlinear parabolic equations such as

$$|u_t|^{p-2}u_t - \Delta u = \lambda u \quad \text{for} \quad x \in \Omega$$

and

$$|u_t|^{p-2}u_t - \Delta u + |u|^{p-2}u = \lambda u \quad \text{for} \quad x \in \Omega$$

for $x \in \Omega \subset \mathbb{R}^N$, $t > 0$, where $p > 2$ and $\lambda \in \mathbb{R}$. Our method of analysis is based on classical separation of variable method as well as variational method.

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Mathematical Modeling of Mitochondrial Swelling

Sabine Eisenhofer
Helmholtz Zentrum München, Germany

(F. Toókos, B. A. Hense, S. Schulz, F. Filbir, H. Zischka)

The permeabilization of mitochondrial membranes accompanied by mitochondrial swelling is a decisive event in apoptosis or necrosis culminating in cell death. Swelling of mitochondria is measured by means of decreasing light scattering values at a fixed wave length. Our aim is to understand and describe the experimental results mathematically. For that we introduce two approaches, which model the Ca^{2+}-induced swelling from different perspectives. The first model considers the swelling performance of single mitochondria and translates this behavior to a whole population. The second one gives insight into the kinetics of mitochondrial swelling and shows the total volume changes of a mitochondrial population. Both models are in accordance with the experimentally determined course of volume increase throughout the whole swelling process. By that, the existence of a positive feedback mechanism is confirmed and we obtain consistent parameter changes with increasing Ca^{2+} concentrations. The second model may be adapted and extended to other inducing conditions or to mitochondria from other biological sources. Both models benefit a better understanding of the mitochondrial permeability transition.

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Dynamic Analysis of a Quasilinear Riser Equation

Jorge Esquivel-Avila
Universidad Autonoma Metropolitana, Mexico

We study some qualitative properties of a quasilinear wave equation of fourth order that models the mechanical vibrations of a marine riser. Our analysis gives sufficient conditions for existence of global and nonglobal solutions with respect to the norm of some Hilbert space $\mathcal{H}$, for any real value of the initial energy. To that end, we employ invariant sets. We conclude that globality implies exponential decay to zero, if initial energy $E_0 < d$, is sufficiently small. Also, we show that nonglobality is due to blow up. We also present conditions either for uniform boundedness or blow up, for higher initial energies, $E_0 \geq d$. Our results are given with respect to the norm of the solution in $\mathcal{H}$.

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Asymptotic Behavior of Solutions for One Dimensional Parabolic Equations with Nonlinear Boundary Conditions

Junichi Harada
Waseda university, Japan

(Mitsuharu Otani)

In this talk, the asymptotic behavior of solutions for $u_t = u'' - au^p$, $\partial_\nu u = u^q$ are discussed. In particular, we prove the nonexistence of solutions which converge to some singular solutions.

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Nonexistence of Weak Solutions of Quasilinear Elliptic Equations with Homogeneous Coefficients in Exterior Domains

Takahiro Hashimoto
Japan Meteorological Agency, Japan

In this talk, we are concerned with the following quasilinear elliptic equations:

$$\text{(E)} \qquad \begin{cases} -\mathrm{div}\left\{|x|^\alpha |\nabla u|^{p-2}\nabla u\right\} = |x|^\beta |u|^{q-2}u \\ \quad \text{in } \Omega, \\ u(x) = 0 \qquad \text{on } \partial\Omega, \end{cases}$$

where Ω is a domain in $\mathbf{R}^N$ $(N \geq 2)$ with smooth boundary.

This problem is related to the well-known Caffarelli-Kohn-Nirenberg inequalities. When $\alpha = \beta = 0$, a complementary result was founded for the problem of interior, exterior and whole space. The main purpose of this talk is to obtain the nonexistence results for (E) with a class of weak solutions to discuss whether the complementary result mentioned above is still valid for this type of equations.

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Some Space-Time Integrability Estimates of the Solution for Heat Equations in Two Dimension

Norisuke Ioku
Tohoku University, Japan

We consider an integrability estimate for a solution of the homogeneous initial boundary value problem for heat equations in a bounded domain with external force. Introducing the Lorentz-Zygmund space, we show the exponential integrability of the Brezis-Merle type for a weak solution of initial boundary value problem of the heat equation. The way of the proof is based on the rearrangement technique. Due to this technique, we can reduce the problem of the integrability of the solution to estimate the singularity near the origin.

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On the Variational Problems Associated with Trudinger-Moser Type Inequalities in Unbounded Domains

Michinori Ishiwata
Fukushima university, Japan

It is known that the classical Trudinger-Moser inequality (in bounded domains) has several $\mathbb{R}^2$-versions. In this talk, we discuss the existence and nonexistence of a maximizer for the associated variational problem and try to reveal the difference between the problem defined in bounded domains. Particularly, we obtain the nonexistence of maximizers as well as the existence, which reveals a striking difference from the problem associated with bounded domains.

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Large Time Behaviors of Hots Spots to a Heat Equation with a Potential Term

Yoshitsugu Kabeya
Osaka Prefecture University, Japan
(Kazuhiro Ishige)

We consider the initial value problem with a linear heat equation with a potential term. We discuss how the behavior of the "hot spots"(the maximum points of a solution) is affected by the potential term as time goes by. We show that the corresponding elliptic problem controls the behavior.

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Mixed Problem for Ott-Sudan-Ostrovskiy Equation

Elena Kaikina
Istituto Tecnologico de Morelia, Mexico

Consider the mixed initial-boundary value problem for the Ott-Sudan-Ostrovskiy (OSO) equation on a half-line

$$\begin{cases} u_t + uu_x - u_{xxx} + \int_0^{+\infty} \frac{\operatorname{sign}(x-y)u_y(y,t)}{\sqrt{|x-y|}} dy = 0, \\ \quad t > 0,\ x > 0, \\ u(x,0) = u_0(x),\ x > 0, \\ u(0,t) + \alpha u_x(0,t) = u_x + \beta u_{xx}(0,t) = 0,\ t > 0, \end{cases}$$

where $\alpha, \beta \in \mathbb{C}$, $|\alpha\beta| \neq 0$. We study traditionally important problems of a theory of nonlinear partial differential equations, such as global in time existence of solutions to the initial-boundary value problem and the asymptotic behavior of solutions for large time.

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L^p Estimates of Solutions to Mixed Problem for Heat Eqations

Kunihiko Kajitani
Tsukuba University, Japan

The aim of this work is to get $L^p - L^q$ estimates of solutions to mixed problem for heat equations in the half space R^n_+. To do so we shall derive $L^\infty - L^1$ estimates and L^p boundedness of solutions and obtain $L^p - L^q$ estimates by use of Riesz-Torin interpolation theorem.

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On a Certain Nonlinear Nonlocal in Space Evolution Equation with Temporally Nonlocal Nonlinearity

Mokhtar Kirane
University of La Rochelle, France
(Ahmad Z. Fino)

In this talk, we present first a new technique to prove, in a general situation, the recent result of Cazenave, Dickstein and Weissler on the blowing-up solutions to a temporally nonlocal nonlinear parabolic equation. Then, we study the blow-up rate and the global existence in time of the solutions. Furthermore, we show necessary conditions for global existence.

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Nonlinear Evolution Equations Associated with Mathematical Models

Akisato Kubo
Fujita Health University, Japan

We investigate the global existence in time and asymptotic profile of the solution of some nonlinear evolution equations with strong dissipation. Applying results obtained above to models of mathematical biology and medicine, we discuss mathematical properties of them.

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On the Critical Nongauge Invariant Nonlinear Schrödinger Equation

Pavel Naumkin
Universidad Michoacana, Mexico
(Isahi Sánchez-Suárez)

We consider the Cauchy problem for the critical nongauge invariant nonlinear Schrödinger equations

$$\begin{array}{c} iu_t + \frac{1}{2}u_{xx} = i\mu\overline{u}^{\alpha}u^{\beta},\ x \in \mathbf{R},\ t > 0, \\ u(0,x) = u_0(x),\ x \in \mathbf{R}, \end{array} \tag{1}$$

where $\beta > \alpha \geq 0$, $\alpha + \beta \geq 2$, $\mu = -i^{\frac{\omega}{2}}t^{\frac{\vartheta}{2}-1}$, $\omega = \beta - \alpha - 1$, $\vartheta = \alpha + \beta - 1$. We prove that there exists a unique solution $u \in \mathbf{C}\left([0,\infty); \mathbf{H}^1 \cap \mathbf{H}^{0,1}\right)$ of the Cauchy problem (1). Also we find the large time asymptotics of solutions.

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Schrödinger Type Evolution Equations with Monotone Nonlinearity

Noboru Okazawa
Tokyo University of Science, Japan
(Yoshiki Maeda)

Existence of unique strong solutions is established for Schrödinger type evolution equations with monotone nonlinearity of non-power type. The proof is based on a perturbation theorem for m-accretive (or maximal monotone) operators in a complex Hilbert space. In particular, if the nonlinearity is of power type, then the result is already known.

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On Verifying Mathematical Models with Diffusion, Transport and Interaction

Stefanie Sonner
Helmholtz Zentrum München, Germany
(Messoud Efendiev)

The solutions of many systems of convection-reaction-diffusion equations arising in biology, physics or engineering describe such quantities as population densities or concentrations of nutrients and chemicals. Hence, a natural property to require for the solutions is positivity.

We present necessary and sufficient conditions for the positivity of quasi-linear and semi-linear systems of parabolic equations. This criterion allows us to verify mathematical models. As an application we consider models arising in the modelling of biofilms.

Moreover, the extension of this result to systems of stochastic partial differential equations will be discussed.

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Global Existence of Solutions for Higher Order Nonlinear Damped Wave Equations

Hiroshi Takeda
Tohoku University, Japan

We consider a Cauchy problem for a polyharmonic nonlinear damped wave equation. We obtain a critical condition of the nonlinear term to ensure the global existence of solutions for small data. Moreover, we show the optimal decay properity of solutions under the sharp condition on the nonlinear exponents, which is a natural extension of the results for the nonlinear damped wave equations. The proof is based on L^p-L^q type estimates and Strichartz type estimate of the fundamental solutions of the linear polyharmonic damped wave equations.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Existence of Solutions for Some Nonlinear Damped Wave Equations

Takahiro Tannai
Waseda University, Japan
(Mitsuharu Ôtani)

$$(E) \begin{cases} u_{tt} - \triangle u + a|u_t|^{m-1}u_t = b|u|^{p-1}u \\ \qquad (t,x) \in [0,T] \times \Omega, \\ u(t,x) = 0 \qquad (t,x) \in [0,T] \times \partial\Omega, \\ u(0,x) = u_0(x), u_t(0,x) = u_1(x) \qquad x \in \Omega, \end{cases}$$

where $a > 0, b \in \mathbb{R}, m \geq 1, p \geq 1$ are parameters, and Ω is a bounded domain in $\mathbf{R}^n (n \in \mathbf{N})$ with smooth boundary $\partial\Omega$. We prove the existence of weak solutions of (E) under the assumption,

$$(A) \begin{cases} 1 \leq p < \infty & n \leq 2, \\ 1 \leq p \leq \dfrac{n}{n-2}\dfrac{2m}{m+1} & n \geq 3. \end{cases}$$

$\longrightarrow \infty \diamond \infty \longleftarrow$

Some Linear System of Partial Differential Equations with Real Characteristic of Constant Multiplicity

Jean Vaillant
University Paris 6, France

We consider a linear system of partial differential equations with real characteristic of constant multiplicity. In previous publications (Japanese Journal of Math., ...), we have defined invariant conditions L for the system. We have stated that these conditions are necessary and sufficient in order that the Cauchy problem is well posed in C^∞ to the multiplicity 5 and also (Taglialatela-Vaillant ...)in some special series of cases. We define here systems "with good diagonalization", which generalize the analogous notion in the scalar situation. We state that, if the system has a good diagonalization, the conditions L are verifeed and we begin the study of the reciprocal theorem. A system with good diagonalisation has similar properties to a scalar equation relatively to the Cauchy problem. So we have a step to the generalization of the previous results.

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Limiting Structure on Eigenfunctions of Linearized Eigenvalue Problems for 1-Dimensional Bistable Reaction-Diffusion Equations

Tohru Wakasa
Meiji University, Japan
(Shoji Yotsutani)

Reaction diffusion equations of the bistable type, which are known as simple models describing phase transition, has been exceedingly studied by many researchers. In this talk we will discuss the linearized eigenvalue problems associated with arbitrary stationary solution when the diffusion rate is so small. It is related with the super slow motion of interface which has been studied by J. Carr-R. L. Pego and G. Fusco-J. K. Hale. Our interest is to understand asymptotic profiles of every eigenfunctions as the diffusion rate goes to zero. In this talk we take the two typical balanced bistable nonlinearities, and give complete asymptotic formulas of every eigenvalues and eigenfunctions for the both linearized problems. For negative eigenvalues related with motion of interfaces, both profiles of eigenfunctions are governed by the similar rule. On the other hand, we will show that profiles of eigenfunctions for positive eigenvalues are different in two cases of nonlinearities.

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Stable Solutions for Semilinear Elliptic PDEs Involving the Biharmonic Operator

Guillaume Warnault
Universié de Tours, France

We consider the class of radial solutions of semilinear equations $\Delta^2 u = \lambda f(u)$ where f is regular. For this equation, we consider Dirichlet boundary conditions in the unit ball $B \subset \mathbb{R}^N$. The class of radial solutions is the class of stable solutions which includes minimal solutions and extremal solution. We establish the regularity of this extremal solution for $N \leq 9$ in the case of regular nonlinearity. Our regularity results do not depend on the specific nonlinearity f. Moreover, we will talk about several Liouville-type results on the fourth order semilinear elliptic equation $\Delta^2 u = f(u)$ in $\mathbb{R}^N$, where f is a smooth nonlinearity. We prove the non-existence of stable solutions which verify decay conditions at infinity.

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G-Invariant Positive Solutions for a Quasilinear Schrödinger Equation

Tatsuya Watanabe
Kyoto Sangyo University, Japan

We consider the existence of standing wave solutions related to some type of quasilinear Schrödinger equation arising in plasma physics. Using the unique global solution of some ordinary differential equation, we convert the quasilinear elliptic equation into a semilinear elliptic equation. We adopt variational method to show the existence of group invarinat solutions for a suitable potential.

$\longrightarrow \infty \diamond \infty \longleftarrow$

On Sign-Changing Solutions of a Damped Wave System and a Reaction-Diffusion System

Yusuke Yamauchi
Waseda University, Japan
(Hiroshi Takeda)

We consider the Cauchy problem for a damped wave system and a reaction-diffusion system. For a damped wave equation and a reaction-diffusion equation, Fujita exponent plays a crucial role in the global existence of the solutions. In this talk, we mainly focus our attention on the existence, nonexistence and asymptotic behavior of the global solutions of each system.

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Global Attractor of a Phase-Field Model of Grain Boundary Motion with Constraint

Noriaki Yamazaki
Kanagawa University, Japan
(Nobuyuki Kenmochi)

We consider a model of grain boundary motion with constraint. In composite material science it is very important to investigate the grain boundary formation and its dynamics. In this talk we study a phase-field model of grain boundaries with constraint, which is a modified version of the one proposed by R. Kobayashi, J. A. Warren and W. C. Carter. The model is described as a system of a nonlinear parabolic partial differential equation and a nonlinear parabolic variational inequality. The main objective of this talk is to study the asymptotic behavior of solutions to our system from the viewpoint of attractors. Moreover we discuss the characterization of global attractor to our model.

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Special Session 64: Variational Methods for Non-Smooth Functions and Applications

Salvatore A. Marano, University of Catania, Italy
Siegfried Carl, Martin Luther University of Halle-Wittenberg, Germany
Dumitru Motreanu, University of Perpignan, France

Introduction: Variational methods for C^1-functions f in a real Banach space X are by now well established. Applications - chiefly to differential equations - are countless, and their number always increases. However, many variational problems, which arise in the modelling of important mechanical and engineering questions, naturally lead to consider functionals lacking the smoothness properties usually required for the use of classical results. As an example, we only mention both variational inequalities and elliptic equations with discontinuous nonlinearities. Concerning the first case, the indicator function of some convex closed subset of X must appear in the expression of f; for the second case, f turns out to be locally Lipschitz continuous at most. The main purpose of this special session is to gather researchers interested in non-smooth variational methods and applications.

Existence and Multiplicity Theorems for a Second Order Nonautonomous System with a Nonsmooth Potential

Giuseppina Barletta
Mediterranea University of Reggio Calabria, Italy

The aim of this talk is to show some recent existence and multiplicity theorems for a second order nonautonomous system with a nonsmooth potential. The results are obtained using the nonsmooth critical point theory developed essentially in the books of L. Gasinski and N. S. Papageorgiou (Nonsmooth Critical Point Theory and Nonlinear Boundary Value Problems, Chapman Hall/ CRC Press, Boca Raton, FL., 2005) and D. Motreanu and V. Radulescu (Variational and Non Variational Methods in Nonlinear Analysis, and Boundary Value Problems, Kluwer, Dordrecht, 2003). We prove two existence theorems both in cases where the Euler functional is coercive and the solution is a minimizer of it, or when it is unbounded. Finally, for the multiplicity result we employ a nonsmooth version of the local linking theorem.

$\longrightarrow \infty \diamond \infty \longleftarrow$

A Local Minimum Theorem for Differentiable Functions Via Non-Smooth Variational Methods

Gabriele Bonanno
University of Messina, Italy

An existence theorem of a local minimum for continuously Gâteaux differentiable functions, possibly unbounded from below, is presented. The approach is based on the Ekeland's Variational Principle applied in a non-smooth variational framework by using also a novel type of Palais-Smale condition which is more general than the classical one.

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Existence and Multiplicity of Critical Point Theorems for Non-Smooth Functionals and Applications

Pasquale Candito
Mediterranea University of Reggio Calabria, Italy

The aim of this talk is to point out some recent existence and multiplicity of critical point theorems for functionals of the type $\Phi - \lambda\Psi$, where Φ and Ψ are two suitable locally Lipschitz functionals defined in a reflexive Banach space and λ is a real parameter. In this framework, some versions of basic methods

for proving the existence and obtaining the localization of local minima will be showed. In addition, a Mountain Pass Lemma involving a non standard Palais-Smale condition will be treated. Moreover, the existence of bounded Palais-Smale sequence for such class of functionals will be discussed. Applications will be given to hemivariational inequalities, partial differential equations with discontinuous nonlinearities and difference equations.

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Sub-Supersolution Method for Nonsmooth Variational Problems

Siegfried Carl
University of Halle, Germany

We present a general framework of the sub-supersolution method for multi-valued variational inequalities. This method combined with variational methods for nonsmooth functionals is an effective tool in the study of existence, comparison, and multiplicity results for nonsmooth variational problems.

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Multiplicity Results for a Discontinuous Dirichlet Problem Involving the $p(x)$-Laplace Operator

Antonia Chinnì
University of Messina, Italy

We investigate the existence and multiplicity of weak solutions for a Dirichlet problem with discontinuous nonlinearities involving the $p(x)$-Laplace operator. The results are based on a multiple critical points theorem for nonsmooth functionals established by G. Bonanno and P. Candito [Non-differentiable functionals and applications to elliptic problems with discontinuous non linearities, J. Differential Equations, 244, 3031-3059, (2008)].

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Infinitely Many Solutions for a Class of Nonlinear Elliptic Variational-Hemivariational Inequalities

Giuseppina D'Aguì
University of Messina, Italy

The aim of this talk is to present some results on the existence of infinitely many solutions for a class of nonlinear elliptic variational-hemivariational inequalities. The approach is based on a result of infinitely many critical points due to G. Bonanno and G. Molica Bisci [Infinitely many solution for a Boundary Value Problem, Bound. Value Probl.2009, 1-20] which is a more precise version of Theorem 1.1 of S. A. Marano and D. Motreanu, [Infinitely many critical points of Non-differentiable Functions and applications to a Neumann problem involving the p-Laplacian, Journal of Differential equations, 182, 108-120 (2002)]. Moreover as a special case, a result of existence of infinitely many solutions to an elliptic Neumann problem involving the p-Laplacian is pointed out [G. Bonanno and G. D'Aguì, On the Neumann problem for elliptic equations involving the $p-$Laplacian, J. Math. Anal. Appl., 358, (2009), 223-228].

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Multiple Solutions to Fourth-Order Boundary Value Problems

Beatrice Di Bella
University of Messina, Italy

The talk deals with existence and multiplicity of solutions for a fourth-order boundary value problem: $u^{iv} + Au'' + Bu = \lambda f(t,u)$ in [0, 1], subject to Dirichlet boundary value condition. The study of the problem is based on variational methods and critical point theory. In particular, under a suitable growth condition of the antiderivative of the nonlinear term, the existence of three solutions is established. While, under a convenient oscillatory behaviour of the nonlinear term, the existence of a sequence of pairwise distinct solutions is ensured. The approach is based on some very recent critical point results.

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Infinitely Many Solutions for the p-Laplacian with Non-Smooth Boundary Conditions

Francesca Faraci
University of Catania, Italy

In this talk we will deal with a quasilinear elliptic equation gathered with non-smooth Neumann boundary conditions. By using a direct method due to Saint Raymond [On the multiplicity of solutions of the equation $-\triangle u = \lambda \cdot f(u)$, J. Differential Equations, 180 (2002) 65-88] we will prove the existence of infinitely many bounded solutions for our problem. We follow a recent paper by Faraci, Iannizzotto and Varga [Infinitely many bounded solutions for the p-Laplacian with nonlinear boundary conditions, Monatsh. Math.].

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Nonlinear Elliptic Equations with Singular and Concave Terms

Leszek Gasinski
Jagiellonian University, Poland
(N. S. Papageorgiou)

We consider nonlinear elliptic Dirichlet problems with a singular term, a concave term and a Caratheodory perturbation. We study the cases where the Caratheodory perturbation is $(p-1)$-linear and $(p-1)$-superlinear near infinity. Using variational techniques based on the critical point theory together with truncation arguments and the method of upper and lower solutions, we show that if the coefficient of the concave term is small enough, the problem has at least two nontrivial smooth solutions.

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Non-Differentiable Embedding of Lagrangian Systems and Partial Differential Equations

Isabelle Greff
University of Pau, France
(Jacky Cresson)

In this talk, we develop the non-differentiable embedding theory of differential operators and Lagrangian systems using a new operator on non-differentiable functions. We then construct the corresponding calculus of variations and we derive the associated non-differentiable Euler-Lagrange equation, and apply this formalism to the study of PDEs. We then extend the characteristics method to the non-differentiable case. We prove that non-differentiable characteristics for the Navier-Stokes equation correspond to extremals of an explicit non-differentiable Lagrangian system.

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Three Solutions for a Differential Inclusion Via Critical Point Theory

Antonio Iannizzotto
University of Catania, Italy

Differential inclusions offer a general pattern for problems of several types, as differential equations with discontinuous nonlinearities (see Chang, J. Math. Anal. Appl. 80 (1981)) or hemivariational inequalities (see Kyritsi, Papageorgiou, Nonlinear Anal. 61 (2005)). Such problems can be studied via a variational approach, using the critical point theory developed by Clarke for locally Lipschitz functionals (see Ribarska, Tsachev, Krastanov, Nonlinear Anal. 43 (2001)). Here we present an extension of a recently achieved three critical points theorem (see Ricceri, Nonlinear Anal. 71 (2009)) to the non-smooth framework and apply it to a differential inclusion with Dirichlet boundary conditions, depending on two positive parameters and involving u.s.c. multivalued nonlinearities. Under certain technical assumptions, we prove the existence of at least three solutions to the inclusion problem, whose norms are less than a constant independent on the parameters. Further applications to problems involving unilateral constraints will be discussed.

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Asymptotically Critical Problems on Higher-Dimensional Spheres

Alexandru Kristaly
Babes-Bolyai University, Romania

We study the equation $-\Delta_h u = f(u)$ on the standard unit sphere (S^d, h) where the continuous nonlinearity $f:\mathbb{R}\to\mathbb{R}$ oscillates either at zero or at infinity having an asymptotically critical growth. By using a group-theoretical argument and an appropriate variational approach, we establish the existence of $[d/2]+(-1)^{d+1}-1$ sequences of sign-changing weak solutions in $H_1^2(S^d)$ whose elements in different sequences are mutually symmetrically distinct whenever f has certain symmetry and $d \geq 5$. Although we are dealing with a smooth problem, we are forced to use a non-smooth version of the principle of symmetric criticality. The L^∞- and H_1^2-asymptotic behaviour of the sequences of solutions are also fully characterized.

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Supersolution in the Theory of Wavefronts

Ruediger Landes
University Of Oklahoma, USA

We will present supersolutions to nonlinear parabolic boundary problems in order to get upper bounds on the propagation speed of the phase transitions modeled by these PDEs. We also use supersolutions (and subsolutions) to describe stable (and unstable) initial configuration.

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Some Recent Developments in Critical Points Theory for Non-Smooth Functions

Roberto Livrea
University of Reggio Calabria, Italy

The aim of this talk is to show some recent existence and multiplicity critical point results for non-differentiable functionals defined in a Banach space and which are the sum of a locally Lipschitz term and of a convex, proper semicontinuous function.

More precisely, starting from the theory developed in R. Livrea - S. A. Marano, *Existence and classification of critical points for nondifferentiable functions*, Adv. Differential Equations 9 (2004), 961-978, where a general min-max principle established by Ghoussoub is extended to functions having the above mentioned structure, it will be pointed out how this approach, in these recent years, allowed us to generalize to the non-smooth case several well known results obtained in the C^1 case. Moreover, the theoretical results will be applied to study some classes of variational or variational-hemivariational inequalities. Finally, the structure of the set of critical points as well as the possibility to weaken the Palais-Smale condition will be also treated.

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Critical Points of Non-Smooth Functions with a Weak Compactness Condition

Salvatore Marano
University of Catania, Italy
(D. Motreanu)

In the framework of nondifferentiable functionals expressed as a locally Lipschitz continuous term plus a convex, proper, lower semicontinuous function, a critical point result is established under a nerw weak Palais-Smale hypothesis, which contains the so-called Cerami condition. Some meaningful special cases are then pointed out.

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Vibrations of Simple Mechanical Systems with Nonsmooth Potentials

Stanisław Migórski
University of Krakow, Poland

Two mathematical models which describe the vibrations of spring-mass-damper systems with contact and friction are considered. The models lead to second order differential inclusions without and with integral term. We provide results on the existence and uniqueness of solutions.

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Infinitely Many Critical Points of Non-Differentiable Functions and Applications

Giovanni Molica Bisci
University of Reggio Calabria, Italy

For a family of functionals in a Banach space X, which are possibly non-smooth, we show the existence of a sequence of critical points by using a recent result of G. Bonanno and G. Molica Bisci [Bound. Value Probl. 2009, 1-20]. Our abstract result is a more precise version of an infinitely many critical points theorem of S. A. Marano and D. Motreanu [Journal of differential Equations, vol. 182, n. 1, 108-120, 2002]. Several special cases and consequences are pointed out. Two applications are then presented. More precisely, under an appropriate oscillating behavior of the potential, the existence of infinitely many solutions for a Dirichlet and Sturm-Liouville boundary value problems is obtained.

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Multiple Solutions with Sign Information for Elliptic Equations and Systems

Dumitru Motreanu
University of Perpignan, France

We prove multiplicity results for quasilinear elliptic equations and systems providing sign information for the solutions. Specifically, we obtain constant sign and sign changing solutions. Our results are established by using a combination of variational, sub-supersolution and topological methods. Elliptic inclusions with nonsmooth potential are also examined.

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Multiple Solutions of Elliptic Problems with Measure Data Via Nonsmooth Analysis

Claudio Saccon
University of Pisa, Italy
(A. Ferrero)

We present some existence and multiplicity results for solutions of elliptic problems of the type $-\Delta u = g(x,u) + \mu$ where μ is a Radon measure. The function $g(x,s)$ is asymptotically linear as $|s| \to +\infty$ and resonant situations are allowed. We also deal with some perturbation results for problems of the same type when the semilinear term $g(x,u)$ depends on some parameter. To properly set the above mentioned problems in a variational framework we are are lead to consider a class of abstract nonsmooth functional defined on Banach spaces and extend to this nonsmooth framework some classical linking theorems.

This is a joint work with A. Ferrero, to appear in Advanced Nonlinear Studies.

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A Variational Approach to Semilinear Elliptic Variational Inequalities with Dependence on the Gradient

Raffaella Servadei
University of Calabria, Italy

In this talk I will consider a semilinear variational inequality with gradient-dependent nonlinear term. Obviously the nature of this problem is non-variational. Nevertheless the problem was studied via variational methods. The existence of a non-trivial non-negative weak solution u is obtained by associating a suitable semilinear variational inequality, variational in nature, with the equation, and by performing an iterative technique. The Lewy-Stampacchia estimates and the regularity theory for elliptic equation allow to show that u is differentiable and its gradient is α-Hölder continuous on $\overline{\Omega}$ for any $\alpha \in (0,1)$. These results appeared in a joint paper with Michele Matzeu.

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Local C^1-Minimizers Versus Local $W^{1,p}$-Minimizers of Nonsmooth Functionals

Patrick Winkert
TU Berlin, Germany

The aim of this talk is the study of nonsmooth functionals $J : W^{1,p}(\Omega) \to R$ with $1 < p < \infty$ involving locally Lipschitz functions $j_1 : \Omega \times R \to R$ as well as $j_2 : \partial\Omega \times R \to R$. We show that local $C^1(\overline{\Omega})$-minimizers of J must be local $W^{1,p}(\Omega)$-minimizers of J.

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Special Session 65: Nonlinear Partial Differential Equations and Applications

Eun Heui Kim, California State University Long Beach, USA
Chung-Min Lee, California State University Long Beach, USA

Introduction: This session will bring together analytical and numerical experts to discuss the current development in nonlinear partial differential equations. The session will cover broad ranges of topics including mathematical problems for conservation laws, mixed-type problems, and various applications.

Existence of Multiple Positive Solutions to Some Semipositone Systems

Maya Chhetri
UNC Greensboro, USA
(Sarah Raynor and Stephen Robinson)

In this paper we use the method of upper and lower solutions combined with degree theoretic techniques to prove the existence of multiple positive solutions to some semipositone, superlinear elliptic systems of the form

$$\begin{aligned} -\Delta u &= g_1(x,u,v) \\ -\Delta v &= g_2(x,u,v) \end{aligned}$$

on a smooth, bounded domain $\Omega \subset R^n$ with Dirichlet boundary conditions, under suitable conditions on g_1 and g_2. Our techniques apply generally to subcritical, superlinear problems with a certain concave-convex shape to their nonlinearity.

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Existence of Algebraic Vortex Spirals

Volker Elling
University of Michigan, Ann Arbor, USA

In incompressible inviscid flow, vortex sheets are curves across which the tangential velocity is discontinuous, with zero vorticity away from the curve. They occur in many important applications, for example from the trailing edge of wings in accelerating aircraft. Most such flows have rollup of vortex sheets into algebraic spirals. Although exact solutions with logarithmic spirals have been known since Prandtl, all attempts at rigorous existence proofs of the more relevant algebraic spirals have been unsuccessful. A proof for at least one example will be presented in this talk.

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Well-Balanced High Resolution Fv Schemes for the Simulation of Wave Propagation in 3-Dimensional Non-Isothermal Stratified Magneto-Atmospheres and the Effects of Radiation

Franz Fuchs
CMA, University of Oslo, Norway
(Andrew D. McMurry, Siddhartha Mishra, Nils H. Risebro, Knut Waagan)

We simulate and study wave propagation in stellar atmospheres, in particular the atmosphere of the Sun. An important feature of the solar atmosphere

is that the temperature decrease from the core is reversed in the outer atmosphere. The heating of the atmosphere seems to violate the second law of thermodynamics and is known as the coronal heating problem. This phenomenon is only just now becoming understood, using data from modern solar observation satellites, and particularly from the results of numerical modeling. There are two important energy transport mechanisms in the solar atmosphere; convection generated waves and radiative transfer.

Wave propagation. One of the important energy carriers in the solar atmosphere are convection generated waves from the inner layers of the sun. They transport and deposit energy in the overlaying chromospheric and coronal plasmas. The waves interact with complex magnetic fields generated by the plasma, and these interactions effect the qualitative as well as the quantitative features of the energy transfer. We model this configuration based on the ideal magneto-hydrodynamic equations (MHD), augmented by a gravitational source term.

The simulation of wave propagation in stratified non-isothermal magnetic atmospheres is far from trivial. One obvious reason is the exponentially decaying structure of the pressure of the underlying steady state, giving rise to near vacuum states. Consequently, standard numerical schemes were observed to crash due to slight oscillations, leading to states of negative pressure.

In this talk, we present a robust high-resolution, well-balanced finite volume based scheme along with appropriate boundary conditions, implemented in a massively parallel code. Furthermore, we describe numerical experiments in both two and three space dimensions involving a realistic steady state temperature and magnetic field dstributions. Diverse phenomena like mode mixing, wave acceleration at the transition region, wave focusing due to the magnetic fields and movement of the transition region are highlighted. The numerical experiments also include a simulation based on observed magnetic fields and boundary conditions and illustrate the robustness of the new computational framework.

Radiative Transfer. The equations of MHD model the processes in the photosphere and chromosphere fairly well. But in order to have an appropriate model for the corona one has to account for the effects of radiation, playing a vital energy transfer mechanism in this part of the sun. Our model is a combination of the M_1-model of radiative transfer and the MHD equations, where a new source term is added to the energy equation. Even though the resulting system is hyperbolic, the design of numerical schemes is hindered by the fact that the maximum eigenvalues are of the order of the speed of light. Thus, in the new system waves can move much faster than the fast magneto-sonic waves in MHD. Therefore, appropriate semi-implicit schemes have to be devised. This investigation clarifies the importance of the effects of radiation in the corona and is of vital importance in the understanding of the processes in stellar atmospheres in general and of the Sun's corona in particular.

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Complete Synchronization of the Particle and Kinetic Kuramoto Model

Seung Yeal Ha
Seoul National University, Korea
(Taeyoung Ha and Jongho Kim)

In this talk, we will discuss complete phase-frequency synchronization for the particle and kinetic Kuramoto phase models. We present sufficient conditions for initial configurations leading to the exponential time-decay toward completely synchronized states characterized by initial configurations and natural frequencies. For the kinetic model, we provide the global existence of measure-valued solutions and their asymptotic behavior. This is a joint work with Taeyoung Ha and Jong-Ho Kim.

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Singular and Quasi-Regular Solutions to Protter Problem for Weakly Hyperbolic Equations

Tsvetan Hristov
University of Sofia, Bulgaria
(Nedyu Popivanov (University of Sofia, Bulgaria) and Manfred Schneider (University of Karlsruhe, Germany))

In 1952, at a conference of AMS, M. Protter formulated new boundary value problems for a class of hyperbolic-elliptic equations, which are 3-D analogue of the Morawetz-Guderley plane transonic problem. For weakly hyperbolic equations they are 3-D analogues of the Darboux problems on the plane. In the contrast of the well-posedness of the Darboux problem in 2-D case, the new problems are strongly ill-posed. We study such Protter problem for weakly hyperbolic equation with power-type degeneration in 3-D simply connected domain, bounded by two characteristic surfaces and by a region in the plane of parabolic degeneration of the equation. Under Protter condition for lower order terms we consider the BVP, with the Dirichlet's condition on the outward characteristic surface and third BV data on non-characteristic part of the boundary. We find a smooth right-hand side function for which the corresponding unique generalized solution has strong power-type singularity isolated at the vertex of the inner characteristic sur-

face. This singularity does not propagate along the bi-characteristics and does not depend on power of degeneration. Further we find sufficient conditions for uniqueness of quasi-regular solution to the considered problem. This work is partially supported by the Bulgarian NSF under Grant DO 02-115/08.

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Vorticity Discontinuities and a Burgers-Hilbert Equation

John Hunter
University of California at Davis, USA
(Joseph Biello)

We analyze the motion of discontinuities in vorticity in two-dimensional, inviscid, incompressible flows. An example is the boundary of a vortex patch. Such discontinuities are linearly stable and support surface waves whose linearized frequency is independent of their wavenumber. We derive an asymptotic equation for the weakly nonlinear evolution of a discontinuity and show that an inviscid Burgers-Hilbert equation provides an effective equation for its motion. The formation of singularities in this equation coincides with the filimentation of the discontinuity, and we compare the asymptotic solutions with contour dynamics solutions of the full problem.

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The Sonic Line as a Free Boundary: Stability under Perturbations

Barbara Keyfitz
The Ohio State University, USA
(Allen Tesdall, Nedyu Popivanov, Daniela Lupo and Kevin Payne)

The study of self-similar solutions of multidimensional conservation laws leads to systems of equations that change type. Change of type occurs either across a transonic shock or at a sonic line. Often the sonic line appears as a free boundary in the formulation of the problem. Some recent numerical (and experimental) discoveries of a new kind of shock reflection ('Guderley Mach reflection') lead to interesting and still unresolved questions concerning the nature of the self-similar solutions in this generic case.

In this talk, I will present some analysis of a simple model for this phenomenon, using the transonic small disturbance equation. The simplified problem seems amenable to analysis, but we are just beginning to make progress. This is a report on joint work with Allen Tesdall, Nedyu Popivanov, Daniela Lupo and Kevin Payne.

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Transonic Problems Arising in Multi-Dimensional Conservation Laws

Eun Heui Kim
California State University Long Beach, USA

We discuss transonic problems, which give rise free boundary and degenerate (sonic) boundary conditions, arising in two dimensional conservation laws.

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Direct Numerical Simulation on Inertial Particles in Strained Turbulent Flows

Chung-Min Lee
California State University Long Beach, USA
(A. Gylfason, P. Perlekar and F. Toschi)

We will discuss the influence of straining in turbulent flows on inertial particles using the direct numerical simulation with the Rogallo algorithm. The numerical methods for simulating strained turbulent flows and for tracking inertial particles will be described. Some preliminary acceleration statistics of particles with varied Stokes numbers will be presented and discussed.

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Non-Existence of Solutions for Nonlinear Equations of Mixed Elliptic-Hyperbolic Type

Daniela Lupo
Politecnico di Milano, Italy
(Kevin R. Payne (Universitá di Milano) and Nedyu Popivanov (University of Sofia))

Non-existence results of solutions to semilinear equations of mixed type that satisfy Dirichlet boundary conditions on all or part of the boundary will be discussed. Several classes of domains and equations will be presented. The results represent an ongoing collaboration with Kevin R. Payne (Universitá di Milano) and Nedyu Popivanov (University of Sofia).

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Existence of Weak Solutions for Nonlinear Equations of Mixed Elliptic-Hyperbolic Type

Kevin Payne
Universitá di Milano, Italy
(Daniela Lupo (Politecnico di Milano) and Dario Monticelli (Universitá di Milano))

Abstract: The question of existence of weak solutions to semilinear equations of mixed type that satisfy Dirichlet boundary conditions on all or part of

the boundary will be discussed. In particular, the use of variational and topological methods based on the availability of suitable information on the linear part will be examined. The results represent an ongoing collaboration with Daniela Lupo (Politecnico di Milano) and Dario Monticelli (Universitá di Milano).

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Semi-Fredholm Solvability of $(3+1)-D$ Protter Problems

Todor Popov
University of Sofia, Bulgaria

(Nedyu Popivanov (University of Sofia) and Rudolf Scherer (University of Karlsruhe))

We discuss four-dimensional boundary value problems for the nonhomogeneous wave equation, that were proposed by M. H. Protter as multidimensional analogues of Darboux problems on the plane. In the frame of classical solvability the Protter problem is not Fredholm, since it has infinite-dimensional cokernel. Alternatively, the notion of generalized solution was introduced. It is known that the generalized solution may have a strong power-type singularity at one boundary point. This singularity is isolated at the vertex of the characteristic cone and does not propagate along the cone. Explicit asymptotic expansion of the generalized solution is found for the case when the right-hand function is a harmonic polynomial. In the general case necessary and sufficient conditions for the existence of regular and bounded solution are given. When these conditions are not fulfilled, a right-hand side function of the wave equation is constructed such that the generalized solution has exponential type singularity.

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High-Resolution Solutions for Shock Formation in Transonic Flow

Allen Tesdall
City Univ. of N.Y., College of Staten Island, USA

Shock waves that form as the result of an interaction of a rarefaction wave with a sonic line are a generic feature of solutions of transonic flow problems. Examples include (i) the sequence of shocks that occur in Guderley Mach reflection, (ii) the shock that forms at the rear of a supersonic bubble on an airfoil in a slightly subsonic free stream flow, and (iii) the shock wave that forms when a supersonic flow hits the corner of an expanding duct. Whether the shock forms on the sonic line or inside the supersonic region appears to be an open question. We present high-resolution numerical solutions of problems for the steady and unsteady transonic small disturbance equations that describe examples (ii) and (iii) above. Our solutions show that the shock forms strictly inside the supersonic region. These results appear to be the first that clearly show the supersonic nature of the shock formation point.

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Special Session 66: Discrete Integrable Systems

Stephane Lafortune, College of Charleston, USA
Kenichi Maruno, The University of Texas-Pan American, USA

Introduction: Essentially, integrability is the field that studies nonlinear equations for which the dynamics can be determined for long periods of times. While the property of integrability for differential equations has been widely studied, the theory of integrable discrete systems is a relatively new field of research. Just like differential equations however, finite-difference equations appear in a wide range of application. The goal of this mini-symposium is to bring together specialists of this field of research. We will discuss different existing notions of integrability in the discrete and ultra-discrete cases and see how these can be used for studying equations arising in applications.

Quadrilateral Lattices, Desargues Maps, and Their Affine Weyl Group Symmetry

Adam Doliwa
University of Warmia and Mazury, Poland

I will present new incidence geometric description of the (non-commutative) Hirota-Miwa system in terms of the so-called Desargues maps, and its equivalence to the quadrilateral lattice theory. The symmetry of the Desargues theorem, which is the cornerstone of the multidimensional compatibility of the map naturally brings to light the A_N affine Weyl group symmetry of the system.

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Spectral Curves and the Lagrangian Description of Discrete Integrable Systems

Anton Dzhamay
University of Northern Colorado, USA

I will discuss some recent joint work with I. Krichever on understanding the Lagrangian structure of discrete integrable systems in terms of the spectral curve data associated to the discrete Lax representation of such systems. Specifically, we consider certain elementary transformations on the space of rational Lax matrices, give geometric characterization of such transformation using the spectral curve, and then use it to find their generating Lagrangian function.

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The Semi- and Full-Discretizations for the Short-Wave Model of the Camassa-Holm Equation

Baofeng Feng
The University of Texas-Pan American, USA
(Ken-ichi Maruno and Yasuhiro Ohta)

In this talk, semi- and full-discretizations for the short-wave model of the Camassa-Holm equation (SCHE) will be proposed based on Hirota's bilinear method. The above equation is of importance in physics, and admit cuspon solutions. First, we will show the link between these soliton equations and the two-dimensional Toda-lattice through the hodograph transformations. Then, the integrable semi-discretizations will be derived from the semi-discretization of 2DTL. Further, the full-discretizations will be deduced by using the Bäcklund transformations of 2DTL.

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On the Noncommutative and Discrete BKP Equations

Claire Gilson
Glasgow University, Scotland

We introduce the notion of the quasi-pfaffian which is a non-commutative form of a pfaffian and describe its properties. These are of interest because they are solutions to the (discrete) BKP equations, but, unlike ordinary pfaffians they offer the prospect of allowing solutions to the non-commutative extensions of these equations. We report progress toward this goal.

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Discrete Integrable Systems and the Self-Dual Yang-Mills Equations

Rod Halburd
University College London, England

The self-dual Yang-Mills equations are sometimes referred to as the master integrable system as very many integrable differential equations can be obtained by reduction (such as KdV, NLS, Ernst, and the Painlevé equations). Methods for obtaining discrete integrable systems from the self-dual Yang-Mills equations will be discussed.

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Hirota's Direct Method and the Three-Soliton Condition

Jarmo Hietarinta
University of Turku, Finland
(Da-jun Zhang)

Integrable PDEs are characterized by many interesting properties, one of which is the existence of multi-soliton solutions. The construction of these solutions is particularly simple using Hirota's direct method. It turns out that although many equations have one and even two-soliton solutions, the existence of three-soliton solutions is in practice equivalent to integrability. This provides a method for searching for integrable equations. We briefly review the situation in the continuum case and present some new results in the discrete case.

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Lax Pairs and Spectral Curves for Somos Sequences

Andrew Hone
University of Kent, UK, England

Somos sequences are integer sequences generated by bilinear difference equations. The aim of this talk is to explain how these bilinear recurrences are related to maps of QRT type and their higher order analogues. In general, an arbitrary bilinear recurrence of this kind does not correspond to an integrable map. However, all known integrable cases can be obtained as reductions of discrete KP (the Hirota-Miwa equation) or discrete BKP (Miwa's equation), and this leads to Lax pairs and spectral curves associated to Somos sequences.

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Integrable Systems Via Conservation Laws

Peter Hydon
University of Surrey, England
(Claude-M. Viallet)

Integrable difference equations commonly have more low-order conservation laws than occur for nonintegrable difference equations of similar complexity. We use this empirical observation to sift a large class of difference equations, in order to find candidates for integrability. The candidates can all be written in affine linear form, so we have tested their integrability by calculating their algebraic entropy.

In this way, we have found several integrable equations, none of which are in the Adler-Bobenko-Suris classifications. Indeed, one of the equations seems to be entirely new. A useful by-product of this method is a complete classification of single-tile conservation laws.

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Integrable Discretizations of the Short Pulse Equation

Kenichi Maruno
University of Texas-Pan American, USA
(Bao-Feng Feng, Yasuhiro Ohta)

The short pulse (SP) equation was derived recently as a model equation for the propagation of ultra-short optical pulses in nonlinear media. The SP equation admits various interesting exact solutions such as loop soliton solutions and breather solutions.

Discrete integrable systems have received much attention recently because of many applications to numerical algorithms, computer visualization, mathematical physics, etc. In the study of discrete integrable systems, finding integrable discretizations of continuous integrable systems such as soliton equations is one of keys to explore the world of discrete integrable systems. Although integrable discretizations for many of soliton equations such as the KdV, mKdV, NLS equations were found, integrable discretizations of some classes of soliton equations having solutions with singularities (e.g. the SP equation) are still missing.

In this talk, we propose new integrable semi-discretization and fully discretization of the SP equation by using the method proposed by us recently. These discrete equations have various exact solutions which correspond to exact solutions of the continuous SP equation. Using the semi-discrete analogue of the SP equation, we perform numerical computations of loop solitons.

⟶ ∞ ⋄ ∞ ⟵

Constructing Two Dimensional Integrable Mappings Which Possess Invariants of High Degree

Junta Matsukidaira
Ryukoku University, Japan
(Hironori Tanaka, Atsushi Nobe, Teruhisa Tsuda)

We propose a method for constructing two-dimensional integrable mappings that possess invariants with degree higher than two. Such integrable mappings are obtained by making a composition of a QRT mapping and a mapping that preserves the invariant curve of the QRT mapping except for changing the integration constant involved. We show several concrete examples whose integration constants change with period 2 and 3.

⟶ ∞ ⋄ ∞ ⟵

Bidifferential Calculus and Discretizations of Matrix NLS Equations

Folkert Mueller-Hoissen
MPI for Dynamics and Self-Organization, Germany
(A. Dimakis)

In the bidifferential calculus (or bidifferential graded algebra) approach to integrable partial differential or difference equations (see A. Dimakis and F. Müller-Hoissen, Discr. Cont. Dyn. Systems Suppl. 2009, 208-219), we describe matrix NLS equations and their (Ablowitz-Ladik-type) discretizations. The step from the bidifferential calculus for the continuous NLS equation to a corresponding discrete version turns out to be very simple. Applying a general result to the continuous and discrete NLS cases then quickly generates a large class of exact solutions, including the matrix solitons in the focusing NLS case.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Ultradiscrete Analogue of Determinant Solution for Soliton Equations

Hidetomo Nagai
Waseda University, Japan

A new form of solutions to some ultradiscrete soliton equations are presented recently. It is called ultradiscrete permanent, which is defined by ultradiscretizing the permanent, that is, signature-free determinant. In this talk, we show properties and relations of the ultradiscrete permanents. Moreover, we prove ultradiscrete soliton solutions by means of an ultradiscrete analogue of Plücker relation.

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Sufficient Number of Integrals of the Lyness Equation of Arbitrary Order

Reinout Quispel
La Trobe University, Australia

We present a sufficient number of explicit first integrals for the Lyness equation of arbitrary order. We first use the staircase method to construct integrals of a derivative equation of the Lyness equation. Closed-form expressions for the integrals are given based on a non-commutative Vieta expansion. The integrals of the Lyness equation then follow directly from these integrals. Previously found integrals for the Lyness equation arise as special cases of our new set of integrals.

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On Some Particle Systems and Ultradiscretization

Daisuke Takahashi
Waseda University, Japan

Some continuous systems are directly related to particle systems through ultradiscretization. For example, we obtain the elementary cellular automaton of rule 184, the box and ball system and the ultradiscrete Toda equation by ultradiscretizing the Burgers equation, the KdV equation and the Toda equation respectively. In my talk, I will report some particle systems of cellular automaton type. The space and time variables are both discrete and the dependent variable is binary in the systems. Their common features are the conservation of the number of particles and the direct relation to the continuous systems, for example, a system of ordinary differential equations or a partial differential equation. Moreover, behavior of solutions, mathematical features and physical aspect of each system will also be discussed.

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Toropical Geometry and Integrable Ultra-Discrete Dynamical Systems

Tomoyuki Takenawa
Tokyo University of Marine Sci. and Tech., Japan

I will review the recent developments of toropical geometry for the study of integrable ultra-discrete dynamical systems, including the box and ball system and ultra-discrete Toda lattice. Especially I will talk about the tropical version of Fay's trisecant identity for toropical theta functions.

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Special Session 68: Differential, Integral Equations and Their Application

Onur Alp Ilhan, Erciyes University, Turkey
Mokhtar Kirane, Universite de La Rochelle, La Rochelle, France
Nasser-Eddine Tatar, King Fahd University, Saudi Arabia

On Some Parabolic Equations Involving $p(x)$-Laplacian

Goro Akagi
Shibaura Institute of Technology, Japan

We treat some parabolic equations involving the $p(x)$-Laplacian given by

$$\Delta_{p(x)}\varphi(x) := \operatorname{div}\left(|\nabla\varphi(x)|^{p(x)-2}\nabla\varphi(x)\right)$$

for $x \in \Omega \subset \mathbb{R}^N$ with a function $p : \Omega \to [1, \infty)$. This talk is based on the joint work with Kei Matsuura (Waseda University, Tokyo).

$\longrightarrow \infty \diamond \infty \longleftarrow$

On Pullback Attractors for Two-Dimensional Turbulent Shear Flows with Tresca Law

Mahdi Boukrouche
LaMUSE, Saint-Etienne University, France
(G. Lukaszewicz)

We consider a two-dimensional Navier-Stokes shear flow with time dependent boundary driving and subject to Tresca law. We establish the existence of a unique global in time solution of the considered problem and then, using a recent method based on the concept of the Kuratowski measure of noncompactness of a bounded set, we prove the existence of the pullback attractor for the associated cocycle. This research is motivated by a problem from lubrication theory.

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Volterra Equations and Anisotropic Image Denoising

Eduardo Cuesta
University of Valladolid, Spain
(M. Kirane and S. A. Malik)

A new approach to anisotropic image denoising/filtering based on Volterra type equations is considered. The new approach lies in a closed mathematical framework, i. e the well posedness and numerical solvability have largely studied and this do not represent a gap in our model. Moreover our model enjoys very good filtering properties as show the experiments we will provide. These properties are inherited from the inner nature of the Volterra equation we consider which is based on a generalization of time fractional derivatives and integral.

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On the Solvability Conditions for One Partial Integral Equation

Onur Alp Ilhan
Erciyes University, Turkey

A Volterra type integral equation in Hilbert space with an additional linear self-adjoint operator L and a spectral parameter depending on time is proposed. In the case where the value of the parameter at some moment coincides with an isolaled of the spectrum of the operator L sufficient conditions for solvability are established. The obtained results are applied to the partial integral equations associated with a contact problem of the theory of elasticity.

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Dynamic Boundary Conditions as Limit of Singularity Perturbed

Angela Jimenez-Casas
Universidad Pontificia Comillas, Spain
(Anibal Rodriguez-Bernal)

We obtain a dynamic boundary conditions for a problem as a limit of a parabolic problem with flux null and a singularity in the variation on the boundary.

Let Ω be an open bounded smooth set in R^N with a C^2 boundary $\Gamma = \partial\Omega$. Define the strip of width ε and base Γ as

$$\omega_\varepsilon = \{x - \sigma\vec{n}(x),\ x \in \Gamma,\ \sigma \in [0, \varepsilon)\}$$

for sufficiently small ε, say $0 \le \varepsilon \le \varepsilon_0$, where $\vec{n}(x)$ denotes the outward normal vector. We note that for small ε, the set ω_ε is a neighborhood of Γ in $\bar{\Omega}$, that collapses to the boundary when the parameter ε goes to zero and we denote by $\mathcal{X}_{\omega_\varepsilon}$ the characteristic function of the set ω_ε.

We consider the family of parabolic problems (1) where a term is concentrated in a neighborhood of Γ and this neighborhood shrinks to Γ as a parameter ε goes to zero

$$(P_\varepsilon) \equiv \begin{cases} \dfrac{1}{\varepsilon}\mathcal{X}_{\omega_\varepsilon} u_t - \Delta u + \lambda u = 0 & \\ & \text{in } \Omega \times (0,\infty) \\ \dfrac{\partial u}{\partial n} = 0 & \text{on } \Gamma \times (0,\infty) \\ u(x,0) = u_0(x). & \end{cases} \quad (1)$$

We prove the limit in "some sense" of problem (1) when the parameter ε goes to zero, is

$$(P_0) \equiv \begin{cases} -\Delta u + \lambda u & = 0 \quad \text{in } \Omega \times (0,\infty) \\ u_t + \dfrac{\partial u}{\partial n} & = 0 \quad \text{on } \Gamma \times (0,\infty) \end{cases} \quad (2)$$

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On Operators of Higher Order and Their Application to Non-Linear Differential and Integral Equations

Piotr Kasprzak
Adam Mickiewicz University, Poznan, Poland

It appears that solutions to many differential and integral equations which describe some physical phenomena are functions of bounded variation. On the spaces of real-valued functions of bounded variation we will consider so-called mappings of higher order, that is mappings satisfying the following condition: for any positive number L there exists a closed ball centred at zero on which the mapping in question satisfies the Lipschitz condition with the constant L. We will present some general theorems concerning the behaviour of mappings of higher order as well as we will show some necessary and sufficient conditions for autonomous Niemytzki operator and Hammerstein integral operator to be such mappings. Furthermore, we will provide examples of applications of mappings of higher order to uniqueness and existence results for perturbed differential and integral equations in the spaces of functions of bounded variation.

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A Boundary Value Problem for the Einstein Equations

Marcus Khuri
Stony Brook University, USA

We will discuss existence and regularity for a geometrically motivated boundary value problem associated with the static vacuum Einstein equations. Physical applications will also be highlighted.

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Nonlinear Elliptic Equations with General Potentials in $\mathbb{R}^N$

Athanasios Lyberopoulos
University of the Aegean, Greece

We present existence results for bound states and ground states for a class of nonlinear elliptic equations with potentials which may be unbounded or decaying to zero at infinity.

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Definition of t-Patterns and Their Detection in Behavior, Neuronal Interactions, and DNA

Magnus Magnusson
Human Behavior Lab., University of Iceland

The detection of hidden repeated patterns in real-time nonverbal and verbal streams of behavior whether in humans or animals are a challenge requiring pattern detection algorithms at least as powerful as used in DNA analysis. Pattern type hypotheses are obviously a precondition for such algorithms. A pattern-type, called a t-pattern, with derived types has been defined together with detection algorithms implemented in available software (Theme; www.noldus.com) that has been used for the detection of human, animal and neuronal interaction patterns. The data type, called t-data, is a set of time point series within an observation interval where each series represents the occurrence of a particular type of event, which in the case of behavior is typically the onset or offset of a particular act or action by some agent. In recent studies of neuronal firing (behavior) in a population of neurons each t-data series is the firing of a particular neuron. In both human, animal, and neuronal behavior and interactions complex repeated patterns have been detected while similarities between t-patterns and motives and genes in DNA are striking. Questions regarding kinds of systems that would generate such patterns have not yet been answered. For a list of publications see www.hbl.hi.is.

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General Decay for the Solution of a Viscoelastic Wave Equation with a Nonlinear Damping

Salim Messaoudi
KFUPM, Saudi Arabia

In this paper we consider the nonlinearly damped viscoelastic equation of the form

$$u_{tt} - \Delta u + \int_0^t g(t-\tau)\Delta u(\tau)d\tau + a(x)|u_t|^{m-2}u_t = 0$$

in a bounded domain. Here $a > 0$ is a constant and g is the relaxation function We discuss the rate of decay when $m > 1$ and g is uniformly decaying. In particular, we show, for certain class of relaxation function, that the rate of decay of the energy is exactly the rate of decay of g provided that $m \geq 2$. This result improves some existing results, in which g is only of exponential or polynomial decay rate.

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On Integral Equations and Convolutional Decomposition of Certain Density Functions

Katsuo Takano
Ibaraki University, Japan

The speaker will talk on integral equations and a property of Bessel functions and convolutional decomposition of certain density function. The speaker is interested in the behavior of kernel function of an integral equation of the following type: For a given probability density function $g(x)$ which is defined on the interval $(0, \infty)$, $g(x)$ satisfies

$$xg(x) = \int_0^x g(x-y)k(y)dy, \ x > 0.$$

If the density function $g(x)$ is a simple function we want to obtain the function $k(y)$. In the case of the Student t distrbution with degree of freedom 5, there appears a function given by the left hand side of the following

$$\frac{2}{\pi^2 v(J^2_{5/2}(v) + Y^2_{5/2}(v))} = \frac{1}{\pi}\{1 - \frac{3^2 + 3v^2}{3^2 + 3v^2 + v^4}\}$$

for $v > 0$, where $J_{5/2}(v)$, $Y_{5/2}(v)$ are the Bessel functions of first kind and second kind with order $5/2$. From the right hand side, a convolutional decomposition of the Cauchy density function can be shown and we find a convolutional decomposition of the heigher dimensional Cauchy density function.

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Slow Down of the Decay in Viscoelasticity

Nasser-Eddine Tatar
King Fahd University, Saudi Arabia

It is known that when we add a viscoelastic damping to a frictional damping acting in the domain we might lose the property of exponential stability of the system. Moreover, a necessary condition for a system to be sub-exponentially stable is that the kernel itself must be sub-exponentially decaying to zero. Having this in mind, a natural question to be asked is when is this necessary condition also sufficient. In this talk we will discuss this matter and prove that this is the case for a fairly large class of kernels.

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Factorization of Schrödinger Operators and Pseudoanalytic Functions

Sébastien Tremblay
University of Québec (Trois-Rivières), Canada
(Vladislav V. Kravchenko)

Using the theory of pseudoanalytic functions developped independently by Bers and Vekua, we consider the stationary two-dimensional Schrödinger equation. With the aid of any of its particular solutions, we construct a Vekua equation (generalization of the Cauchy-Riemann equations) possessing the following special property. The real parts of its solutions are solutions of the original Schrödinger equation and the imaginary parts are solutions of an associated Schrödinger equation with a potential having the form of a potential obtained after the Darboux transformation. Using Bers' theory of Taylor series for pseudoanalytic functions, we obtain a locally complete system of solutions of the original Schrödinger equation which can be constructed explicitly for an ample class of Schrödinger equations.

In this talk we will present some new results for the "standard" two-dimensional pseudoanalytic functions and for higher dimensional pseudoanalytic functions using quaternions and Clifford algebras.

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Special Session 69: Non-local Equations in Biology

Yuan Lou, Ohio State University, USA
Salome Martinez, Universidad de Chile

About Stability Analysis for a Diffusion Equation Parameterized by Nonlocal Interactions

Armel Andami Ovono
University of Picardie, France

We consider a diffusion model in which the dstribution is dependent on the extent of long-range interactions and address the question of asymptotic behaviour of the population. We prove firstly the existence, uniqueness and L^∞ estimate from L^p estimate of the parabolic equation by using Moser iterations. Secondly we study the associated stationary equation and generalize a result of Chipot-Lovat determining the number of stationary solutions. We finish our study by giving in some cases the asymptotic behavior of the solution for large time.

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Asymptotic Behavior for a Non-Local Diffusion Equation in an Exterior Domain

Carmen Cortazar
P. Universidad Catolica de Chile
(M. Elgueta, F. Quiros and N. Wolanski)

We study the asymptotic behavior, as t goes to infinity, of the solution of a non-local diffusion equation, with Dirichlet boundary conditions, in the exterior of a bounded domain.

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Evolutionary Stability of Ideal Free Nonlocal Dispersal

Chris Cosner
University of Miami, USA
(J. Dávila and S. Martínez)

This talk will present results on the evolutionary stability of nonlocal dispersal strategies that can produce ideal free population dstributions, that is, dstributions where all individuals have equal fitness and there is no net movement of individuals at equilibrium. We find that the property of producing ideal free dstributions is necessary and often sufficient for evolutionary stability. The results extend those already developed for discrete diffusion models on finite patch networks to the case of nonlocal dispersal models based on integrodifferential equations. The analysis is based on the use of comparison methods and the construction of sub- and supersolutions.

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Harnack Inequality for Nonlocal Operator

Jerome Coville
INRA- Avignon (BIOSP), France

I will present some Harnack's inequalities satisfied by any positive solution of some linear nonlocal equation that arise in the modelling of the dispersal of the individuals through their environment. As a first application of these estimates, I will present the construction of the principal eigenfunction for some particular nonlocal eigenvalue problem.

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Periodic Fronts for Nonlocal Equations

Juan Davila
U. de Chile - CMM, Chile
(Jerome Coville and Salome Martínez)

We construct periodic fronts for the evolution equation

$$u_t = J * u - u + f(x,u) \qquad t \in \mathbb{R},\ x \in \mathbb{R}^N$$

where J is a smooth, non-negative symmetric kernel in $\mathbb{R}^N$ with compact support and mass one, and f is periodic in x, and KPP in u with some additional assumptions. We assume that the corresponding stationary problem has a positive periodic solution. Then for any unit vector $e \in \mathbb{R}^N$ there is $c^*(e)$ such that for $c \geq c^*(e)$ there is pulsating front propagating in the direction $-e$ with effective speed c, that is, a solution satisfying

$$u(t + k \cdot e/c, x) = u(t, x+k) \quad \forall t \in \mathbb{R},\ x \in \mathbb{R}^N \quad \forall k \in \mathbb{Z}^N$$

and

$$u(t,x) \to 0 \quad \text{as } t \to -\infty, \quad \text{for all } x$$

$$u(t,x) \to p(x) \quad \text{as } t \to +\infty, \quad \text{for all } x$$

where p is the positive stationary solution.

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Asymptotic Behavior of Solutions to Inhomogeneous Nonlocal Diffusion Problems

Jorge Garcia-Melian
Universidad de La Laguna, Spain

(Carmen Cortazar, Manuel Elgueta, Salome Martínez)

We consider the nonlocal evolution Dirichlet problem

$$\begin{cases} u_t(x,t) = \displaystyle\int_\Omega J\left(\frac{x-y}{g(y)}\right)\frac{u(y,t)}{g(y)^N}dy - u(x,t) \\ \qquad x \in \Omega,\ t > 0 \\ u = 0 \qquad x \in \mathbb{R}^N \setminus \Omega,\ t \geq 0 \\ u(x,0) = u_0(x) \qquad x \in \mathbb{R}^N, \end{cases}$$

where Ω is a bounded domain in $\mathbb{R}^N$, J is a Hölder continuous, nonnegative, compactly supported function with unit integral and $g \in C(\overline{\Omega})$ is assumed to be positive in Ω. We discuss existence, uniqueness and asymptotic behavior of solutions as $t \to +\infty$. Moreover, we prove the existence of a positive stationary solution when the inequality $g(x) \leq \delta(x)$ holds at every point of Ω, where $\delta(x) = \text{dist}(x, \partial\Omega)$. The behavior of positive stationary solutions near the boundary is also analyzed.

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Single Phytoplankton Species Growth with One Nutrient and Light in a Water Column

Sze-Bi Hsu
National Tsing-Hua University, Taiwan

(Lou Yuan)

In this talk we shall present a mathematical model of a single phytoplankton species consuming two complementary resources, a nutrient R and light I in a water column. We shall do the steady state analysis for the model. First we consider the case that the species u is I-limited on the water column $[0,1]$. Then we consider the species u is R-limited in $[0,1]$. The spatial effect will be discussed. The bifurcation will be discussed in the parameters region in R_0 and I_0 where R_0 is the nutrient input from the bottom of the water column and I_0 is the light intensity at the surface of the water column.

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Random Dispersal vs. Non-Local Dispersal

Yuan Lou
Ohio State University, USA

Random dispersal is essentially a local behavior which describes the movement of organisms between adjacent spatial locations. However, the movements and interactions of some organisms can occur between non-adjacent spatial locations. To address the question about which dispersal strategy can convey some competitive advantage, we consider a mathematical model consisting of one reaction diffusion equation and one integrodifferential equation, in which two competing species have the same population dynamics but different dispersal strategies: the movement of one species is purely by random walk while the other species adopts a non-local dispersal strategy. Our results suggest that for spatially periodic heterogeneous environments, the competitive advantage may belong to the species with much lower effective rate of dispersal, but not necessarily so for random dispersal strategy with either zero Dirichlet or zero Neumann boundary condition compared with non-local dispersal strategy with hostile surroundings. This talk is based on joint works with Chiu-Yen Kao and Wenxian Shen.

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Sign Changing Solutions for a Nonlocal Problem

Salome Martinez
Universidad de Chile

We consider the following nonlocal equation

$$u_t(x,t) = \int_\mathbb{R} J\left(\frac{x-y}{g(y)}\right)\frac{u(y,t)}{g(y)}dy - u(x,t)$$
$$x \in \mathbb{R},\ t \geq 0,$$

where J is an even, compactly supported, Hölder continuous probability kernel, g is a continuous function, bounded and bounded away from zero in $\mathbb{R}$. If $u(x,t)$ is thought of as the density of a species at the point x in time t then $\frac{1}{g(y)}J(\frac{x-y}{g(y)})$ denotes the probability density of moving from point x to y, thus the dispersal is nonhomogeneous. We will study the steady state solutions of this equation, establishing the existence of a positive solution $p(x)$, and the existence of a sign changing steady state solution $q(x)$. This solution satisfies $\kappa_1 \leq \frac{|q(x)|}{|x|} \leq \kappa_2$ for positive constants κ_1, κ_2 and large $|x|$. This work is in collaboration with Carmen Cortázar, Manuel Elgueta and Jorge García Melián.

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A Chemotaxis System with Porous Medium Diffusion and Logistic Source

Youshan Tao
Dong Hua University, Peoples Rep. of China

This talk is mainly concerned with a 2×2 parabolic-parabolic chemotaxis system with porous medium diffusion $\triangle u^m$ and logistic source $\mu u(1-u)$ for the cell density equation. Under the assumption that $m > 3/2$ and $\mu > 0$, the existence and uniform-in-time boundedness of weak solutions is proved in

three space dimensions by establishing some delicate a priori estimates of solutions to an approximate system with non-degenerate diffusion.

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Competition of Microorganism for a Single Limited Resource with Structure of Cell Quota

Feng-Bin Wang
National Tsing-Hua University, Taiwan
(James P. Grover and Sze-Bi Hsu)

In this talk, we first review the following equation:

$$\frac{\partial n(t,q)}{\partial t} + \frac{\partial (g(q)n(t,q))}{\partial q} \qquad (1)$$
$$= -\mu(q)n(t,q) - b(q)n(t,q) + 4b(2q)n(t,2q).$$

Here t denotes time, q stands for the size of an individual cell. n is the population density function, that is, $\int_{q_1}^{q_2} n(t,q)dq$ represents the number of cells with size between q_1 and q_2 at time t. The functions $\mu(q)$, $b(q)$ and $g(q)$ are the rates at which cells of size q die, divide and grow, respectively. The second term at the left hand side (the first term at the right hand side) denotes changes due to the growth (death or dilution) of cells. The last two terms describe the reproduction process. We note that equation (1) has been discussed by Diekmann, Heijmans, and Thieme.

The system (1) can be used to consider a model which describes the competition of n microorganisms for a single limited resource with internal storage structure. After using some properties of semigroup, it can be reduced into a early work by S. B. Hsu. We prove that the Competitive Exclusion Principle holds for our system. Finally, we add the spatial diffusion to our model (with $n = 2$) and it seems that coexistence occurs.

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Special Session 70: Structure and Dynamics of Biochemical Reaction Networks

Maya Mincheva, Northern Illinois University, USA
Maria Leite, University of Oklahoma, USA

Strongly Connected Inflation of Coupled Cell Networks

Nikita Agarwal
University of Houston, USA

A coupled cell system is a network of interacting dynamical systems. The dynamics of large coupled cell systems can be extremely complex and difficult to analyze. One way of approaching the study of such networks is to start with a simple, well-understood but interesting small network and to investigate how the simple network can be naturally embedded in a larger network in such a way that the dynamics of the small network appears in the dynamics of the larger network. This process is termed as inflation (of a network). Strong connectivity plays an important role in understanding the dynamics of coupled cell systems such as synchronization of cells. In this talk, we give necessary and sufficient conditions for the existence of a strongly connected inflation of a strongly connected network. We will describe a simple algorithm for the construction of a strongly connected inflation as a sequence of simple inflations. We will conclude the talk with some examples.

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Switching in Replicator Dynamics

Manuela Aguiar
University of Porto, Portugal

Replicator equations describe evolutionary dynamics and arise very frequently as a model for networks of interacting species in a broad variety of different contexts like theoretical biology and dynamical game theory. The special case of second order replicator equations (dynamically equivalent to Lotka-Volterra equations) is of particular interest. Although their simple form, the dynamics of this equations, restricted to the unit simplex, can be extremely rich. In particular, heteroclinic networks occur as robust phenomena. We consider heteroclinic networks in the boundary of the simplex with equilibria at the vertices. We prove the existence of complicated dynamics – switching dynamics – induced by this kind of networks, that translates in terms of the coexistence of the species along time. This includes joint work with Sofia Castro (University of Porto) and Alexandre Rodrigues (University of Porto).

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Switching in Mass Action Networks Based on Linear Inequalities

Carsten Conradi
MPI Magdeburg, Germany
(Dietrich Flockerzi)

Switching and bistability has been recognized as an important feature of dynamical systems originating in Systems Biology. In the majority of cases bistability of the dynamical system is established numerically. However, parameter uncertainty is a predominant issue in Systems Biology: The dynamical systems consist of a large number of states and parameters, while measurement data are often very noisy and data points and repetitions are usually few. Hence techniques allowing the analytic computation of parameter vectors where a given system exhibits bistability are desirable. Here we present a new sufficient condition for a large class of mass action networks. This condition takes the form of linear inequality systems. And it is constructive in the sense that solutions to one of the inequality systems determine state and parameter vectors where the underlying mass action systems – under explicitly stated genericity conditions – undergo a saddle-node bifurcation.

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Chemical Dynamics and Algebraic Geometry

Markus Eiswirth
Fritz-Haber-Insitut der MPG, Berlin, Germany
(Sonja Sauerbrei)

The kinetic equations describing chemical kinetics typically lead (or can be transformed) to systems of nonlinear differential equations with polynomial expressions on the right-hand side, which are difficult to solve. It is shown that by the use of methods from linear algebra (stoichiometric network analysis) as well as algebraic geometry (polynomial rings, ideal theory, factorization theory) all stationary solutions of such systems can be obtained, and a stability analysis can be carried out. The procedure is illustrated using examples from catalysis and electrochemistry. The applicability of the methods is discussed.

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Periodic Forcing of Systems Near Hopf Bifurcation

Martin Golubitsky
MBI / Ohio State University, USA
(Yanyan Zhang)

The small amplitude periodic forcing of systems near Hopf bifurcation occurs in a number of areas including feedforward chains of identical coupled cells. Such problems are difficult because of the existence of three small parameters: the amplitude of the forcing ε, the deviation of the system from Hopf bifurcation λ, and the deviation of the forcing frequency from the Hopf frequency ω. In this talk I will focus only on the amplitude a of periodic solutions to the forced system that have the same frequency as the frequency of the forcing and discuss the resulting bifurcation diagrams of a as a function of ω for fixed λ and ε. I will also describe some of the complications that occur in feedforward chains due to the complexity of these bifurcation diagrams.

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Generalized Modeling of Biological Networks

Thilo Gross
MPI for the Physics of Complex Systems, Germany

Generalized modeling is a new modeling approach by which some insights on the dynamics of complex nonlinear networks can be gained much more efficiently than with conventional models. A generalized model is a system of differential equations in which the basic structure of gain and loss terms is fixed, but these terms are not restricted to specific functional forms. A single generalized model thus describes a large class of different conventional models. Using a simple normalization procedure the Jacobian matrices corresponding to all possible steady states in this class of models can be parameterized. The generalized model thus identifies a set of parameters that capture the local dynamics close to all possible steady states in the class of models. The parameters are constructed in such the way that they generally have an intuitive interpretation in the context of the model. I illustrate this approach of generalized modeling by several examples, including the mitochondrial TCA cycle.

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Dynamics of Coupled Feedback Loops: Multistability, Oscillations, and Bifurcations

Maria Leite
University of Oklahoma, USA
(Yunjiao Wang)

Complex networks of genes, proteins and enzymes play an important role in modern cellular biology. One of main topics studied in this context is the connection between sub-network structure $-$ *motifs* and their corresponding biological functions. Recently, feedback loops motifs are found to play an important role in the dynamics of complex networks and they have received extensive attention. We will

discuss the relation between the topology of coupled feedback loops and interesting dynamical features, such as multistability and oscillations.

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Reliable and Unreliable Behavior in Oscillator Networks

Kevin Lin
University of Arizona, USA
(Eric Shea-Brown, Lai-Sang Young)

I will talk about the reliability of driven oscillator networks, focusing on an analysis of networks that can be decomposed into modules connected by acyclic graphs. Here, reliability means that an input elicits essentially identical responses upon repeated presentations. For networks in this class, I will discuss how the source of unreliability can be localized, and address questions concerning downstream propagation of unreliability once it is produced. This is joint work with Eric Shea-Brown and Lai-Sang Young.

⟶ ∞ ◇ ∞ ⟵

The Design of Oscillatory Biological Circuits Using Inverse Methods

James Lu
Johann Radon Institute (RICAM), Austria

Synthetic biology is an emerging field which aims to construct biological circuits out of a set of molecular components, in order to achieve certain functionalities. Many mathematical and computational challenges arise in formulating the right framework for developing and analyzing complex circuit designs. Of these, one important problem is the mapping of the (often qualitative) functional requirements to circuit topologies and conditions on the kinetic rates. In this presentation, we study the problem in the context of oscillator design. There have been previous work using evolutionary optimization strategies to match time-course of synthetic networks to the desired temporal functions. In our approach, we focus on the limit cycle solution itself and consider its discretization using a collocation scheme. In considering the inverse problems associated with the desired qualitative features, we formulate the appropriate adjoint systems on the unit cycle. The design problems being ill-posed, we utilize and compare regularization strategies according to the application of interest.

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Oscillations in Biochemical Reaction Networks

Maya Mincheva
Northern Illinois University, USA

Mathematical models of biochemical networks often lead to complicated dynamical systems with many unknown parameters. Graphs associated with biochemical networks can be used to predict oscillations or multistability in a dynamic model without knowing the parameter values. We will discuss general graph-theoretic conditions for oscillations associated with a positive cycle or a negative cycle.

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Patterns of Synchrony in Lattice Networks with Nearest Neighbour Coupling

Eliana manuel Pinho
Centro de Matemática / Faculdade de Arquitectura Universidade do Porto, Portugal
(Ana Dias)

We consider lattice differential equations with architectures given by general n-dimensional lattice networks with nearest neighbour coupling. Patterns of synchrony are finite-dimensional flow-invariant subspaces of the lattice differential equations that depend only on the corresponding lattice network. We present several results concerning the existence and enumeration of spatially periodic patterns of synchrony of lattice differential equations.

⟶ ∞ ◇ ∞ ⟵

Multiple Roles of Negative Feedback in Immune Response to Virus Attack

David Swigon
University of Pittsburgh, USA
(Saishuai Tang)

The basic response of host system to virus infection, when described using a set of differential equations, leads to a competitive dynamical system which trajectories converge to either a healthy equilibrium state, chronic equilibrium state, or a limit cycle, depending on parameter values. The limit cycle is undesirable for the host because the number of host cells may drop below sustainable threshold. We show that the immune response of the host modifies the trajectories of the system so as to avoid cell extinction and stabilize the healthy equilibrium state. We also show that the limit cycle is avoided if the stochastic nature of the problem is taken into consideration.

⟶ ∞ ◇ ∞ ⟵

Special Session 71: Qualitative aspects of nonlinear differential equations

Marta Garcia-Huidobro, Pontificia Universidad Católica de Chile
Raul Manasevich, CMM and DIM, Universidad de Chile
James Ward, University of University of Alabama – Birmingham, USA

Complex Structure for Radial Solutions of a Semi-Linear Equation with Mixed Sobolev Growth

Matteo Franca
Marche Polytechnic University, Ancona, Italy

In this talk we analyze the structure of positive radial solutions for equations of the form

$$\Delta u + f(u, |x|) = 0$$

where $x \in \mathbb{R}^n$ and f has subcritical behavior for u small and supercritical for u large. We stress that the opposite case is widely studied and understood, but much less is now in this particular setting.

We start from a prototypical family of nonlinearities f for which the set of positive solutions undergoes to two different sequences of bifurcations, thus generating an unexpectedly complex structure. Then we see which of the results are maintained in a more general framework. We exploit dynamical systems techniques and we propose a method which allows to unify the results obtained by Bamon et al. for $f(u, |x|) = u^{q_u-1} + u^{q_s-1}$ and by Alarcon and Quaas for $f(u, |x|) = (|x|^a + |x|^{-b})u^{q-1}$.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Positive Solutions for a Class of Equations with a p-Laplace Like Operator and Weights

Marta Garcia-Huidobro
Pontificia Universidad Catolica de Chile
(P. Drabek, R. Manasevich)

We study the problem of existence of positive solutions to the problem

$$(D)\quad \begin{cases} (a(r)\varphi(u'))' + b(r)g(u) = 0, \text{ a.e. in } (0, R), \\ \lim\limits_{r\to 0} a(r)\varphi(u'(r)) = 0, \ \ u(R) = 0, \end{cases}$$

where φ is an odd increasing homeomorphism of $\mathbb{R}$ and $g \in C(\mathbb{R})$ is such that $g(z) > 0$ for all $z > 0$ with $g(0) = 0$. The functions a and b, that we will refer to as weight functions, satisfy $a(r) > 0$, $b(r) > 0$ for all $r \in (0, R]$ and are such that $a,\ b \in C^1(0, R] \cap L^1(0, 1)$.

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Ground States for a Semilinear Elliptic Equation with Mixed Subcritical and Supercritical Growth

Ignacio Guerra
Universidad de Santiago de Chile
(J. Dávila, M. del Pino)

We consider the semilinear elliptic equation $\Delta u + u^p + \lambda u^q - u = 0$, $u > 0$ in $\mathbb{R}^N$, where $\lambda > 0$, $N \geq 3$ and $1 < q \leq p$. We present a numerical description of solutions for the problem in the case of q subcritical and p supercritical. We also prove existence of solutions when q is subcritical and p is supercritical and sufficiently close to the critical exponent $(N + 2)/(N - 2)$. Multiplicity, uniqueness and nonexistence of solutions is also discussed.

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Traveling Wave Laminations and the Jacobi-Toda System

Michal Kowalczyk
Universidad de Chile
(M. del Pino and J. Wei)

In this talk I will discuss existence of multiple front traveling wave solutions for the Allen-Cahn equation in the entire space. These traveling fronts form a lamination of a neighborhood of the eternal, rotationally symmetric solution of the mean curvature flow and their asymptotic profiles are determined by solving the associated Jacobi-Toda system.

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Oscillatory Entire Solutions of a Biharmonic Equation with Power Nonlinearity

Monica Lazzo
University of Bari, Italy
(Paul G. Schmidt)

It is well known that the equation $\Delta^2 u = u|u|^{p-1}$ with $p \in (1, \infty)$ has positive entire radial solutions if and only if the exponent p is critical or supercritical; also the asymptotic behavior of these solutions is well understood. In this talk, we summarize our recent results regarding the existence and uniqueness, up to scaling and symmetry, of oscillatory entire radial solutions in the subcritical case and discuss work in progress regarding their asymptotic behavior.

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Eigenvalues and Bifurcations for One-Dim p-Laplacian Problems

Yong-Hoon Lee
Pusan National University, Korea

In this talk, we consider the eigenvalues of one-dim p-Laplacian problems with singular weights. We classify those weights having discrete eigenvalues or continuous ones. Based on this study, we consider the global bifurcation phenomena for related nonlinear p-Laplacian problems.

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Some Remarks on Existence of Radial Positive Solutions to a p-q Laplace System in the Subcritical Case

Raul Manasevich
University of Chile

In this talk we will revise the problem of existence of positive radially solutions for the Dirichlet problem

$$\begin{aligned} -\Delta_p u &= |v|^{\delta-1} v, \quad x \in B \\ -\Delta_q v &= |u|^{\mu-1} u, \quad x \in B \\ u &= v = 0, \quad x \in \partial B, \end{aligned} \tag{1}$$

in a subcritical situation and by using a shooting method. Here B is the ball of radius $R > 0$ centered at the origin in $\mathbb{R}^N$, $p, q > 1$, $\Delta_p u = \operatorname{div}(|\nabla u|^{p-2}\nabla u)$, and $\delta,\ \mu > 0$.

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Radially Shaped Solutions of Some Semilinear Elliptic Equations

Filomena Pacella
University of Roma "La Sapienza", Italy

We considear a semilinear elliptic problem with a power nonlinearity in an expanding bounded domain with a hole, diffeomorphic to an annulus. We will show some results about the existence of a positive solution which is close to the positive radial solution in the corresponding diffeomorphic annulus. The proof relies on a careful analysis of the spectrum of the linearized operator at the radial solution as well as on a delicate analysis of the nondegeneracy of suitable approximating solutions. As a consequence of this analysis also bifurcation results from the radial solution can be obtained.

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Fundamental Solutions and Liouville Type Theorems for Nonlinear Integral Operators

Alexander Quaas
Universidad Santa Maria, Chile

(Patricio Felmer)

In this article we study basic properties of a class of nonlinear integral operators deeply related to the existence of fundamental solutions for the operator. Our goal is to establish Liouville type Theorems: non-existence Theorems are for positive entire solutions for $\mathcal{I}u \le 0$ and for $\mathcal{I}u + u^p \le 0$, $p > 1$.

We prove the existence of fundamental solutions and use them, via comparison principle, to prove the theorems for entire solutions. The non-local nature of the operators poses various difficulties in the use of comparison techniques, since usual values of the functions at the boundary of the domain are replaced here by values in the complement of the domain. In particular, we are not able to prove the Hadamard's Three Sphere Theorem, but we still obtain some of its consequences that are sufficient for the arguments.

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Large Radial Solutions of Polyharmonic Equations with Power Nonlinearities

Paul Schmidt
Auburn University, USA

(Ildefonso Díaz and Monica Lazzo)

Most regular radial solutions of the equation $\Delta^m u = u|u|^{p-1}$ with $m \in \mathbb{N}$ and $p \in (1, \infty)$ have finite exit radius and diverge to ∞ or $-\infty$. Generalizing a well-known result for the second-order case, we show that all such solutions blow up at the same, explicitly known rate if $m = 2$. In higher-order cases (at least for $m \ge 6$), we find more complex blow-up behavior.

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Uniqueness of Positive Radial Solutions of Superlinear Elliptic Equations in a Ball

Satoshi Tanaka
Okayama University of Science, Japan

The following Dirichlet problem is considered: $\Delta u + K(|x|)u^p = 0$ in B; $u = 0$ on ∂B. Here, $B = \{x \in R^N : |x| < 1\}$, $N \ge 3$, $p > 1$, $K \in C^1[0, 1]$ and $K(r) > 0$ for $0 \le r \le 1$. According to the well-known result of Gidas, Ni and Nirenberg, every positive solution is radially symmetric if $K'(r) \le 0$ for $0 \le r \le 1$. It is also well-known that if $1 < p < (N + 2)/(N - 2)$, then there exists at least one positive radial solution. In this talk, a new kind

of sufficient conditions for the uniqueness of positive radial solutions is presented.

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Asymptotic Forms of Slowly Decaying Solutions of Quasilinear Ordinary Differential Equations with Critical Coefficient Functions

Hiroyuki Usami
Gifu University, Japan

Let us consider the quasilinear ODE $(t^\beta |u'|^{\alpha-1}u')' + t^\sigma(1 + o(1))u^\lambda = 0$, where α, β, λ, and σ are constants satisfying $\lambda > \alpha > 0, \beta > \alpha$, and $\sigma \in R$. It is important to analyze the asymptotic behavior of positive solutions u satisfying $\lim_{t\to\infty} u(t) = 0$ and $\lim_{t\to\infty} t^{(\beta-\alpha)/\alpha}u(t) = \infty$. Such positive solutions u are called slowly decaying solutions. When $(\beta - \alpha) - 1 < \sigma < (\lambda/\alpha)(\beta - \alpha) - 1$, every slowly decaying solution u satisfies $u(t) \sim Ct^{-\gamma}$ for some constants $C > 0$ and $\gamma \in (0, (\beta - \alpha)/\alpha)$ which depend on α, β, σ and λ. In this talk we consider the same problem for the critical case $\sigma = \beta - \alpha - 1$. We can show that, in this case, every slowly decaying solution u satisfies $u(t) \sim A(\log t)^{-\rho}$ for some constants $C > 0$ and $\rho \in (0, (\beta - \alpha)/\alpha)$ which depend on α, β, σ.

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Contributed Sessions

Contributed Session 1: Hamiltonian Systems

Holomorphic Hamiltonian Systems Via Pullbacks: Geometry, Dynamics and Visualization

Alvaro Alvarez-Parrilla
Universidad Autónoma de Baja California, Mexico
(Jesús Muciño Raymundo, Carlos Yee Romero)

Certain hamiltonian systems give rise to hamiltonian vector fields which are directly related to complex analytic vector fields, hence understanding the geometry and dynamics of complex analytic vector fields is of interest.

Some recent results concerning the study of complex analytic vector fields are presented, in particular a complete characterization of complex analytic vector fields as pullbacks of holomorphic vector fields via ramified coverings over the Riemann sphere is presented. Using this characterization we proceed to show how the dynamics and geometry of complex analytic vector fields can be understood, including an efficient visualization scheme.

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Separation of Variables and Explicit Theta-Function Solution of the Classical Steclov-Lyapunov Systems: A Geometric and Algebraic Geometric Background

Inna Basak Gancheva
Spain
(Yuri Fedorov, Inna Basak)

The work studies the explicit integration of classical Steclov-Lyapunov system, which was first made by F. Kotter in 1900 by means of separation of variables. Namely, we give a geometric explanation of such variables and, then, applying the root functions, obtain a complete theta-function solution to the problem.

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Homoclinic and Heteroclinic Motions in Scar Theory

Florentino Borondo
Universidad Autonoma de Madrid, Spain
(R. M. Benito)

The pioneering work of Poincaré and others at the turn of the twentieth century unveiled the possibility of chaotic motion in dynamical systems. Moreover, he demonstrated the importance of periodic orbits (PO), and its homoclinic and heteroclinic connections, in the hierarchical organization of the associated tangle. In 1984, Heller published his seminal work [1] on scar theory, in which the importance of PO was also demonstrated for quantum dynamics. Recently, and using a technique previously developed by us to construct scarred functions [2], we have demonstrated that the information concerning the associated homoclinic and heteroclinic motions is also contained in the quantum mechanics of the system [3]. This completes Hellers work in the sense that cares about the fate of quantum probability not circulating along the main scarring path due to the PO, but pushed away, in the first instance by the Lyapunov dynamics, and moving along homoclinic and heteroclinic circuits reinforcing the scarring power of the PO.

[1] E. J. Heller, Phys. Rev. Lett. 53, 1515 (1984).
[2] G. G. de Polavieja, F. Borondo, and R. M. Benito, Phys. Rev. Lett. 73, 1613 (1994); ibid.80, 944 (1998).
[3] D. A. Wisniacki, E. Vergini, R. M. Benito, and F. Borondo, Phys. Rev. Lett. 94, 054101 (2005); ibid.97, 094101 (2006); E. L. Sibert III, E. Vergini, R. M. Benito and F. Borondo, New J. Phys.10, 053016 (2008).

$\longrightarrow \infty \diamond \infty \longleftarrow$

Modelling of the MPW under Condition of Superconductivity Destruction

Lyudmyla Grygor'yeva
National University of Kyiv, Ukraine
(Vasyl' Kozorez, Mstislav Fedorchuk)

Non-typical MPW-force (Magnetic Potential Well force) in the form of “saw teeth” obtained experimentally is investigated. It is obtained for the samarium-cobalt disc magnet of 40mm diameter and 8mm thickness and coaxial to the magnet thin niobium ring of 40mm diameter. After positioning the ring on the magnetic pole at room temperature, the magnet-ring pair is cooled in a liquid helium cryostat and the magnetic force as a function of separa-

tion between coaxial magnet and ring is measured. This result can be used to explain some properties of high temperature superconductor levitation such as “sticking in an invisible heap of sand”. If such a specimen can be represented as a plurality of loops different in size and orientation, floating like “sticking” can be explained as a consequence of the saw-shape MPW-force. According to this, the continuous range of stable positions ascribed to levitation of high temperature superconductors is an approach from a set of discrete (quantized) equilibrium positions to the continuity of these positions. This is possible for big amount of zero resistance loops different in size and orientation, and operating nearby the critical magnetic field. The paper is devoted to mathematical modeling of magnetic system presenting non-typical MPW-force interaction.

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Generic Properties of 4-Dimensional Hamiltonian Dynamical Systems

Joao Lopes Dias
Universidade Tecnica de Lisboa, Portugal
(Mario Bessa)

We will discuss some properties of 4-dimensional Hamiltonian dynamical systems that hold C2-generically. In particular, we show the following typical dichotomies: 1) energy surfaces are uniformly hyperbolic or else have zero Lyapunov exponents a. e; 2) energy surfaces are uniformly hyperbolic or else elliptic periodic orbits are dense.

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A Renormalization Scheme for the Focal Decompositions of a Family of Mechanical Systems

Diogo Pinheiro
Technical University of Lisbon, Portugal
(C. A. A. de Carvalho, M. M. Peixoto, A. A. Pinto)

Galileo observed that the small oscillations of a pendulum seem to have constant period. In fact, the Taylor expansion of the period map of the pendulum is constant up to second order in the initial angular velocity around the stable equilibrium. It is well known that, for small oscillations of the pendulum and small intervals of time, the dynamics of the pendulum can be approximated by the dynamics of the harmonic oscillator. We study the dynamics of a family of mechanical systems that includes the pendulum at small neighbourhoods of the equilibrium but after long intervals of time so that the second order term of the period map can no longer be neglected. We characterize such dynamical behaviour through a renormalization scheme acting on the dynamics of this family of mechanical systems. The main theorem states that the asymptotic limit of this renormalization scheme is universal: it is the same for all the elements in the considered class of mechanical systems. As a consequence we obtain an universal asymptotic focal decomposition for this family of mechanical systems. Furthermore, we obtain that the asymptotic trajectories have a Hamiltonian character and compute the action of each element in this family of trajectories.

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40 Years of the Toda Lattice: A Look at the Finite Non-Periodic Systems

Barbara Shipman
University of Texas at Arlington, USA
(Yuji Kodama)

In 1967, Japanese physicist Morikazu Toda published his seminal papers exhibiting soliton solutions to a chain of particles with nonlinear interactions between nearest neighbors. In the decades that followed, Toda’s system of particles has been generalized in different directions, each with its own analytic, geometric, and topological characteristics that sets it apart from the others. This talk will describe and compare several versions of the finite non-periodic Toda lattice from the perspective of their geometry and singularities, and list some open questions.

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Integrability of the Geodesic Equations on a Certain Class of Surfaces

Thomas Waters
NUI Galway, Ireland

The equations defining geodesic curves on surfaces admit a Hamiltonian formulation. A natural question which arises is that of integrability. By combining numerical and analytical techniques we are able to prove that the geodesic equations of a class of surfaces defined in terms of the spherical harmonics are not integrable, and that there are chaotic regions of phase space. This talk will combine elements of differential geometry, dynamical systems, and differential Galois theory.

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Contributed Session 2: ODEs and Applications

Existence and Quasilinearization in Banach Spaces

Khalid Alshammari
KFUPM, Saudi Arabia
(M. A. El-Gebeily, Donal O' Regan)

We establish some existence results for the nonlinear problem $Au = f$ in a reflexive Banach space V, without and with upper and lower solutions. We then consider the application of the quasilinearization method to the above mentioned problem. Under fairly general assumptions on the nonlinear operator A and the Banach space V, we show that this problem has a solution that can be obtained as the strong limit of two quadratically convergent monotone sequences of solutions of certain related linear equations.

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On Qualitative Characteristics of Solutions to Nonlinear Ordinary Differential Equations of Higher Order

Irina Astashova
Moscow State Universities, Russia

Qualitative characteristics of solutions to quasi-linear ordinary differential equations of the higher order are obtained. In particular, to the equation

$$y^{(n)} + \sum_{j=0}^{n-1} a_j(x)\, y^{(j)} + p(x)\, |y|^k \operatorname{sgn} y = 0$$

with $n \geq 1$, real (not necessary natural) $k > 1$, and continuous functions $p(x)$ and $a_j(x)$, uniform estimates for solutions with the same domain, sufficient conditions for existence of non-oscillatory solutions, a criterion for existence of non-oscillatory solutions with non-zero limit at infinity, sufficient conditions for existence of solutions equivalent to those of the related linear differential equation are formulated. In the case of even order and positive potential $p(x)$, a criterion is obtained for all solutions to be oscillatory.

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Matrix Sturm-Liouville Problems with Applications to the Beam Differential Equations

Grzegorz Bartuzel
Warsaw University of Technology, Poland
(Andrzej Fryszkowski)

In the talk we deal with the Matrix Sturm-Liouville equation

$$Z'' - A^2 Z = F, \qquad (*)$$

where matrix $A \in \mathbb{C}^{n\times n}$ and function $F \in L^1([0,1], \mathbb{C}^n)$. We construct the Green kernel for $(*)$ with boundary conditions

$$Z'(0) = Z'(1) = 0.$$

Obtained result we apply to the beam differential equation

$$\mathcal{D}y = y'''' + 4k^4 y = f$$

defined on $[0,1]$ with boundary conditions

$$y(0) = y(1) = y'(0) = y'(1) = 0.$$

This kind of considerations is motivated by differential inclusions of the type

$$\mathcal{D}y = y'''' + 4k^4 y \in F(t,x).$$

$\longrightarrow \infty \diamond \infty \longleftarrow$

Non-Monotone Approximation Schemes for Solutions of Boundary Value Problems for ODE

Marija Dobkevica
Daugavpils University, Latvia

We consider problem $x'' = f(t,x)$, $x(a) = A$, $x(b) = B$. It is known that in presence of upper and lower functions there exist monotone iterations converging to a solution of the BVP. We introduce types of solutions and show that monotone iterations converge to solutions of 0-type. We consider solutions of non-zero types and discuss the possibility of constructing converging (generally non-monotone) iterations.

[1] M. Dobkevich, F. Sadyrbaev. *Types of solutions and approximation of solutions of second order non-linear boundary value problems.* In: Amer. Inst. Phys. Conference Proceedings Volume 1168. Numerical Analysis and Applied Mathematics: International Conference on Numerical Analysis and Applied Mathematics 2009: Vol. 1, Rethymno, Crete (Greece), 18 - 22 September 2009, p.260 - 263.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Pointwise Estimates for Retractions on the Solution Set to Lipschitz Differential Inclusions

Andrzej Fryszkowski
Warsaw University of Technology, Poland
(Tadeusz Rzezuchowski)

Denote by $S'_F(\zeta)$ the set of derivatives of all absolutely continuous solutions of a Lipschitz differential

inclusion

$$\begin{cases} x' \in F(t,x), & t \in [0,1] = I \\ x(0) = \zeta. \end{cases}$$

It is known that the set $S'_F(\zeta)$ is an absolute retract. We improve that result and present the following

Theorem: *For every $\varepsilon > 0$ there exists a continuous mapping $r : X \times L^1 \to L^1$ such that for every $\zeta \in X$ the map $r(\zeta, \cdot)$ is a retraction of L^1 onto $S'_F(\zeta)$ and for all $(\zeta, u) \in X \times L^1$ and almost all $t \in I$ we have a Filippov type pointwise estimates*

$$|r(\zeta,u)(t) - u(t)| \le \varepsilon (1 + l(t)) \|p(\zeta,u)\| + l(t) \int_0^t e^{m(t)-m(s)} p(\zeta,u)(s)ds + p(\zeta,u)(t),$$

where

$$p(\zeta,u)(t) = \operatorname{dist}\left(u(t), F\left(t, \zeta + \int_0^t u(\tau)d\tau\right)\right)$$

a.e. in I, and the functions l and m are related with the Lipschitz condition.

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Limit Cycle Bifurcations and Geometric Applications

Valery Gaiko
National Academy of Sciences of Belarus, Belarus

We develop the global bifurcation theory of planar polynomial dynamical systems and suggest a new geometric approach to solving Hilbert's Sixteenth Problem on the maximum number and relative position of their limit cycles in two special cases of such systems. First, using geometric properties of four field rotation parameters of a new canonical system, we present the proof of our earlier conjecture that the maximum number of limit cycles in a quadratic system is equal to four and their only possible dstribution is (3 : 1). Then, by means of the same geometric approach, we solve the Problem for Liénard's polynomial system (Smale's Thirteenth Problem). Besides, generalizing the obtained results, we present the solution of Hilbert's Sixteenth Problem on the maximum number of limit cycles surrounding a singular point for an arbitrary polynomial system and, applying the Wintner–Perko termination principle for multiple limit cycles, we develop an alternative approach to solving the Problem. By means of this approach we complete also the global qualitative analysis of a generalized Liénard cubic system, a neural network cubic system, a Liénard-type piecewise linear system and a quartic dynamical system which models the population dynamics in biomedical and ecological systems.

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Double Resonance with Landesman-Lazer Conditions for Planar Systems of ODEs

Maurizio Garrione
SISSA - Trieste, Italy
(Alessandro Fonda)

We consider the planar system

$$\dot{u} = F(t,u),$$

with T-periodic boundary conditions. We assume that

$$F(t,u) = -\gamma(t,u)J\nabla H_1(u) - (1-\gamma(t,u))J\nabla H_2(u) + r(t,u),$$

being J the standard 2×2 symplectic matrix, $0 \le \gamma(t,u) \le 1$ and $|r(t,u)| \le h(t) \in L^2(0,T)$. Moreover, we suppose that H_1 and H_2 are positively homogeneous functions (of order 2) both at resonance. Under some kind of Landesman-Lazer conditions at both sides, we prove the existence of a solution to the above problem. As particular cases, we recover the results proved by Lazer and Leach (1969), Frederickson and Lazer (1969), and Fabry (1995).

$\longrightarrow \infty \diamond \infty \longleftarrow$

Dynamics and Stability of Multibody Magnetic Systems in Magnetic Potential Well (MPW)

Lyudmyla Grygor'yeva
National University of Kyiv, Ukraine

The obtained mathematical models for dynamic magnetic multibody systems based on the Magnetic Potential Well (MPW) phenomenon are discussed. These are the results of modeling for: (i) dynamic problem of magnetic pendulum, (ii) levitation problem of the system with magnetic interaction of the "ring-dipole" type, (iii) dynamic problem of magnetic multibody "garland"-type system in the gravitational field, etc. The approach for construction of mathematical model is based on analytical electromechanics, taking into account magnetic interaction, gravity forces, constancy of full magnetic fluxes coupled with superconductive elements, six degrees of freedom of free rigid magnetically interacting bodies, Newton-Euler and Lagrange methods for deriving equations of motion. Static and dynamic stability problems are investigated. The stability sufficient conditions in the parameter space of the magnetic multibody system are derived. The last can be satisfied by the MPW phenomenon manifestation and relevant selection of geometrical and magnetic parameters of the system.

The derived dynamic equations of motion for systems (i)-(iii) are essentially nonlinear. We review

the results of numerical analysis of these nonlinear systems which is held by means of procedures built on the basis of computer algebra system. Dynamic behavior of the said systems demonstrates features that have not been discussed before.

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Analysis of Grazing Bifurcations Within a Discontinuity-Geometry Framework

Neil Humphries
National University of Ireland, Galway, Ireland
(P. T. Piiroinen)

Impacting systems have two fundamental components - smooth dynamics (sometimes with an analytic solution) that describes the system between impacts and a reset rule to model the behaviour of the system at impact. After a brief description of discontinuity geometry methodology, applied to a harmonic impact oscillator, we will use the geometry of the discontinuiy surface to analyse the existence of periodic orbits in the vicinity of grazing bifurcations. We will also see how the methodology can be extended to systems with two discontinuity surfaces.

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Hunting Ducks and Nondeterminism in Nonsmooth Dynamics

Mike Jeffrey
University of Bristol, England

Discontinuous differential equations are often though to be crude approximations of what is, underneath, a smooth dynamically evolving world. However, in recent years it has become apparent that discontinuous dynamics represents something more subtle. In the context of systems with multiple timescales, or different components interacting, discontinuities can reveal the geometry at the heart of many sudden behaviours in real world systems. We will discuss models from neuroscience and mechanics, and will present a striking prediction from discontinuous dynamics that has been verified in experiments on a superconductor. Our interest will center around sliding – when a systems sticks to and slides along the locus of discontinuity itself. Regions where sliding motion is unstable have been largely overlooked before, but we will show that they lead to the most catastrophic of discontinuity induced bifurcations. They provide a new window into some well studied phenomena, such as the 'canards' of slow-fast systems, but they also generalises them. The predictions they make pose problems for our understanding of physical dynamics (particularly mechanics and electronics), providing the conditions for a nondeterministic form of chaos.

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On the Infinite Product Representation of the Solution of Sturm-Liouville Equation with Dirichlet-Neumann Condition

Aliasghar Jodayree Akbarfam
Tabriz University, Iran
(Mohammad Dehghan)

The purpose of this paper is studying the infinite product representation of solution of boundary value problem with Dirichlet-Neumann conditions. It must be mentioned that the Potential function in this problem has one zero of order of integer number which is called turning point. The Neumann condition is held on the fixed point which it may be at before or after turning point. We consider the boundary value problem $L_2 = L_2(\varphi^2(t), q(t), x)$ of the form

$$ly := \quad -y'' + q(t)y = \lambda\varphi^2(t)y, \qquad 0 \le t \le 1$$

with Dirichlet-Neumann conditions

$$y(0) = y'(x) = 0$$

where $\lambda = \rho^2$ is the spectral parameter, x is a fixed point in the interval $(0,1)$ and the functions $\varphi^2(t)$ and $q(t)$ satisfy:

1. $\varphi^2(t) = (t-t_0)^l\varphi_0(t)$ is real and has one zero, t_0, so called turning point of order $l \in N$ in $[0,1]$ and also $\varphi_0(t)$ is positive and twice continuously differentiable.
2. $q(t)$ is bounded and integrable on $[0,1]$.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Exact Solutions of Certain Nonlinear Autonomous Ordinary Differential Equations of the Second Order

Michail Markakis
University of Patras, Greece

By considering autonomous equations of polynomial structure for dy/dx, with coefficients of a not necessarily polynomial form for y (see for example the Langmuir equation), in the present work we investigate analytically this generalized polynomial form, aiming at the construction of proper techniques capable of removing the difficulties arising in the derivation of general solutions.

A significant part of the relevant search in procedures of this kind deals with the use of appropriate transformations. However, the classic transformation ${y'}_x = q(y)$, usually applied to autonomous nonlinear ordinary differential equations of the second order results in Abel equations of the second kind, which in general cannot be solved analytically,

except in special cases, most of which accept only parametric solutions. Hence, in order to construct a more efficient analytical technique, concerning two general subclasses of the autonomous equations under consideration, we use another, properly modified, general transformation, to obtain Abel equations of the first kind ((Ab-1) equations). By considering a general form of this equation, as well as a specific, solvable in closed-form Abel equation involving arbitrary functions, we further apply a transformation provided by Kamke and finally obtain an implicit solution of the general (Ab-1) equation, together with the associated sufficient condition.

Then application of the splitting technique to the original reduced (Ab-1) equation, combined with the derived solution, yield exact one- parameter families of solutions of the original nonlinear autonomous equation. Finally we apply the developed analytical technique to five specific cases of Liénard equations.

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Classical Analysis and Discontinuity Geometry of an Impact Oscillator with Two Discontinuity Surfaces

Joanna Mason
MACSI, University of Limerick, Ireland
(Neil Humphries and Petri Piiroinen)

An impacting model is used to describe a pair of meshing gears, where the discontinuities in the model arises from the backlash between the gear teeth. We find that there is a complex interplay between both smooth and discontinuity-induced bifurcations

A classical approach of basin of attraction computations, explicit solution construction, bifurcation diagrams and manifold computations is combined with a discontinuity geometry methodology to provide insight into the extremely rich dynamical behaviour observed.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Noether, Partial Noether Operators and First Integrals for Two Linearly Coupled Nonlinear Duffing Oscillators

Imran Naeem
Lahore University of Management Sc., Pakistan

We investigate Noether and partial Noether operators corresponding to a Lagrangian and a partial Lagrangian for a system of two linearly coupled nonlinear Duffing oscillators. Then the first integrals with respect to Noether and partial Noether operators are obtained explicitly by utilizing Noether and partial Noether theorems for the system under consideration. We show how the first integrals can be constructed without a variational principle. Moreover, if the partial Euler-Lagrange equations are independent of derivatives then the partial Noether operators become Noether symmetry generators for such equations. The difference arises in the guage terms due to Lagrangians being different for respective approaches. This study points new ways of constructing first integrals for nonlinear equations without regard to a Lagrangian.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Generalized Lipschitz Condition for ODEs

Christine Nowak
University of Klagenfurt, Austria

In [1] and [2] it is shown that the Lipschitz uniqueness theorem due to Picard-Lindelöf remains (locally) true for the 1-dimensional Cauchy problem $y' = f(x,y), y(x_0) = y_0$, if f is a Lipschitz function with respect to the first argument and $f(x_0, y_0) \neq 0$. This result is a special case of a generalized Lipschitz condition given in [3] which allows to estimate $f(x,y)$ also in other directions (not only in the direction of the x- or y-axis). In the talk generalizations with respect to Kamke-type uniqueness criteria and higher dimensions are presented.

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$\longrightarrow \infty \diamond \infty \longleftarrow$

The C^1 Hypothesis in the Closing Lemma

Charles Pugh
University of Toronto, Canada

The usual hypothesis for the closing lemma is that the perturbation is C^1-small. An example of Carlos Gutierrez shows that the standard semi-local perturbation proof cannot work for C^2-small perturbations. I mention that a closing lemma with $C^{1+\alpha}$-small perturbations is a possibility.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Average Conditions for Extinction in Nonautonomous Kolmogorov Systems

Joanna Petela
Wrocław University of Technology, Poland

In this talk we consider a nonautonomous Kolmogorov system

$$u_i' = u_i f_i(t, u), \tag{K}$$

where

$$\frac{\partial f_i}{\partial u_j} \qquad 0.$$

They gave algebraic criteria on the parameters which guarantee that all but one of the species are driven to extinction.

Jiandong Zhao and Jifa Jiang generalized these results, namely, they gave average conditions on the coefficients to guarantee that r of the species in the system are permanent while the remaining $N-r$ are driven to extinction.

I extend their results to Kolmogorov systems. With the help of lower and upper averages of a function I give the average conditions which insure that all but one of the species are driven to extinction. Moreover, I proved that if $u(t) = (u_1(t), \ldots, u_N(t))$ is a positive solution of (K) then $u_1(t) \to U_1(t)$ as $t \to \infty$, where $U_1(t)$ is a solution of the equation

$$U_1'(t) = U_1(t) f_1(t, U_1(t), 0, \ldots, 0).$$

Then I show that for any $r \leq N$ the average conditions guarantee that r of the species in the system are permanent while the remaining $N-r$ are driven to extinction.

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Particular Solution to the Euler-Cauchy Equation with Polynomial Non-Homogeneities

Adnan Sabuwala
California State University, Fresno, USA

The Euler-Cauchy differential equation is one of the first, and simplest, forms of a higher order non-constant coefficient ordinary differential equation that is encountered in an undergraduate differential equations course. For a non-homogeneous Euler-Cauchy equation, the particular solution is typically determined by either using the method of variation of parameters or transforming the equation to a constant-coefficient equation and applying the method of undetermined coefficients. In this talk, we demonstrate the surprising form of the particular solution for the most general n^{th} order Euler-Cauchy equation when the non-homogeneity is a polynomial by presenting a complete proof of the form of the particular solution. In addition, a formula that can be used to compute the unknown coefficients in the form of the particular solution will be presented.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Properties of a Nonlinear Asymmetric Oscillator with Description of Spectra

Felix Sadyrbaev
LU MII and Daugavpils University, Latvia
(A. Gritsans)

We consider a nonlinear asymmetric oscillator described by equation

$$x'' = -\lambda f(x^+) + \mu g(x^-), \tag{1}$$

where nonlinear functions f and g are continuous, positive valued for $x > 0$, $f(0) = g(0) = 0$, λ, μ are parameters.

We mainly look for nontrivial solutions which satisfy the boundary conditions

$$x(0) = 1, \quad x(1) = 0. \tag{2}$$

If the normalization condition $|x'(0)| = \alpha$ is imposed, the problem (1), (2) has a two-dimensional spectrum. We provide description of the spectra and, in particular, discuss some peculiar features.

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On the Third Order Three Point Nonlinear Boundary Value Problems

Sergey Smirnov
Daugavpils University, Latvia

Boundary value problem for equation

$$x''' = f(x) \tag{1}$$

together with boundary conditions

$$x(a) = x(b) = x(c) = 0 \tag{2}$$

is considered, where $f(x)$ is strictly increasing, continuous function such that $f(0) = 0$ and $a < b < c$.

The structure and properties of solutions of equation (1) are discussed. Estimates of the number of solutions to boundary value problem (1), (2) are established.

Illustrative examples and figures are provided.

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On Nonlinear Perturbations of Second Order Linear Nonoscillatory Differential Equations

Tomoyuki Tanigawa
Kumamoto University, Japan
(Akihito Shibuya)

In this talk, we present that any second order

nonoscillatory linear differential equation can be converted into an oscillating system by applying a “sufficiently large” nonlinear perturbation.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Harmonic Limits of Dynamical Systems

Tobias Wichtrey
Universität Augsburg, Germany

In this talk, we analyze the rotational behaviour of dynamical systems, particulary of solutions of ODEs. With rotational behaviour we mean the existence of rotational factor maps, i.e. semi-conjugations to rotations in the complex plane. In order to analyze this kind of rotational behaviour, we introduce harmonic limits $\lim_{T\to\infty} \frac{1}{T}\int_0^T \mathrm{e}^{\mathrm{i}t\omega} f(\Phi_t x)\mathrm{d}t$. We discuss the connection between harmonic limits and rotational factor maps, and some properties of the limits, e.g., existence under the presence of an invariant measure by the Wiener Wintner Ergodic Theorem. Finally, we look at linear differential equations (autonomous and periodic), and show the connection between the frequencies of the rotational factor maps and the eigenvalues of the system matrix (or of the Floquet exponents in the periodic case).

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Contributed Session 3: Delay and Difference Equations

State-Dependent Neutral Delay Differential Equations in Population Dynamics

Maria Vittoria Barbarossa
Technische Universität München, Germany
(K. P. Hadeler, C. Kuttler)

The classical model for a population with age-structure and density dependent birth and death rates is the Gurtin-MacCamy system (1974), that has the form of either a first order PDE with an asymmetric boundary condition or of a set of renewal equations.

Hadeler et al. (2000-2008) derived a class of population models in the form of (neutral) delay equations from the Gurtin-MacCamy system. In their approach, juveniles and adults are distinguished by a critical age $\tau > 0$.

In this work we focus on what happens when the length of the juvenile period depends on the size of the adult population $\tau = \tau(u(t)) > 0$ (e.g., via competition for food or defense against predators). Starting from the birth-law, we find by a rigorous derivation a nonlinear (neutral) DDE with state-dependent delay. With birth and death rates chosen as piecewise functions of the age, we get a state-dependent version of the blowfly equation (Perez et al., 1978):

$$\dot{u}(t) = \frac{b_1(u(t-\tau))u(t-\tau)e^{-\mu_0\tau} - \mu_1(u(t))u(t)}{1+\tau'(u(t))\, b_1(u(t-\tau))u(t-\tau)e^{-\mu_0\tau}}.$$

A peak b_2 in the birth rate $b(a)$ yields the following nonlinear state-dependent neutral delay equation:

$$\dot{u}(t) = \frac{g(u(t-\tau),\dot{u}(t-\tau)) - \mu_1 u(t)}{1+\tau'(u(t))g(u(t-\tau),\dot{u}(t-\tau))}, \qquad (1)$$

where

$$g(p,q) = \left[b_1 p + b_2 \frac{q+\mu_1 p}{1-\tau'(p)q}\right] e^{-\mu_0\tau}.$$

The properties of the general system (1) have been investigated by analytical and numerical methods. With a particular choice of the parameters a periodic solution was found.

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A Delay Differential Equation and the Method of Generating Tree

Fatima Fabião
CEMAPRE - ISEG UTLisbon, Portugal
(Paulo Brito, Leonor Pinto and Antonio Staubyn)

Recently, a new method for solving a very simple linear delay differential equation (DDE) was presented and it is an alternative to the Method of Steps Algorithm (MSA). This new Method of Generating Tree (MGT) is based on the existence of a specific class of polynomials in the delay. Computational experiments to evaluate the performance of both methods, conducted using C++, are reported. We also analyse the dynamic of the Solow growth model, an economic growth model, introducing a delay parameter in the technological progress variable $A(t)$. With this new assumption the model can exhibit oscillating solutions.

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Solution of First Boundary Problem of Thermal Conductivity Equation with Delay

Denys Khusainov
National University of Kyiv, Ukraine

We give the approach to obtain the solution of first boundary problem. Using a special function, called delay exponential we obtain the solution of the boundary problem for thermal conductivity equation with constant delay

$$u_t(x,t) = a_1^2 u_{xx}(x,t) + a_2^2 u_{xx}(x,t-\tau) + c_1 u(x,t) + c_2 u(x,t-\tau) + f(x,t)$$

on $0 \le x \le l$, $t \ge 0$. This is first boundary problem with initial condition

$$u(x,t) = \varphi(x,t), \ 0 \le x \le l, \ -\tau \le t \le 0,$$

and boundary conditions $u(0,t) = \mu_1(t), u(l,t) = \mu_2(t), t \ge -\tau$.

For solving the boundary value problem we use the method of separation of variables. And the solution is represented as a row.

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Implicit Difference Methods for Parabolic Fde on Cylindric Domains with Initial Boundary Conditions of Robin Type

Karolina Kropielnicka
University of Gdańsk, Poland

Implicit difference schemes for parabolic functional differential equations are presented. Benefits of implicit methods are pointed. The attention is focused here on cylindric domains and on initial condition of Robin type. A complete convergence analysis for methods is presented. Nonlinear estimates of the Perron type for given functions with respect to functional variables are used. Results obtained in the paper can be applied to differential integral problems and to equations with deviated variables. Numerical examples display the results of our investigations.

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Representation of the Solution of the First Boundary Problem for a System of Two Second Order Delay Equations

Oleksandra Kukharenko
National University of Kyiv, Ukraine

The system of two linear homogeneous second order partial differential equations with constant coefficients and constant delay has been considered:

$$\frac{\partial u(x,t)}{\partial t} = a_{11}\frac{\partial^2 u(x,t-\tau)}{\partial x^2} + a_{12}\frac{\partial^2 v(x,t-\tau)}{\partial x^2},$$
$$\frac{\partial u(x,t)}{\partial t} = a_{21}\frac{\partial^2 u(x,t-\tau)}{\partial x^2} + a_{22}\frac{\partial^2 v(x,t-\tau)}{\partial x^2}.$$

The matrix of linear part of the system has the real and different roots of the characteristic equation. By using Fourier method, the first boundary problem has been reduced to solving the Sturm-Liouville problem for sequence of second order differential equations with pure delay. A special function called the delay exponential has been introduced. With its help a constructive form of the solution of linear differential equations with pure delay has been received. A comparative analysis of the representation of solution for this class of systems with and without delay has been realized.

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Stability Investigation of Hybrid Difference Systems with Delay

Olena Kuzmych
Volyn National University, Ukraine
(D. Khusainov, J. Diblik)

We investigate the hybrid system with time law of switching. It is represented by a set of linear discrete subsystems with constant coefficients and with time lags

$$x(k+1) = A_i x(k) + B_i x(k-m), \ \ i = 1, 2, \ldots, N.$$

Estimations of solutions of linear subsystems depending on initial conditions are obtained in the case when we deal with stability or with unstability. Finally we give estimation of solutions of initial hybrid system on the whole interval considered.

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Absolute Robust Stability of the Delay Differential Systems

Andriy Shatyrko
National University of Kyiv, Ukraine

The non-linear regulator system with the interval defined linear part and the time-delay argument is considered:

$$\dot{x}(t) = (A+\Delta A)x(t) + (B+\Delta B)x(t-\tau) + bf(\sigma(t)),$$
$$\dot{\sigma}(t) = c^T x(t) - \rho f(\sigma(t)).$$

The matrices of the linear part of the system take on their values from the given fixed intervals. Nonlinear function of one argument $f(\sigma(t))$ is located in the defined linear sector. Using Lyapunov-Krasovskiy functional

$$V[x(t),\sigma(t)] = x^T(t)Hx(t) + \int_{t-x}^{t} e^{-\zeta(t-s)} x^T(s)Gx(s)ds + \int_0^{\sigma(t)} f(\xi)d\xi,$$

the conditions of the absolute robust stability have been obtained and the coefficients of the exponential decay of solutions have been calculated.

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Stability, Boundedness and Uniform Boundedness of Solutions Solutions of Nonlinear Delay Differential Equations

Cemil Tunc
Yuzuncu Yil University, Turkey

In this paper, we establish some new sufficient conditions to ensure the stability, boundedness and uniform boundedness of solutions of some non-linear delay differential equations of higher order. By defining appropriate Liapunov functionals, we obtain new results on the subject and give some examples to illustrate the theoretical analysis in this work. By this work, we extend and improve some important result in the literature.

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Contributed Session 4: Modelling and Math Biology

Modelling the AH1N1 Virus Epidemic in Mexico City: The First Predictions and Analysis

Gustavo Cruz-Pacheco
National University of Mexico
(L. Esteva, A. A. Minzoni, P. Panayotaros)

We use a time dependent modification of the Kermack and McKendrick model to study the evolution of the influenza A(H1N1)v epidemic reported in the Mexico City area under the control measures used during April and May 2009. The model illustrates how the sanitary measures postponed the peak of the epidemic and decreased its intensity. It provides quantitative predictions on the effect of relaxing the sanitary measures after a period of control. We show how the sanitary measures reduced the maximal prevalence of the infected population from 10% to less than 6% of the total population. We also show how the model predicts the time of maximal prevalence and explains the effect of the control measures. Finally, we present some results on the geographic spreading of the infection.

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Existence of a Hopf Bifurcation for the Nowak-Bangham Model in CTL Dynamics

Chikahiro Egami
Numazu National College of Technology, Japan

In this talk, we discuss the local bifurcation probrems of the CTL response model published in [Nowak & Bangham, Science 272 (1996)]. The Nowak-Bangham model can have three equilibria depending on the basic reproduction number, and generates a Hopf bifurcation through two bifurcations of equilibria. The main result shows a sufficient condition for the interior equilibrium to have a unique bifurcation point at which a simple Hopf bifurcation occurs. For this proof, some new techniques are developed in order to apply the method established by [Liu, J. Math. Anal. Appl. 182 (1994)]. Moreover, to demonstrate the result obtained theoretically, some bifurcation diagrams are presented with numerical examples.

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Explaining Pulse Responses in Models of Intracellular Calcium Dynamics

Emily Harvey
University of Auckland, NZ, New Zealand

A key feature of intracellular calcium dynamics is that some physiological processes occur much faster

than others. This leads to models with variables evolving on very different time scales. Using geometric singular perturbation techniques it is possible to exploit this separation in timescales to analyse these models. In cases where there are two slow variables, existing geometric singular perturbation theory can be applied to explain the observed dynamics including the complicated oscillatory patterns known as mixed mode oscillations. I will present some new results that extend the existing theory to the case of three slow variables, and use these results to explain an anomalous delay seen in models when a cell is pulsed with a secondary messenger chemical, inositol trisphosphate. These results are relevant to the design of an experimental protocol for determining the physiological mechanisms underlying intracellular calcium oscillations.

⟶ ∞ ⋄ ∞ ⟵

Theory of Reentrant Wave Termination in Excitable Media – Utilizing a Special Case of the Eikonal Relationship

Marcel Hörning
Department of Physics, Kyoto University, Japan
(A. Isomura, Z. Jia, E. Entcheva, K. Yoshikawa)

Spiral waves are observed in diverse physical, chemical and biological systems. Beating heart has been recognized as one of the important examples in which spiral waves occur. Spiral waves in the heart, known as ventricular tachycardia (VT), are a precursor of ventricular fibrillation (VF), which is one of the most prevalent diseases in the world. To date, most research has been directed toward elucidating the origin of the formation of VT. However, it is still poorly understood how to terminate spiral waves anchored to heterogeneities.

In the present paper we report, that obstacle-anchored vortices can be terminated by the application of high-frequency wave-trains in excitable media. We theoretically derived the dependency between the obstacle radius and the maximum unpinning period by utilizing a special case of the well-known Eikonal equation. Additionally it is shown that the critical velocity of high-frequency wave trains decreases linearly with a decrease in the size of the obstacle. Our theoretical result was confirmed by experiments with cardiomyocyte monolayers. This result may be useful for improving the stimulation protocol of implantable cardiac pacemakers.

⟶ ∞ ⋄ ∞ ⟵

A Multiscale Approach to Cell Migration in Tissue Networks

Jan Kelkel
Universität Stuttgart, Germany

We use multiscale mathematical methods to investigate the migration of tumour cells through the surrounding tissue. Migration of tumour cells is decisive in the formation of secondary or metastatic tumours. The contact with the surrounding tissue both enables the cells to move along tissue fibers and stimulates the production of proteolytic enzymes that dissolve fibers of the tissue network, thus enhancing cell migration. The product of tissue degradation is seen as a chemotactic signal influencing the cell motility. Existing models for the migration of tumor cells account for the interactions of the cells with their environment, but neglect biochemical processes taking place inside or on the surface of the cell. The latter are only handled by ODE or PDE models for single cells, without connection to directional cell movement. It is however very important to include these processes in a model for cell migration, since the dynamics of receptors on the cell surface as well as the cytoskeleton structure are decisive in determining both the speed of the cell and the secretion of proteolytic enzymes. We present a model in which cell movement is described by a kinetic equation involving these subcellular mechanisms. This equation is then supplemented by a reaction-diffusion equation for the chemoattractant, along with an integro-differential equation for the tissue fibers. Analytical and numerical challenges for this strongly coupled and high dimensional model are then addressed.

⟶ ∞ ⋄ ∞ ⟵

The Size, Shape, and Dynamics of Cellular Blebs

Fong Yin Lim
Inst. of High Performance Computing, Singapore
(Keng-Hwee Chiam)

Cellular blebs are spherical cell membrane protrusions powered by cytoplasmic flow. The phenomenon of cells developing such protrusions is recently gaining attention as one of the mechanisms contributing to cell migration. However, how blebbing could contribute to cell movement is not well studied yet. To understand the dynamics of cellular blebs, we developed a model to study how a bleb develops when a portion of the cell membrane detaches from the underlying cytoskeleton. We calculated the minimum cytoplasmic pressure and minimum unsupported membrane length for a bleb to nucleate and grow. We also showed how a bleb may travel around the periphery of the cell. Such traveling

blebs do not necessarily contribute to directed cell migration; however, understanding their dynamics should help us to gain further understanding of the mechanisms for bleb-associated cell movement.

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Periodic Oscillations in Systems with Nearly Elastic Obstacles

Irina Martynova
Voronezh State Academy of Technology, Russia

The talk studies the following differential equation:

$$\ddot{u}+\varepsilon c\dot{u}+\varepsilon^2 au+\varepsilon^2(u-1)^+ +\varepsilon^2(u+1)^- = \varepsilon^2\gamma\cos\,\omega t,$$

where $u^+ = \max\{u,0\}$, $u^- = \max\{-u,0\}$, a, c, γ, ω are fixed and ε is small. This equation models dynamics of a two-sides constrained oscillating process and commonly found in applied sciences (e.g. harmonic oscillator with collisions in mechanics). Our choice of small parameter reflects essential role of friction. If $a \neq 0$ then periodic oscillations can be examined by means of the standard averaging theory, which is no longer applicable if $a = 0$. Under the latter assumption we prove the following: for any $\delta > 0$ there exists $\varepsilon_0 > 0$ such that for all $\varepsilon \in (0, \varepsilon_0]$ the differential equation has at least one $\frac{2\pi}{\omega}$-periodic solution u_ε with the initial condition $(u_\varepsilon(0), \dot{u}_\varepsilon(0)) \in B_\delta([-1,1]) \times B_\delta(0)$.

The research is partially supported by RFBR grants 09-01-00468, 09-01-92003 and by the RF President young researcher grant MK-1530.2010.1.

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Simulation of Microscale Model for Dengue Infection

Nuning Nuraini
Institut Teknologi Bandung, Indonesia
(H. Tasman)

A model of viral infection of monocytes population by Dengue virus is formulated here. The model can capture phenomena that dengue virus is quickly cleared in approximately 7 days after the onset of the symptoms. The model takes into account the immune response and also the hematocrit. It is shown that the quantity of free virus is decreasing when the viral invasion rate is increasing. The basic reproduction ratio of model without immune response is reduced significantly by adding the immune response. Numerical simulations indicate that the growth of immune response and the invasion rate are very crucial in identification of the intensity of infection. For further development we would like to simulate the plasmaleakage phenomena on Dengue infection model.

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Stoichiometric Population Models: Coexistence of Two Producers and One Consumer

Bruce Peckham
University of Minnesota Duluth, USA
(Laurence Lin, Harlan Stech, John Pastor)

We consider a stoichiometric population model of two producers and one consumer. Stoichiometry involves keeping track of nutrient content as well as biomass (carbon). The model is closed for nutrient but open for carbon. It is a generalization of the Rosenzweig-MacArthur population growth model, which is a one-producer, one-consumer population model without stoichiometry. The generalization involves two independent steps: 1) introducing stoichiometry, and 2) adding a second producer which competes with the first. Both generalizations introduce new bifurcations. We focus on bifurcations which are a result of enrichment: the primary parameters we vary are the growth rates of both producers. Secondary variable parameters are the total nutrient in the system, and the producer nutrient uptake rates. The possible equilibria are: no-life, one-producer, coexistence of both producers, the consumer coexisting with either producer, and the consumer coexisting with both producers. We observe limit cycles in the latter three coexistence combinations. Bifurcation diagrams along with corresponding representative time series summarize the behaviors observed for this model. Especially interesting are various forms of transcritical bifurcations bounding regions of coexistence of all three populations.

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Synchronization of Interconnected Systems with Applications to Biochemical Networks

Luca Scardovi
Technische Universität München, Germany
(M. Arcak, E. Sontag)

I will present a formalism to analyze synchronization in networks of dynamical systems where each component of the network (referred to as a compartment) itself consists of subsystems (referred to as species) represented as nonlinear operators. The input to the operator includes the influence of other species within the compartment as well as a diffusion-like coupling term between identical species in different compartments. The synchronization conditions are provided by combining the input-output properties of the subsystems with information about the structure of the network. The model is motivated by cellular networks where signaling occurs both internally, through interactions of species, and externally, through intercellular signaling. The

theory is illustrated providing synchronization conditions for networks of genetic oscillators.

⟶ ∞ ⋄ ∞ ⟵

A PDE Approach to Maximal Packing Motivated by Invariances in Phyllotactic Tilings

Patrick Shipman
Colorado State University, USA
(Alan Newell)

Phyllotaxis refers to the arrangement of primordia (the first stage in the development of a structure such as a leaf) on plants, and *phyllotactic planforms* refers to the shapes of the primordia in a phyllotactic arrangement. As the van Iterson parameter Γ–a measurement of the ratio of the size of the annular generative region at the plant tip where the patterns form to primordium area–varies, certain properties of observed planforms are invariant. We demonstrate *discrete invariance* in phyllotactic planforms, by which we mean a similarity in the planform under a scaling $\Gamma \to \Gamma\varphi^n$, where φ is the golden number, and n is an integer. *Continuous invariance* in planforms is then motivated by examples in which the shapes of primordia are homogeneous as n varies over the real numbers. We also show how continuous invariance results from classical number-theoretical theorems on the approximation of irrational numbers (such as φ) by rational numbers. We define these notions first for the underlying phyllotactic lattice, and then for primordium shapes and amplitude equations resulting from partial differential equation models. These results then motivate a more general PDE approach to maximal packing problems.

⟶ ∞ ⋄ ∞ ⟵

Modeling Cell Dispersion: Some New and Classical Approaches

Christina Surulescu
IANS, University of Stuttgart, Germany
(Nicolae M. Surulescu)

Starting from the classical description of cell motion relying on velocity jump processes, we propose some ways to enhance the realism of modeling and to account for interesting features like allowing for resting phases, moving through a heterogeneous medium or stochastically switching between biased and unbiased motion. The behavior of the cell population is assessed upon applying a nonparametric method, the performance of which is then illustrated by comparison with numerical simulations based on the method of moments.

We also propose and analyze a multiscale model for bacterial motility in the more classical framework of partial differential equations.

⟶ ∞ ⋄ ∞ ⟵

Impact of Toxins on Anaerobic Digestion

Marion Weedermann
Dominican University, USA

This study addresses the impact of toxins on two stages of anaerobic digestion, acetogenesis and methanization. The basic model includes two microorganisms, a nutrient chain and inhibition in case of abundance of one of the nutrients. Under some conditions the unique internal equilibrium is globally stable. However, the inhibition may cause bistability of two equilibria. We then study the impact of toxins. For externally introduced toxins conditions are given under which the global stability is preserved. Bistability of two equilibria continues to exist. Numerical results indicate a possible bistability of a periodic orbit and an internal equilibrium. Syntrophic interactions between cohorts of bacteria are modeled by means of an internally allocated toxin. Under additional conditions on the growth functions, global stability of an internal equilibrium is obtained.

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Stability Analysis of an Isolation Epidemic Model with Nonlinear Incidence Rate

Xiuxiang Yang
Weinan Normal University, Peoples Rep. of China

Abstract: Stability of an epidemic model SIQS with isolation and nonlinear incidence rate is analyzed in this paper. Derived is a threshold value R, which determines the existence of the infectious disease. We show that the disease free equilibrium point is locally asymptotically stable when $R > 1$, and the endemic equilibrium point is locally asymptotically stable when $R < 1$. With the help of Liapunov function and Dulac function, we have shown under some extra conditions that the disease free and endemic equilibrium points are also globally stable, whenever $R > 1$ respectively $R < 1$.

⟶ ∞ ⋄ ∞ ⟵

Travelling Waves in Calcium Models

Wenjun Zhang
University of Auckland, New Zealand
(Vivien Kirk, James Sneyd)

Intracellular calcium (Ca2+) plays an important part in various cell types involved in the process of delivering external signals to the inside of the cell. Often signaling takes place through variation

of the concentration of calcium inside the cell, i.e., through intracellular calcium waves. In this talk, I will explain how bifurcation analysis can be used to give insight into the mechanisms underlying calcium waves for various mathematical models of intracellular calcium dynamics. A technique I will focus on is exploitation of the presence of multiple timescales.

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Contributed Session 5: Stability

Investigation of Problem Stability of Solutions of Stochastic Dynamic Equations

Irada Dzhalladova
Kyiv National Economical University, Ukraine

The paper is devoted to the research of dynamical system with Non Markov influence stability and optimization. The consideration is given to the technique of analytic continuous and discrete process. At first there are constructed moment equations for different classes of differential and difference equations with random coefficients and random transformations of solutions. There are obtained necessary and enough conditions of stability of solutions for these classes of dynamic systems.

In this work we develop the Lyapunov functions methods for systems of non linear differential and difference equations with right-hand part depending on Semi Markov process. Construction of the Lyapunov functions through moment equations for stochastic equations is performed. There are obtained equations for synthesis of optimal control through the Lyapunov functions for the systems of linear and non linear equations with random influence. That equation is more general than Riccati equation in deterministic case.

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Internal Dynamics Stabilization of Nonlinear Non-Minimum Phase Supersonic Flight Vehicle

Behrouz Ebrahimi
Aerospace Research Institute, Iran
(M. Bahrami, M. Asadi and M. Hashemi)

It is known that a nonlinear system is non-minimum phase if it's internal or zero dynamics are unstable. The non-minimum phase characteristic of a plant restricts direct application of nonlinear control techniques such as feedback linearization, back-stepping, sliding mode control and etc. In general, exact tracking in causal nonlinear non-minimum phase systems seems to be impossible for arbitrary reference signals even in absence of plant uncertainties and external disturbances due to large Mach-altitude fluctuations and due to aerodynamic coefficient uncertainties resulting from inaccurate wind-tunnel measurements. The system dynamics is non-autonomous and highly agile. For the normal acceleration as the desired output tracking profile, the internal dynamics of the system is demonstrated to be unstable. The zero dynamics that obtained by finding a relevant control to nullify the output and its successive derivatives are unstable and almost for all initial conditions, exhibits somersault and oscillation motion for the vehicle. In this research stable system center is invoked to get an ideal stable internal dynamics for the output-reference profile which converges to a solution on the center manifold asymptotically. It is shown that in stable system center method, state reference profiles are identified such that state-tracking problem with compensated unstable internal dynamics contributes to output-tracking problem, thus allowing direct application of nonlinear control schemes.

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Reduction Principle in the Theory of Stability for Strongly Nonlinear Equations

Andrejs Reinfelds
University of Latvia, Riga, Latvia

Consider the following system of differential equations in small neigbourhood of origin in $\mathbb{R}^{n+k}$

$$\begin{cases} \dot{x} = & X(x,y), \\ \dot{y} = & Y_m(x,y) + g(x,y), \end{cases} \tag{1}$$

where $Y_m(\lambda y) = \lambda Y_m(y)$ $(\lambda \geq 0, m > 1)$, $F(x,y) = o((|x| + y|)^m)$ and $g(x,y) = o((|x| + y|)^m)$. We find sufficient conditions of the existence Lipschitzian map u in small neighbourhood of origin that the trivial solution of

$$\dot{x} = X(x, u(x)))$$

is stable, asymptotically stable or nonstable if and only if the trivial solution of difference equation (1) is stable, asymptotically stable or nonstable.

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Frequency-Domain Conditions for Convergence to the Stationary Set in Coupled PDEs

Volker Reitmann
St. Petersburg State University, Russia
(N. Yu. Yumaguzin)

We investigate the global stability of the stationary set of a coupled nonlinear hyperbolic-parabolic system arising in microwave heating processes. The system described by Maxwell's equations and the heat equation will be considered as control system with monotone nonlinearities. Using the transfer operator of the linear part of the system we derive frequency-domain conditions for the existence of Lyapunov functions which guarantee some convergence properties of solutions to the stationary set.

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Spectral Intervals and Their Associated Leading Directions for Differential-Algebraic Equations

Linh Vu
Vietnam National University, Hanoi, Vietnam
(Volker Mehrmann)

This talk is devoted to the spectral theory of linear time-varying differential-algebraic equations (DAEs). Concepts and numerical technique that have been known for ordinary differential equations (ODEs) are extended to DAEs. First, the analysis of spectral intervals and their associated solution subspaces is given. Then, we discuss numerical methods that are based on smooth matrix decompositions for approximating all or only some spectral intervals and their associated leading directions.

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Contributed Session 6: Control and Optimization

A Special Generalized Network Polyhedron Arising from Eco-Development

Yaw Chang
UNC-Wilmington, USA

A generalized network is a network with arc flows subject to multiplicative factors. The graph-theoretic characterization of a vertex of the constraint polyhedron for a generalized network is known as a one-tree. In this talk, we will present a generalized network model arising from eco-development. We will give a graph-theoretic characterization of a vertex of the constraint polyhedron for this special model. We will also present a general algorithmic approach to exploiting this property in vertex-enumeration.

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Invariant Feedback Design for Control Systems with Lie Symmetries: A Kinematic Car Example

Carsten Collon
Technische Universität Dresden, Germany
(Joachim Rudolph, Frank Woittennek)

Invariant tracking control design for control systems in state representation with classical Lie point symmetry is considered. The relevance of the invariance aspect is motivated by an exemplary control design for the kinematic car. Introducing the perpendicular distance between the position of the rear axle's midpoint and the reference trajectory and the contouring error as tracking errors an invariant feedback w.r.t. the action of SE(2) is derived. In fact, the use of compatible tracking errors allows the application of well-known feedback design approaches yielding an error dynamics that is invariant w.r.t. the group action. To this end we outline a normalization procedure from which invariant tracking errors can be derived and give a geometric interpretation w.r.t. the group orbits.

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Differential Equations of Ellipsoidal State Estimates in Nonlinear Control Problems under Uncertainty

Tatiana Filippova
Russian Academy of Sciences, Russia

The properties of set-valued states of differential control systems with uncertainties in the initial data are considered. It is assumed that the dynamical system has a special structure, in which the nonlinear terms in the right-hand sides of related differential equations are quadratic in the state coordinates. The model of uncertainty considered here is deterministic, with set-membership description of uncertain items which are taken to be unknown but bounded with given bounds. The solution to the uncertain differential system is introduced and studied in the framework of the theory of uncertain dynamical systems through the techniques of trajectory tubes of related differential inclusions.

The main problem of the paper is the problem of estimating the trajectory tubes and their time cross-sections (reachable sets) which are set-valued analogies of the real state of uncertain control system. For estimation of reachable sets of nonlinear differential inclusion we use results of the modern theory of ellipsoidal calculus and of the theory of evolution equations for dynamic systems under uncertainty. We construct the outer and inner ellipsoidal set-valued estimates of reachable sets of nonlinear control system and formulate related discrete-time numerical algorithms. Moving from discrete to continuous case we find differential equations of proposed ellipsoidal estimates of reachable sets of nonlinear control system. Numerical simulation results related to the procedures of set-valued approximations of trajectory tubes of nonlinear control systems are also given.

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Optimal Earth-Moon Transfers Using Low-Propulsion

Gautier Picot
Institut of Mathematics of Burgundy, France

In this talk, we will describe the principle of recent computations of optimal transfers between quasi-Keplerian orbits in the Earth-Moon system when low propulsion is applied. The spacecraft's motion is modelled by the equation of the controlled restricted 3-body problem and we based our work on previous studies concerning the orbit transfer in the 2-body problem where geometric and numeric method were developped to compute optimal solutions. More precisely, using numerical simple shooting and continuation methods connected with fundamental results from control theory, as the Pontryagin Maximum Principle and the second order optimality conditions related to the concept of conjugate points, we compute time-minimal and energy-minimal trajectories between the geostationary initial orbit and a final circular one around the Moon, passing through the neighborhood of the libration point L_2. The pertinence of our results is illustrated by comparing them to the practical details of the Smart-1 mission from the european space agency, during wich

electro-ionic engines were used. Our computations give simple trajectories which can be compared to the one of Smart-1 mission and improve the transfer time and the applied maximal thrust.

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Optimal Control of Pollutant Stock as Hierarchical Differential Game between Manufacturer and the State

Ellina Grigorieva
Texas Woman's University, USA
(Evgenii Khailov)

A model of the interaction between a manufacturer and the state is investigated where the manufacturer produces a single product and the state controls the level of pollution. In addition to pollution fines, the state imposes taxes on the sales revenue and on the total annual profit. The model is described by a nonlinear system of two differential equations with three bounded control. The interaction between the manufacturer and the state is considered as the hierarchical differential game in which the state is the leader and the manufacturer is the follower. The state starts the game by establishing the maximum allowed level of the pollution (control parameter) and other taxation. The manufacturer responds to the game using its three controls: the portion of the profit that is invested into production, the portion of the profit that is invested into pollution control, and the amount of the loan that the manufacturer takes in order to expand production. The goal of the state is minimize total environment pollution. The goal of the manufacturer is to maximize the total profit over a given horizon. Optimal policies for each player are obtained analytically with the use of Stackelberg differential game and Pontryagin Maximum Principle.

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On Polyhedral Estimates for Trajectory Tubes of Dynamical Discrete-Time Systems with Multiplicative Uncertainty

Elena Kostousova
Russian Academy of Sciences, Ekaterinburg, Russia

The paper is devoted to the state estimation problem in control theory under set-membership uncertainty. There are many approaches to solving these important problems. Some of them use techniques of ellipsoidal or polyhedral (parallelepiped-valued) estimates. Most of the results in this direction were obtained for linear systems with the uncertainty in initial states and additive controls. Here we study the case of dynamical discrete-time linear systems with a multiplicative (or bilinear) uncertainty when the system matrix is uncertain too. This, in turn, leads to additional difficulties due to nonlinearity of the problem (in particular, reachable sets may be non-convex even in the case of convex constraints).

We describe algorithms for constructing external and internal polyhedral estimates for reachable sets and trajectory tubes of such systems. Similarly to ellipsoidal calculus, we elaborate techniques for set-valued operations using parallelepipeds as basic sets. In particular, an important auxiliary problem was solved, namely the problem of finding an internal estimate for the set which is obtained by multiplying an interval matrix on a parallelepiped. All proposed parallelepiped-valued estimates can be calculated by explicit formulas. The results of numerical simulations based on the described algorithms are presented.

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Simulation of Diffusion Experiments in Environment Water Polluted

Tadeusz Kwater
University of Rzeszow, Poland
(Krutys Pawel, Bartman Jacek, Pekala Robert)

In the paper the mathematical model of distributed pollution concentration in flows with diffusion phenomena. The model consists of partial differential equations with boundary conditions. It can represent such objects as rivers and reservoirs with inflows. In this model concentrations of pollution can represent Biochemical Oxygen Demanded (BOD) and Dissolved Oxygen (DO). The numerical solution of the model with stability condition is also proposed. Simulations were carried out for different values of model parameters to investigate object behavior. Calculations were done for boundary conditions according to natural cases. These research are part of the project of the system monitoring the quality of water in big reservoirs.

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A Comparison of the Optimal Inventory Policies with and without Considering Differential Equations in Model Development

A. Mirzazadeh
Islamic Azad University, Karaj Branch, Iran
(A. Nosrati)

The inventory models are derived with considering: (1) The Differential Equations in Analysis of the Inventory System Behavior for calculation of Inventory System Costs such as holding and shortages costs and (2) other methods in Analysis of the Inventory System Behavior. Recently, the first method has been considered more than the other methods.

This paper compares the optimal ordering policies determined by these methods under some actual situations such as uncertainty. The deteriorating items with allowable shortages have been considered. The numerical examples are used to illustrate the theoretical results.

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The Optimal Ordering Policy with Using Differential Equations of Inventory System Behavior under Uncertain Conditions

A. Mirzazadeh
Islamic Azad University, Karaj Branch, Iran

This paper presents a new inventory model under inflation with considering practical and real situations. The inventory level has been calculated with using the differential equations. The available items deteriorate over time with the known rate and shortages are allowable. The total cost consists of the inventory holding cost, backordering cost, ordering cost purchasing cost is formulated as an optimal control problem. The practical experiences reveal that it is difficult to forecast the inflation with a well known value. Therefore, in this paper, the inflation rate has been considered stochastic. The numerical example and sensitivity analysis have been provided for evaluation and validation of the theoretical results.

Key words: Inventory, Differential Equations, Inflation, Uncertainty.

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New Regularizing Approach to Determining the Influence Coefficient Matrix for Gas-Turbine Engines

Sergey Yunusov
Transport and Telecomm. Institute, Riga, Latvia
(Sh. E. Guseynov)

As is well known, the influence coefficient matrix method is widely used for diagnostics of the gas turbine engines in during operational process. The essence of this method consists in construction and analysis of the diagnostic matrix $C = A^{-1}B$ on the basis of the minor deviations equations describing the gas turbine engine, where the matrix A contains the design parameters values; the matrix B contains the measured parameters values. The problem of construction the diagnostic matrix C for the engines with complicated structures involves the problem of finding the inverse design-identification matrix A that is, as a rule, a sparse matrix, i.e. it is an ill-conditioned matrix.

In present work the regularizing algorithm for stable determination of the diagnostic matrix C permitting to finding the characteristics of the gas-turbine bypass engine. In the basis of proposed algorithm underlies Tikhonov regularization method, where the regularizing parameter α from the Tikhonov smoothing functional $M^{\alpha}\left[z.u_{\delta}\right]$ is determined by some functional criterion.

The proposed regularizing algorithm is universal in a certain sense, and it can be apply (with the minimal modification) for determining the diagnostic matrices of different gas turbine engines with complicated structures in the case, when necessary information about some engine parameters is incomplete or absent. Such problems belong to the parametric uncertainty identification problems, and their investigation is still actual.

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Inverse Problems for Linear Ill-Posed Equations with Uncertain Parameters

Sergiy Zhuk
INRIA Paris-Rocquencourt, France

The talk describes a guaranteed state estimation approach for abstract linear ill-posed equations with uncertain parameters in Hilbert space. Duality theorems are presented, provided the uncertain parameters belong to some convex bounded closed set G. Tikhonov regularization is applied in order to derive Euler-Lagrange system, describing the state estimation. The results are then applied to the problem of the set-membership state estimation for linear differential-algebraic equation with rectangular non-stationary coefficients.

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Contributed Session 7: Scientific Computation and Numerical Algorithms

Visualization of Complex Analytic Vector Fields

Alvaro Alvarez-Parrilla
Universidad Autónoma de Baja California, Mexico
(Carlos Yee Romero, Selene Solorza Calderón)

We present a method that enables us to visualize complex analytic vector fields without having to use numerical integration techniques, we also prove that the method provides information on the parametrization of the solution, hence in fact solving the differential equation associated to the flow of the vector field.

We discuss the theory behind the method, its implementation, comparison with some integration-based techniques, as well as examples of the visualization of complex vector fields on the plane, on the sphere and on the torus (including some cases with essential singularities, and accumulation points of poles and zeros).

The proposed method is a first, and fundamental, step towards some possible generalizations that include visualization of vector fields in R^n.

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Convergence of Discontinuous Time-Stepping Schemes for the Gradient Minimization Boundary Control Problem

Konstantinos Chrysafinos
National Technical University of Athens, Greece

The minimization of the energy functional having states constrained to semi-linear parabolic PDEs is considered. The controls act on the boundary and are of Robin type. The discrete schemes under consideration are discontinuous in time but conforming in space. Stability estimates are presented at the energy norm and at arbitrary time points. The estimates are derived under minimal regularity assumptions and are applicable for higher order elements. Convergence of the discrete control is established. A discrete optimality system (first order necessary conditions) is derived and convergence of the corresponding discrete solutions is demonstrated.

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Improving the Accuracy of Geometric Numerical Schemes for Ordinary Differential Equations

Jan Cieśliński
University of Białystok, Poland
(Bogusław Ratkiewicz)

Geometric numerical integration consists in preserving geometric, structural and physical properties of a considered differential equation. We show how to enhance the accuracy of some standard geometric integrators (including the discrete gradient method) without losing their excellent qualitative properties.

In some cases it is possible to obtain the exact numerical integrator which yields the exact solution up to round-off errors. We use exact integrators to construct improved geometric schemes in more general cases (e.g., by the so called locally exact modification).

This is an extension and continuation of our recent work published in Physical Review E 81 (2010) 016704.

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Information from Partial Sums of the Riemann Zeta Function

Paul Deignan
Adavis LLC, USA
(Ellina Grigorieva)

Algorithmic improvements for the solution of a particular problem from data may be gained by the more efficient use of existing information present therein, i.e. in the utilization of inherent structure in what otherwise was considered to be meaningless variation. This report presents a preliminary demonstration of the possibility of improving upon the state-of-the-art algorithms for the estimation of the zeros of Riemann's Zeta function through an exploration of the structure of the partial sums of the zeta function.

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Numerical Solution of an Able Singularly Perturbed Volterra Integral Equation Presented by a Fractional Differential Equation

G. Hussian Erjaee
Qatar University
(M. Alnasr and H. Taghvafard)

Three different numerical methods are used to solve singularly perturbed Able Volterra integral equation as presented by a fractional differential equation. Convergence and stability analysis together with the results of these methods are compared and contrasted when applied to an Abel equation for the high thermal loss problem.

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Rigorous Analytic Validation of Invariant Tori on the Verge of a Hyperbolicity Breakdown

Jordi-Lluis Figueras
Universitat de Barcelona, Spain
(Alex Haro)

We implement computer assisted proofs for the existence of (normally hyperbolic) invariant tori in quasi-periodically forced systems. The proofs are based on a version of the Newton-Kantorovich theorem. We design a Fourier model for managing truncated Fourier series, providing rigorous estimates of the remainders. With these tools, we obtain rigorous and sharp bounds of the errors in the approximations of invariant tori, proving that those invariant tori do exist. We apply these techniques in different scenarios, with special emphasis to validate invariant tori close to their breakdown.

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A Numerical Method for Soft Landing on Mars

Seyed Hamed Hashemi Mehneh
Aerospace Research Institute, Iran
(H. Moghadaspour, B. Ebrahimi)

In this paper an optimal control problem dealing with soft landing on Mars is studied. One of the important issues in Mars or moon landing is to guide the surface lander in such a way that it reaches to a specified position with zero contact velocity. Referred as soft landing, this problem has obviously different solutions without imposing a performance index. One of the most useful performance indices one may apply here is fuel consumption. Due to requiring fuel for coming back to the parking orbit after mission, it will be useful if we could control the lander to the Mars surface with minimum fuel consumption. Therefore, the problem is modeled as to control a time-varying dynamical system from a given position and velocity to a desired state with zero contact velocity. Because of the time varying coefficients in differential equations, the analytical methods fail to find a solution and there are interests in numerical methods. For such an applicable problem that the initial conditions are unknown before launch and they are determined in descent stage, the required time for solving the problem is very critical. In this paper, the method of iterative dynamical programming (IDP) is used to solve the problem. Once the suitable initial conditions for landing are determined, the flight computer will find the control commands for thrusters in a reasonable time. To check the efficiency of the method we use hardware in the loop simulation (HITLS). A graphical user interface is developed to set the value of initial conditions, to see the resulting effect on the Mars surface lander in a visual environment and to compare the fuel consumption in two cases where optimal control is used and is not used. Case studies and HITLS show the efficiency of the proposed method for driving control commands. The method is also implemented parallel on flight computers with multiple microcontrollers to reduce the time of running IDP subroutine as well.

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Numerical Solution of Linear Elliptic Problems with Robin Boundary Conditions by a Least-Squares/Fictitious Domain Method

Qiaolin He
University of Science and Technology, Hong Kong
(Roland Glowinski)

Motivated by the numerical simulation of particulate flow with slip boundary conditions at the interface fluid/particles, our goal, in this presentation, is to discuss a fictitious domain method for the solution of linear elliptic boundary value problems with Robin boundary conditions. The method is of the virtual control type and relies on a least-squares formulation making the problem solvable by a conjugate gradient algorithm operating in a well chosen control space. Numerical results are presented; they suggest optimal orders of convergence for the finite element implementation of our fictitious domain method.

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Some Results on the Stability Analysis and Strong Approximation of Parabolic SPDEs

Seyed Mohammad Hosseini
Tarbiat Modares University, Tehran, Iran
(Minoo Kamrani)

A stochastic partial differential equation (SPDE) describes the dynamics of a stochastic process defined on a space-time continuum. In this paper we present an investigation of some new methods of solving SPDEs. At first for the approximate solution of the following SPDE

$$\begin{aligned} u_t(x,t) + au_{xx}(x,t) + bu_x(x,t) + cu(x,t) \\ +(du_x(x,t) + \gamma u(x,t))\dot{W}(t) = 0 \end{aligned}$$

the stability and application of a class of finite difference method with regard to the coefficients of the equation is analyzed. Then an error estimate of Galerkin approximation for stochastic parabolic equation is presented.

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The Optimality of Euler-Type Algorithms for Approximation of Stochastic Differential Equations with Discontinuous Coefficients

Paweł Przybyłowicz
AGH University of Science and Technology, Poland

We investigate pointwise approximation via Euler algorithms of scalar stochastic differential equations (SDEs) of the form

$$\begin{cases} dX(t) = \sigma_1(t)a(X(t))dt + \sigma_2(t)b(X(t))dW(t), \\ \qquad t \in [0,T], \\ X(0) = \eta, \end{cases} \tag{1}$$

where we allow existence of singularities for coefficients $\sigma_1, \sigma_2 : [0,T] \to \mathbb{R}$.

We first consider the regular case, when σ_1, σ_2 belong to the class of Hölder continuous functions with parameter $\varrho \in (0,1]$. In the additive noise case ($b \equiv const$), we show that the classical Euler algorithm X^E has the optimal worst case error $\Theta(n^{-\varrho})$, $\varrho \in (0,1]$, among all adaptive algorithms. In the multiplicative noise case, we prove that the algorithm X^E has the error $O(n^{-\min\{1/2,\varrho\}})$, which is optimal if $\varrho \in (0,1/2]$.

In the singular case, we consider a class of functions σ_1, σ_2 that are piecewise Hölder continuous, except for a finite number of unknown singular points. We investigate error of the Euler algorithm X^E and conclude that only singularities of σ_2 have influence on its accuracy. This will allow us to show that, in the additive and multiplicative noise case, the algorithm X^E has the error $O(n^{-\min\{1/2,\varrho\}})$. Next, we show that any nonadaptive algorithm with respect to σ_2 has the error which is no less than $n^{-\min\{1/2,\varrho\}}$, even if there is at most one unknown singular point. In order to preserve the same order of convergence as for regular functions, we therefore turn to adaptive algorithms. In the additive noise case, when coefficient σ_2 has at most one singularity, we construct the adaptive Euler algorithm $\bar{X}^E$ which locates singularity for σ_2 and preserves the optimal error $\Theta(n^{-\varrho})$, known from the regular case. We also consider the case of multiple singularities.

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A Computational Methodology for Calculation of Green's Function Matrix Operator for Representing Surface Deformations Due to Faulting with Reference to the San Andreas Fault Near Parkfield, California

Kaveh Shakiba Nia
K. N. Toosi University of Technology, Tehran, Iran
(H. Ghasemzadeh)

Green's function is a type of function utilized to solve inhomogeneous differential equations subject to boundary conditions. Green's function uses a linear, self-adjoint differential operator which can not usually be calculated for representing surface deformations due to an earthquake using conventional methods. This paper aims at developing a computational methodology for calculation of Green's function matrix operator for representing surface deformations due to faulting in plane-strain condition. The inverse of this matrix can be used to predict magnitude and location of fault movements for any measured surface displacements. The San Andreas Fault near Parkfield, California is simulated using finite element method. This methodology can be used to compute Green's function matrix operator in order to predict magnitude and location of fault movements or surface deformations due to earthquakes.

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Numerical Approach to Repeated Support Splitting and Merging Phenomena in Some Nonlinear Diffusion Equation

Kenji Tomoeda
Osaka Institute of Technology, Japan

The interaction between diffusion and absorption suggests a remarkable property in the behavior of the support of the solution; that is, after support splitting phenomena appear, the support merges, and thereafter the support splits again. As a mathematical model which describes such phenomena we consider the porous media equation with a strong absorption. In this talk, the finite difference scheme which realizes such phenomena is proposed, and the convergence of this scheme is shown. Moreover, *numerically repeated support splitting and merging phenomena* for the initial-boundary value problem are demonstrated.

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FPT Problems in Modeling Complex Systems in Finance: A Monte Carlo Approach

Olena Tsviliuk
JSC Rodovid, Ukraine
(R. Melnik, Di Zhang)

Many examples of complex systems in mathematical modeling are provided by applications in finance and economics areas. Some of intrinsic features of such systems lie with the fact that their parts are interacting in a non-trivial dynamic manner and they can be subject to stochastic forces and jumps. The mathematical models for such systems are often based on stochastic differential equations and efficient computational tools are required to solve them.

Here, on an example from the credit risk analysis of multiple correlated firms, we develop a fast Monte-Carlo type procedure for the analysis of complex systems such as those occurring in the financial market. Our procedure is developed by combining the fast Monte-Carlo method for one-dimensional jump-diffusion processes and the generation of correlated multidimensional variates. As we demonstrate on the evaluation of first passage time density functions in credit risk analysis, this allows us to analyze efficiently multivariate and correlated jump-diffusion processes.

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Eigenvalue Problems for the Zakharov-Shabat System Using Spectral Parameter Power Series

Ulises Velasco
Cinvestav-Queretaro, Mexico
(Vladislav V. Kravchenko)

The Spectral Parameter Power Series (SPPS) method [1] is a powerful tool for obtaining series representations for solutions of the Sturm-Liouville equation under certain restrictions. It also provides an efficient method to obtain numerical solution of spectral Sturm-Liouville problems reducing them to calculation of roots of corresponding polynomials.

In the present work we consider the SPPS method applied to the eigenvalue problems for the Zakharov-Shabat differential system.

[1] V. V. Kravchenko, R. Michael Porter Spectral parameter power series for Sturm-Liouville problems. Mathematical Methods in the Applied Sciences, 2010, v. 33, issue 4, 459-468.

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Adaptive Monte Carlo Methods for SDEs

Georgios Zouraris
University of Crete, Greece
(E. Mordecki, A. Szepessy, R. Tempone)

We propose a general framework to construct adaptive methods for the weak approximation problem of Itô stochastic differential equations, which we apply on variances of the Drift Implicit Euler method.

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Finite Element Approximations for a Linear Cahn-Hilliard-Cook Equation

Georgios Zouraris
University of Crete, Greece
(G. Kossioris)

We construct fully-discrete approximations to the solution of an initial and Dirichlet boundary value problem for a linear Cahn-Hilliard-Cook equation, in one space dimension, forced by the space derivative of a space-time white noise. The proposed method uses, a Galerkin finite element method for the discretization in space, and, the backward Euler method for time-stepping. The convergence of the method is also analyzed by proving strong error estimates.

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Contributed Session 8: Topological Methods and Fixed Points

Fixed Point Theorems in Banach Valued Metric Spaces

Erdal Karapinar
Atilim University, Ankara, Turkey

Banach-valued metric space, namely cone metric spaces (in short CMS), is a generalization of the notion of the metric space that is obtained by replacing real numbers with an ordered real Banach space in the definition of metric (see e.g. [Rzepecki, B.: On fixed point theorems of Maia type, *Publications De L'institut Mathématique* 28, 179-186, (1980)], [Lin, Shy-Der: A common fixed point theorem in abstract spaces, *Indian J. Pure Appl. Math.,* 18(8), 685–690 (1987)] and [Huang Long-Guang, Zhang Xian: Cone metric spaces and fixed point theorems of contractive mappings, *J. Math. Anal. Appl.,* 332, 1468–1476 (2007)]). In Huang Long-Guang, Zhang Xian discussed some properties of convergence of sequences and proved the fixed point theorems of contractive mapping for cone metric spaces: Any mapping T of a complete cone metric space X into itself that satisfies, for some $0 \leq k < 1$, the inequality $d(Tx, Ty) \leq kd(x, y)$, for all $x, y \in X$, has a unique fixed point.

In this talk, some of known results (see, e.g. [Ćirić, L.: Fixed Point Theory – Contraction Mapping principle *Faculty of mechanical Engineering (Beograd)* (2003)] and [Suzuki, T.: Fixed point theorems and convergence theorems for some generalized nonexpansive mappings, *J. Math. Anal. Appl.*, 340, 1088-1095 (2007)]) are extended to cone Banach spaces where the existence of fixed points for self-mappings on cone Banach spaces are investigated.

⟶ ∞ ⋄ ∞ ⟵

On Nonmeasurable Sets

Paula Kemp
Missouri State University, USA

In this paper, it is shown that there are nonmeasurable sets V with outer measure of V being equal to any preassigned positive real number less than or equal to 1. Also, the set of real numbers will be given as a disjoint union of countably many congruent by translation nonmeasurable subsets each of which has a preassigned outermeasure and of inner measure 0. It is also shown that every uncountable set of real numbers has an uncountable subset which is nowhere dense and is independent of the axioms of ZFC in set theory.

⟶ ∞ ⋄ ∞ ⟵

Spherical Designs Via Brouwer Fixed Point Theorem

Maryna Viazovska
Max Planck Institute for Math., Bonn, Germany
(Andriy Bondarenko)

Let S^d be the unit sphere in $\mathbb{R}^{d+1}$ with normalized Lebesgue measure $d\mu_d$. A set of points $x_1, \dots, x_N \in S^d$ is called a *spherical n-design* if

$$\int_{S^d} P(x) d\mu_d(x) = \frac{1}{N} \sum_{i=1}^{N} P(x_i)$$

for all algebraic polynomials in $d+1$ variables and of total degree at most n.

Construction of spherical n-design with minimal cardinality for fixed d and $n \to \infty$ is a difficult analytic problem even for $d = 2$. For each $n \in \mathbb{N}$ denote by $N(d,n)$ the minimal number of points in a spherical n-design. The following lower bounds are known

$$N(d,n) \geq \binom{d+k}{d} + \binom{d+k-1}{d} \text{ for } n = 2k, \quad (1)$$

$$N(d,n) \geq 2\binom{d+k}{d} \text{ for } n = 2k+1.$$

It follows from (1) that $N(d,n) \geq C_d n^d$ and the long standing conjecture is that $N(d,n) \leq c_d n^d$. In [1] we proved the following

Theorem 1. *For each $d \geq 3$ and $n \in \mathbb{N}$ we have $N(d,n) \leq c_d n^{a_d}$, where $a_3 \leq 4$, $a_4 \leq 7$, $a_5 \leq 9$, $a_6 \leq 11$, $a_7 \leq 12$, $a_8 \leq 16$, $a_9 \leq 19$, $a_{10} \leq 22$, and $a_d < \frac{d}{2} \log_2 2d$ for $d > 10$.*

In [2] we suggested a new nonconstructive approach for obtaining new upper bounds for $N(d,n)$. Using area-regular partitions of the sphere and Brouwer fixed point theorem we prove

Theorem 2. *For each $N \geq c_d n^{\frac{2d(d+1)}{d+2}}$ there exists a spherical n-design on S^d consisting of N points.*

[1] A. Bondarenko, M. Viazovska, *New asymptotic estimates for spherical designs,* Journal of Approximation Theory, 152 (2008) 101–106.
[2] A. Bondarenko, M. Viazovska, *Spherical designs via Brouwer fixed point theorem,* arXiv:0811.1416v1 [math. NA].

⟶ ∞ ⋄ ∞ ⟵

How Strange Can an Attractor in a 3-Manifold Look Like?

Jaime Jorge Sánchez-Gabites
University of Warwick, England

In this talk we shall consider whether there are any distinctive topological features to those compact subsets K of 3–manifolds M that are (stable and not necessarily global) attractors for a dynamical system in M. It will be seen that the way K is embedded in M plays a crucial role in this question, linking it with some classical geometric topology from the twentieth century. Drawing on some beautiful work of Alexander, Fox and Artin, a number of surprising examples will be presented.

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Contributed Session 9: PDEs and Applications

Non-Stationary Vibrations of a Thin Viscoelastic Disc

Vitezslav Adamek
University of West Bohemia, Pilsen, Czech Rep.
(Frantisek Vales)

This work deals with the analytical solution of the system of two hyperbolic partial integro-differential equations. This system describes non-stationary wave phenomena in a thin viscoelastic disc of finite radius. Non-stationary in plane vibrations of the disc are investigated for the case of radial excitation acting on the part of the disc boundary. The method of integral transforms and the Fourier method are used for the derivation of the Laplace transforms of required displacement components. With respect to the complexity of integral transforms obtained, nu-

merical approach to the inverse Laplace transform is used. Spatio-temporal dstribution of radial and circumferential displacement components represent main results of this work.

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Nonlinear Differential Equation with Module-Fractional Derivative on a Half-Line

Martin Arciga
Institute of Physics and Math., UMSNH, Mexico

We consider the initial-boundary value problem for a nonlinear differential equation with module-fractional derivative on a half-line. We study the local and global in time existence of solutions to the initial-boundary value problem and the asymptotic behavior of solutions for large time.

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On Some Nonlocal in Time Problems for Evolution Equations

Gia Avalishvili
Tbilisi State University, Rep. of Georgia
(Mariam Avalishvili)

The present paper is devoted to the investigation of nonlocal in time problems for evolution equations of mathematical physics, where instead of classical initial conditions a dependence of the initial values of the solution on its values for later times is given. Nonlocal in time problems are used for mathematical modeling of various processes, particularly, for investigation of radionuclides propagation in Stokes fluid, diffusion and flow in porous media.

In the present paper, we investigate nonclassical problems for abstract first-order and second-order evolution equations with time-dependent coercive operators and the initial conditions defined by general nonlocal operators. We prove the existence and uniqueness theorems for nonlocal in time problems in suitable spaces of vector-valued dstributions; we construct and investigate algorithms of approximation of such type nonclassical problems by classical ones. In addition, we consider applications of the obtained general results to nonlocal in time initial-boundary value problems for parabolic and hyperbolic equations and systems. We show that in the case of hyperbolic equations for a specific nonlocal in time problems, the existence and uniqueness of their solutions depend on the arithmetic properties of expressions containing time moments and geometric characteristics of the space domain.

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On the Investigation of Nonclassical Problems with Nonlocal Boundary Conditions

Gia Avalishvili
Tbilisi State University, Rep. of Georgia
(Mariam Avalishvili)

The boundary and initial-boundary value problems with nonclassical boundary conditions are widely used for mathematical modeling of various processes of technology, ecology and other fields. In nonclassical problems with nonlocal boundary conditions a relationship between the values of the unknown function on the whole boundary or on its part and in internal points of the space domain is given. Nonlocal problems with integral boundary conditions express a conservation of corresponding quantity (mass, heat, momentum and etc.). Nonlocal problems with discrete nonlocal boundary conditions are used for mathematical modeling of decreasing of pollution caused by natural factors of self-purification of the environment.

In the present paper, we study nonlocal boundary value problems for elliptic and hyperbolic partial differential equations and systems. We investigate nonclassical initial boundary value problems with general nonlocal boundary conditions for hyperbolic equations, systems of partial differential equations arising in the theory of elasticity and theory of elastic mixtures. We study nonclassical boundary value problems for elliptic equations with integral nonlocal boundary conditions. Applying variational approach and specific multipliers, we prove the existence and uniqueness of solution in corresponding Sobolev spaces. Moreover, we investigate initial-boundary value problems with integral nonlocal boundary conditions for hyperbolic equations.

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Stable Determination of a Body Immersed in a Stationary Stokes Fluid

Andrea Ballerini
SISSA/ISAS, Italy

We consider the inverse problem of the detection of a single body, immersed in a bounded container filled with a fluid which obeys the Stokes equations, from a single measurement of force and velocity on a portion of the boundary. We obtain an estimate of stability of log-log type.

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Mathematical Analysis of Some Model in Thermo-Visco-Plasticity

Leszek Bartczak
Warsaw University of Technology, Poland

We consider the viscoplasticity equations coupled with the heat equation. This model describes a phenomena in a deformed metal additionally exposed to a heat modification. On the one hand the temperature “controls” the domain of an elastic behaviour of the body, on the other hand the strain and the stress appearing in the body in uences the heat conduction. We prove the existence and the uniqueness of a solution to the considered model.

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One *May* Hear the Shape of a Drum

Mohamed Ben Rhouma
SQU-Oman, Oman

(L. Hermi and M. A. Khabou)

In 1966, M. Kac wrote his famous paper “Can one hear the shape of a drum?” referring to the problem of inferring knowledge about a planar domain Ω from the eigenvalues of the Laplacian operator on Ω subject to Dirichlet boundary conditions. The final answer to this question turned out to be negative and is due to Gordan, Webb and Wolpert who gave in 1992 the first example two isospectral shapes. In this talk, we will show that while the eigenvalues of several linear operators do not uniquely determine the shape of the domain, the ratio of the first few eigenvalues can be used as a reliable tool for shape recognition. In particular, we will discuss the ratios of eigenvalues of the Dirichlet and Neumann Laplacian operators as well as those of the bi-Laplacian and the clamped plate problem. In addition, we will also compare the performance of each of these operators as a tool for shape recognition on a real data set.

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Some Properties of Solutions to the Viscous Navier-Stokes Flows in a Channel

Michal Benes
CTU Prague, Czech Rep.

(Petr Kucera)

We consider the system of the non–steady Navier–Stokes equations with mixed boundary conditions. We study the existence and uniqueness of a solution of this system. We define Banach spaces X and Y, respectively, to be the space of “possible” solutions of this problem and the space of its data. We define the operator $\mathcal{N} : X \to Y$ and formulate our problem in terms of operator equations. Let $u \in X$ and $\mathcal{G}_u : X \to Y$ be the Frechet derivative of $\mathcal{N}$ at u. We prove that $\mathcal{G}_u$ is one-to-one and onto Y. Consequently, suppose that the system is solvable with some given data (the initial velocity and the right hand side). Then there exists a unique solution of this system for data which are small perturbations of the previous ones.

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Oblique Scattering of Water Waves by an Uneven Ocean-Bed in a Two-Layer Fluid with Ice-Cover

Swaroop Bora
Indian Institute of Technology Guwahati, India

(Smrutiranjan Mohapatra)

Within the framework of linearized theory, obliquely incident water wave scattering by an uneven ocean-bed in the form of a small bottom undulation in a two-layer fluid, where the upper layer has a thin ice-cover while the lower one has an undulation, is investigated here. In such a two-layer fluid there exist two modes of time-harmonic waves – the one with lower wave number propagating just below the ice-cover and the one with higher wave number along the interface. An incident wave of a particular mode gets reflected and also transmitted by the bottom undulation into waves of both the modes. A perturbation analysis is employed to solve the first order corrections to the velocity potentials by solving modified Helmholtz equation and using Fourier transform appropriately. The reflection and transmission coefficients are calculated in terms of integrals involving the shape function representing the bottom undulation. For sinusoidal bottom topography, these coefficients are depicted graphically against the wave number. It is observed that when the oblique wave is incident on the ice-cover surface we always find energy transfer to the interface, but for interfacial oblique incident waves there are parameter ranges for which no energy transfer to the ice-cover surface is possible.

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Anisotropic Variational Problems in Fluid Mechanics and Nonlinear Elasticity

Dominic Breit
Saarland University, Germany

(Martin Fuchs)

We discuss partial regularity results concerning local minimizers $u : \mathbb{R}^3 \supset \Omega \to \mathbb{R}^3$ of variational integrals of the form

$$\int_\Omega \{h(|\varepsilon(w)|) - f \cdot w\}\, dx$$

defined on appropriate classes of solenoidal fields, where h is a N-function of rather general type. As a byproduct we obtain a theorem on partial C^1-regularity for weak solutions of certain non-uniformly elliptic Stokes-type systems modelling generalized Newtonian fluids.
In order to model nonlinear elastic material behaviour, in particular the nonlinear Hencky material, we have a look at variational integrals of the form

$$\int_\Omega \left\{a(|\varepsilon^D(w)|) + b(|\mathrm{div}(w)|)\right\}\,dx,$$

where a and b are N-functions of rather general type (note that this integral represents the deformation energy). We prove partial regularity results under quite natural conditions between a and b. Furthermore we can extend this to the non-autonomous situation which finally leads to the study of minimizers of the functional

$$\int_\Omega \left\{(1+|\varepsilon^D(w)|^2)^{\frac{p(x)}{2}} + (1+|\mathrm{div}(w)|^2)^{\frac{q(x)}{2}}\right\}\,dx,$$

where p and q are Lipschitz-functions.

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Cylindrical Vibration of Cusped Reisner-Mindlin Plates

Natalia Chinchaladze
Tbilisi State University, Rep. of Georgia

In the XX I. Vekua (see, e.g., Vekua, I. N., 1985, “Shell Theory: General Methods of Construction”, Pitman Advanced Publishing Program, Boston-London-Melbourne) raised the problem of investigation of cusped plates, i.e. such ones whose thickness on the part of the plate boundary or on the whole one vanishes. The problem mathematically leads to the question of setting and solving of boundary value problems for even order equations and systems of elliptic type with the order degeneration in the statical case and of initial boundary value problems for even order equations and systems of hyperbolic type with the order degeneration in the dynamical case.

Admissible static and dynamical problems are investigated for a cusped Reisner-Mindlin plates. The setting of boundary conditions at the plates ends depends on the geometry of sharpenings of plates ends, while the setting of initial conditions is independent of them.

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Classical Global Solutions for a Class of Eikonal Equations

Jaime Cruz-Sampedro
Universidad Metropolitana, Mexico

Let V be a real-valued function of class C^2 on $\mathbb{R}^n$, $n \geq 2$, which vanishes if $|x| \leq R$ and, for some $\varepsilon > 0$, satisfies $\partial_x^\alpha V(x) = O(|x|^{-\varepsilon-|\alpha|})$, as $|x| \to \infty$, for $|\alpha| \leq 2$. Using a global inverse function theorem of Hadamard, we show that if R is sufficiently large, then the Hamilton-Jacobi equation of eikonal type $|\nabla u|^2 + V(x) = k^2$, with $k > 0$, has a C^1 solution on $\mathbb{R}^n \setminus \{0\}$.

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Cascades of Heteroclinic Connections in Viscous Balance Laws

Julia Ehrt
WIAS & Humboldt Universität Berlin, Germany

The talk investigates the relation between the global attractors of viscous balance laws and hyperbolic balance laws on the circle for vanishing viscosity given by:

$$u_t + [f(u)]_x = \varepsilon u_{xx} + g(u) \qquad \text{(P)}$$

and

$$u_t + [f(u)]_x = g(u). \qquad \text{(H)}$$

The talk focuses on the question of convergence of heteroclinic orbits of the parabolic equation (P) to solutions of the limit equation (H) when $\varepsilon \to 0$. Despite a result of Fan and Hale from 1995 that proves persistence of heteroclinics I will present a surprisingly easy necessary condition for their persistence which proves the Fan Hale result to be wrong. In case of non-convergence to a singular heteroclinic the talk will show convergence to a cascade of heteroclinic orbits of the limiting equation.

The talk closes with some remarks on the consequences for the relation of the global attractors of (P) and (H) for vanishing ε.

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Discrete and Continuous Spectra of Singular Elliptic Operators

Alexei Filinovskiy
Moscow State Technical University, Russia

We study spectral properties of the elliptic singular differential operators with Dirichlet boundary condition in domains $\Omega \subset R^n$, $n \geq 2$.

Our special interest are spectral properties of elliptic operators generated by formally non-self-adjoint differential expressions as weighted Laplace operator $lu = -(1/\rho(x))\Delta u$ where $\rho(x) \in C(\overline{\Omega})$ is a positive function. We shall treat the differential operator in the Hilbert space $L_{2,\rho}(\Omega)$ with norm $\|u\|^2_{L_{2,\rho}(\Omega)} = \int_\Omega |u|^2 \rho\,dx$. Let L be the self-adjoint Friedrichs extension in $L_{2,\rho}(\Omega)$ of the minimal operator generated by differential expression l. There-

fore, L is a positive self-adjoint operator in $L_{2,\rho}(\Omega)$ that is an operator of the Dirichlet problem.

We will investigate location of spectrum on the real axis, density of spectrum on some sets and structure of spectrum for unbounded domains whose boundary satisfy some geometrical conditions. We study the structure of spectrum with respect to the behavior of coefficient ρ and geometry of the boundary. We also investigate the critical asymptotic behavior of ρ providing spectrum to be discrete. Further we give an estimate to the rate of condensation of discrete spectra under the transition to continuous.

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Analysis of a Polymerization Model Involved in Prion Proliferation

Pierre Gabriel
Laboratoire Jacques-Louis Lions, Paris 6, France
(Vincent Calvez, Marie Doumic Jauffret)

Polymerization of proteins is a central event in the mechanism of prion diseases. We model this process by a transport equation with a non local fragmentation term. This PDE is non linearly coupled to an ODE which leads the quantity of monomers. The steady states of this system are investigated through the eigenelements of the PDE. In particular, the dependency of the first eigenvalue on parameters is a key point.

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Pattern Formation Driven by Cross-Diffusion in a Two Dimensional Domain

Gaetana Gambino
University of Palermo, Italy
(Maria Carmela Lombardo, Marco Sammartino)

In this work we describe the mechanism of pattern formation for a reaction-diffusion system with nonlinear diffusion terms (which take into account the self and the cross-diffusion effects). The reaction terms are chosen of the Lotka-Volterra type in the competitive interaction case. The cross-diffusion is proved to be the key mechanism of pattern formation via a linear stability analysis. A weakly nonlinear multiple scales analysis in a 2D spatial domain is carried out to predict the amplitude and the form of the pattern close to the bifurcation threshold. In particular, the studied system supports patterns as rolls, squares and rhombi if the bifurcation occurs via a unique eigenvalue and the Stuart-Landau equation is shown to rule the evolution of the amplitude of the most unstable mode. More complex patterns arise when the bifurcation occurs via a double eigenvalue and two coupled Landau equations for the two amplitudes are found and analyzed. The solutions predicted by the weakly nonlinear analysis both in the cases of single and double eigenvalue are compared with the numerical solutions of the original system and they show a good agreement.

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Motion of Space Curves in Minkowski Space, the Defocusing Nonlinear Schrödinger Equation and the So(2, 1) Spin Equations

Muniraja Gopal
Christian College, Bangalore, India
(M. Lakshmanan)

Modeling of physical systems by curves, surfaces and other differential geometric objects is highly rewarding, for example as the pioneering work of Hasimoto on vortex filaments and the one-dimensional continuum Heisenberg ferromagnetic spin equation by Lakshmanan, Ruijgork and Thompson demonstrate. In both cases the systems were shown to be equivalent to the integrable nonlinear Schrödinger equation (NLSE) of the focusing type. In recent times the relation between differential geometry and certain dynamical systems described by nonlinear evolution equations in (1+1) and (2+1) dimensions, especially the integrable systems, has come into sharp focus. Now it is well known that a class of important soliton equations can be interpreted in terms of moving space curves and the linear eigenvalue problems of the soliton equations can be obtained from the defining Serret-Frenet equations.

The defocusing NLSE, in which the sign of the nonlinear term is negative, was shown to be integrable by the inverse scattering transform method and admits dark soliton solutions. However, there does not seem to be available a simple differential geometric model such as available for the focusing NLSE. We try to address this problem here using the rich geometric structure of the three dimensional Minkowski space.

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On Analytical Solutions of Some Continuous Models for Determining the Volumes of Current Supplies of Divisible Productions

Sharif Guseynov
Inst. of Math. Sci. and Inf. Techn., Riga, Latvia
(E. A. Kopytov, E. Puzinkevich)

In the given paper we investigate the problem of constructing continuous and unsteady mathematical models for determine the volumes of current stock of divisible productions using apparatus and equa-

tions of mathematical physics. It is assumed that time of production dstribution and replenishment is continuous. The constructed models are stochastic, and have different levels of complexity, adequacy and application potentials. The simple models are constructed using the theory of ordinary differential equations, for construction of more complex models it is applied the theory of partial differential equations. Furthermore for some of proposed models we have found an analytical solution in the closed form, and for some of proposed models the discretization is carried out using stable difference schemes.

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Dynamics of the Fluid Stricture for Nonlinear Beam, Interacting with Fluid Flows (Stability Analyses and Simulation)

Akif Ibragimov
Texas Tech. University, Lubbock, USA
(E. Aulisa, Y. Kaya)

In this work we consider the dynamical response of a non-linear beam with changing thickness and viscous damping, perturbed in both the vertical and axial directions interacting with fluid flow. The system is modeled using non-linear momentum equations for the axial and transverse displacements coupled with the fluid flow subjected to laminar law. In particular, we show that under an a priori assumption the dynamics of the coupled fluid structure system is stable with respect to appropriate Lyapunov type functional. It was numerically verified that imposed structural conditions on the the beam thickness dynamics are essential for stability.

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Matrix Equations and Trilinear Commutation Relations

Anvar Juraev
Samarkand State University, Uzbekistan

In this talk we discuss a general algebraic approach to treating static equations of matrix models with a mass-like term. In this approach the equations of motions are considered as consequence of parafermi-like trilinear commutation relations. In this context we consider several solutions, including construction of noncommutative spheres. The equivalence of fuzzy spheres and parafermions is underlined.

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Noncommutative Fields and Actions of Twisted Poincare Algebra

Anvar Juraev
Samarkand State University, Uzbekistan

Within the context of the twisted Poincaré algebra, there exists no noncommutative analogue of the Minkowski space interpreted as the homogeneous space of the Poincaré group quotiented by the Lorentz group. The usual definition of commutative classical fields as sections of associated vector bundles on the homogeneous space does not generalise to the noncommutative setting, and the twisted Poincaré algebra does not act on noncommutative fields in a canonical way. We make a tentative proposal for the definition of noncommutative classical fields of any spin over the Moyal space, which has the desired representation theoretical properties. We also suggest a way to search for noncommutative Minkowski spaces suitable for studying noncommutative field theory with deformed Poincaré symmetries.

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Applications of Finite Differences to the Study of Regularity Properties of Solutions to the Problems from Viscoplasticity Theory

Przemysław Kamiński
Warsaw University of Technology, Poland

We will present how the method of finite differences can be applied to the study of regularity properties of solutions to the equations from viscoplasticity theory, as formulated by H.-D. Alber in his book "Materials with memory". The method works similarly for quasistatic and dynamic problems. Our basic assumption is that the velocity of displacements is in the space $L^2((0,T);H^1(\Omega))$. We will also present how this assumption can be weakened by analyzing the power-type model of Norton-Hoff.

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Singularities in the Geometrical Theory of Differential Equations

Ulrike Kant
University of Kassel, Germany
(Werner M. Seiler)

At first we will give an introduction to the geometrical approach to differential equations using the jet bundle formalism. Here the construction of the Vessiot dstribution will be of special interest. By using this dstribution different types of singular behaviour appearing in the context of differential equations can be understood. We will illustrate this giving suitable examples. If time permits we will discuss how bifurcations can be understood as singularities of associated differential equations.

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Kelvin-Helmholtz Instability Model for Supersonic Adiabatic Jet

Oleg Kelis
Ort Braude College, Israel

The linear Kelvin-Helmholtz instability model for supersonic adiabatic viscid jet is developed for non uniform spatial dstribution of the phon jet's temperature [1]. Dispersion relation based on conservation equations (Navier-Stokes and energy equations) is deduced. The general formula involving Heun functions in simplified cases coincides with various forms of dispersion relation; in resonance cases it reduces to the simple algebraic equations. It is shown that an expanded adiabatic jet is unstable to Kelvin-Helmholtz perturbed modes - they can grow with nonlinear amplitudes. To demonstrate further applications the instability modes have been found and investigated without detailed numerical calculations for complicated hydrodynamic model.

This work was made jointly with Igor Gaissinski (Aerospace Engineering Department, Technion - Israel Institute of Technology, Haifa 3200, Israel) and Vladimir Rovenski (Department of Mathematics Faculty of Science and Science Education, University of Haifa Mount Carmel, Haifa, 31905, Israel).

Keywords: Kelvin-Helmholtz instability, perturbations, viscosity, supersonic adiabatic jet.

[1] Gaissinski I. M., Rovenski V. Yu. and Kelis O. Kelvin-Helmholtz Instability Model for a Supersonic Adiabatic Jet. Int. J. of Pure and Applied Math., 44(4), 2008, 235-248.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Geometric Aspects of the Family of b-Equations

Martin Kohlmann
Leibniz University Hannover, Germany

In recent years, the Camassa-Holm (CH) equation and the Degasperis-Procesi (DP) equation attracted a lot of attention in the mathematical physics literature. Having an integrable structure, these equations are known to be appropriate approximations to the governing equations for the classical water wave problem in the shallow-water medium-amplitude regime. In this talk, I will explain a geometric reformulation of a more general family of evolution equations including CH and DP. Studying those equations as geodesic flow on the diffeomorphism group of the circle, I will present some local well-posedness results. The talk is based on research supported by the DFG Graduiertenkolleg 1463 of the Leibniz University of Hannover.

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On Global Attractors of Nonlinear Hyperbolic PDEs

Alexander Komech
Vienna Univ. and IITP RAS (Moscow), Austria
(Andrew Komech)

We consider Klein-Gordon and Dirac equations coupled to U(1)-invariant nonlinear oscillators. The solitary waves of the coupled nonlinear system form two-dimensional submanifold in the Hilbert phase space of finite energy solutions. Our main results read as follows:

Theorem. Let all the oscillators be strictly nonlinear. Then any finite energy solution converges, in the long time limit, to the solitary manifold in the local energy seminorms.

The investigation is inspired by Bohr's postulates on transitions to quantum stationary states.

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On Asymptotic Stability of Kink for Relativistic Ginzburg-Landau Equation

Elena Kopylova
Inst. for Inf. Transmission Problems RAS, Russia

We prove the asymptotic stability of the kinks for the nonlinear relativistic wave equations of the Ginzburg-Landau type in one space dimension: starting in a small neighborhood of the kink, the solution, asymptotically in time, is the sum of a uniformly moving kink and dispersive part described by the free Klein-Gordon equation. The remainder decays in a global energy norm. Crucial role in the proofs play our recent results on the weighted energy decay for the Klein-Gordon equations.

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Water Flow in Unsaturated Soil with Preisach Hysteresis

Petra Kordulová
Silesian University, Opava, Czech Republic

Unsaturated water flow in porous media is described by a class of PDEs whose model equation is represented by the Richards equation with soil-moisture hysteresis term. We assume that the hysteresis is represented by the Preisach hysteresis operator. Under suitable assumptions on the hysteresis operator we investigate the existence of solution of our problem. An existence result is proved by a method based on an implicit time discretization scheme, a-priori estimates and passage to the limit.

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A Regularity Criterion for an Axially Symmetric Solution to the Navier-Stokes Equations

Adam Kubica
Warsaw University of Technology, Poland

We consider an axially symmertic solutions of the Navier-Stokes equations. In papers [1] and [2] the authors proved the regularity of solution under the assumption that the radial or angular component of velocity satisfy Serrin-type condition. I will present an analogous result, which holds under weighted version of Serrin-type condition, where the weight is a power of the distance from the axis of symmetry of the solution.

[1] J. Neustupa, M. Pokorny, *An interior regularity criterion for an axially symmetric suitable weak solution to the Navier-Stokes equations*, J. Math. Fluid Mech.2 (2000), no. 4, 381-399.
[2] O. Kreml, M. Pokorny, *A regularity criterion for the angular velocity component in axisymmetric Navier-Stokes equations*, Elec. J. Diff. Eq., Vol 2007, No. 08, pp.1-10.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Radial Solutions and the Cessation of Snaking for the Multi-Dimensional Swift-Hohenberg Equation

Scott Mccalla
Brown University, USA
(Bjorn Sandstede)

The bifurcation structure and the cessation of snaking of localized radial patterns of the Swift–Hohenberg equation are explored through numerical computations as the dimension is varied. Our findings elucidate the connection between one-dimensional pulses and 2-pulses to planar spots and rings when the dimension is changed from one to two and then further to three. We also discovered a new class of planar localized spot solutions and discuss an analytical approach to establishing their existence.

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On a Nonlinear Evolution Coupled System with a Non Classical Conditions

Said Mesloub
King Saud University, Saudi Arabia

Abstract: This paper deals with a nonlocal initial boundary value problem for a coupled nonlinear thermoelastic system. The coupling parameter appearing in the system reflects the interation between the two constituent parts of the system: the heat equation and the wave equation with memory effect. Based on an iterative process (Iterative method), we prove the existence and uniqueness of the weak solution in some weighted Sobolev spaces. But in a first step, we prove the existence and uniqueness of a strong generalized solution of the associated linear problem. The a priori estimate method is applied to the linear case and the obtained results are heavily used in the nonlinear case.

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Two Nontrivial Solutions of a Class of Elliptic Equation with Singular Term

Kelly Patricia Murillo
University of Aveiro, Portugal
(J. Chen, E. M. Rocha.)

In this talk, we consider the existence of nontrivial solutions of the following equation

$$-\Delta u - \frac{\lambda}{|x|^2}u = |u|^{2^*-2}u + \mu|x|^{\alpha-2}u + f(x)|u|^\gamma,$$
$$x \in \Omega\backslash\{0\}, \quad u \in H_0^1(\Omega),$$

where $0 \in \Omega$ and $\Omega \subset \mathbb{R}^N$ is a smooth bounded domain, $0 \leq \gamma < 1$ and $f(x)$ is a bounded function on Ω which may be sign-changing. By variational method and Nehari technique, we show that this equation has at least two nontrivial solutions in $H_0^1(\Omega)$ under further restrictions on f and λ.

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Complex Lie Symmetries for Systems of Partial Differential Equations

Imran Naeem
Lahore University of Management Sc., Pakistan
(Fazal Mahomed)

An rth order system of complex partial differential equations (PDEs) with m dependent and $k+1$ independent variables (k space variables and one time variable) is considered. The system can be split into 2m coupled real PDEs which together with $2mk$ Cauchy-Riemann (CR) equations constitute a system of $2m(k+1)$ equations for $2m$ real functions of $2k+1$ real variables. The Lie like operators of this system are the decomposition of Lie symmetries of the given complex PDE system. Further, if a system of complex PDEs have real independent variables, then the system can be decomposed into a system of $2m$ coupled real partial differential equations. The Lie like operators of this system can be obtained from separation of Lie symmetries of the given complex PDE system in the real domain.

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Group Invariant Solution for a Liquid Jet on a Hemi Spherical Shell

Rehana Naz
Lahore School of Economics, Pakistan
(Fazal Mahomed)

When a circular jet of liquid strikes upon shell along its axis then a liquid jet on a hemi spherical shell is formed. This is a new problem. The flow in a liquid jet on a hemi spherical shell is studied here by means of boundary layer theory for the laminar flow. The governing equations are Prandtls momentum boundary layer equation and continuity equation. For the liquid jet on a hemi spherical shell the conserved quantity is also required along with the boundary conditions. The conserved quantity for the liquid jet on a hemi spherical shell is established with the help of a conserved vector. Two conservation laws for the system were obtained and one of these is used to derive the conserved quantity for the liquid jet. A stream function is introduce to reduce the system to a single third order partial differential equation for the stream function. The group invariant solution for the third order partial differential equation for the stream function is constructed by considering the linear combination of Lie point symmetries.

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Biquaternionic Representation for the Time-Dependent Electromagnetic Fields in Chiral Media

Hector Oviedo
ESIME, National Polytechnic Institute, Mexico
(Héctor Oviedo, Vladislav Kravchenko, ESIME IPN and CINVESTAV IPN)

In this work, by means of algebraic properties of biquaternions, we find a representation of solutions of the time-dependent Maxwell equations in chiral media. The Maxwell system for time-dependent electromagnetic fields in a chiral medium is considered. Its equivalent form as a single biquaternionic equation [1] is used for obtaining a general solution in the one-dimensional case. Analysis of the solution leads to conclusions about possible configurations of the field. The dependence of its behavior on the chirality measure is studied. The chiral materials are characterized unlike the common one of the dielectric media till now known by three independent constitutive parameters of the material. Two of these parameters, μ and ε are used in the same way as in the theory of the conventional field. The third one, the chirality β, describes a magnetic and electrical coupling of the fields, that does not appear in the dielectric materials. When a linear polarized wave travels across a chiral sample, this coupling rotates the direction of polarization.

[1] S. M. Grudsky, K. V. Khmelnytskaya, V. V. Kravchenko On a quaternionic Maxwell equation for the time-dependent electromagnetic field in a chiral medium. Journal of Physics A, 2004, v. 37, 16, 4641-4647.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Existence Results for a Non-Monotone Model in Poroplasticity

Sebastian Owczarek
Warsaw University of Technology, Poland

Consolidation of porous media is important in many fields of applications such as chemical and geotechnical engineering. Brittle and granular solid matrices generally fall into the category of porous media, that is, porous solid materials saturated by an arbitrary number of fluids. Existence theory to quasistatic initial-boundary value problem of poroplasticity is studied. This article presents a convergence result for the coercive and monotone approximations to solution of the original non-coercive and non-monotone problem of poroplasticity such that the inelastic constitutive equation is satisfied in the sense of Young measures.

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Final Time Blow-Up Profile of Solutions of Some Supercritical Parabolic Equations

Aappo Pulkkinen
Aalto University, Finland

In this talk I will consider the equation $u_t = \Delta u + f(u)$ on a ball in R^N where the nonlinearity is $f(u) = e^u$ or $f(u) = u^p$. We are interested in solutions that blow-up, i.e. it holds that the L^∞-norm of u tends to infinity as $t \to T < \infty$. Recently H. Matano and F. Merle described how the blow-up rate and the asymptotic selfsimilarity of u (or the local blow-up profile of u) determine the global profile $u(x,T)$ as $x \to 0$, when $f(u) = u^p$ and p is Sobolev supercritical. We demonstrate that if a solution u tends to one of the regular selfsimilar solutions as the blow-up time is approached, then we can find the global blow-up profile $u(x,T)$ as $x \to 0$ also when $f(u) = e^u$. We obtain this by using a rather general method different from that of Matano and Merle. We also show that this implies immediate regularization after the blow-up and that the rate of regularization is selfsimilar.

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Dynamics of Dendrite Growth in a Binary Alloy with Magnetic Field Effect

Amer Rasheed
Inst. National des Sciences Appl., Rennes, France
(Aziz Belmiloudi and Fabrice Mahé)

In this paper, we are interested in the dynamics and structure of free dendrite growth during solidification process of binary alloys. In order to improve the quality and properties of mixtures, the major industrial challenges lie in the possibility to control the metal structure and defects (that occur during the solidification process). It has been observed experimentally that hydrodynamic motions in liquid phase have a considerable influence on structure and dynamic behavior of developing dendrites. Moreover, it has been shown that the velocity and direction of fluid flow can be controlled by applying magnetic field. In order to analyze the dynamic behavior of dendrites under the action of magnetic field, we have developed a new phase-field model which consists of flow, concentration and phase-field systems. The flow equations are of Navier-Stokes type system with Boussinesq approximations and a Lorentz force term which represents the motion in melt along with the applied magnetic field. The phase-field and concentration equations represent the phase and concentration of solid/liquid phase of binary mixtures which are evolutive and nonlinear convection-diffusion type equations coupled with the flow equations in N-dimensions with $N \leq 3$.

We shall present numerical simulations of dentritic solidification in a realistic case, namely the simulation of dentritic growth in Ni-Cu alloy. The influence of magnetic fields, the fluctuations of anisotropic parameter on dynamics and microstructure of dendrites growing during the solidification process are investigated. The results demonstrate that the dentritic growth dynamics can be realistically simulated by using our model.

Key words: Binary alloy, solidification, phase-field model, dendrites, convection, magnetic field.

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Colloidal and Fluid Dynamics in Porous Media Including Electrostatic Interaction

Nadja Ray
FAU Erlangen-Nuremberg, Germany
(C. Eck, P. Knabner, K.-U. Totsche)

We present a pore scale model for transport of charged colloidal particles within a charged porous medium, e.g., the soil. In our model, transport is caused by diffusion, convection and, in addition, electric drift. This results in a modified convection-diffusion equation which is also known as Nernst-Planck equation. We pay special attention to the coupling of the model equations that is induced by electrostatic interaction. Hence fluid flow is described by a modified incompressible Stokes' equations with electric force density as right hand side and concentration dependent viscosity. The resulting system of equations is completed by Poisson's equation for the electrostatic potential. Existence of solution will be shown applying Schauder's fixed point theorem.

After performing an appropriate scaling due to non-dimensionalization, a macroscopic description of the concerned phenomena is derived using homogenization technique. Due to scale separation between size of the pores of the porous medium and size of the porous medium itself two-scale asymptotic expansion is applied. As resulting system of equations we obtain Darcy's law and an averaged convection-diffusion equation on the macro scale supplemented by microscopic cell problems determining coefficient functions as well as the electric potential. Symmetry, positive definiteness and ellipticity are proven for the averaged coefficients permeability and diffusivity, respectively.

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Frequency-Domain Conditions for the Existence of Almost-Periodic Solutions in Coupled PDEs

Volker Reitmann
St. Petersburg State University, Russia
(Yu. N. Kalinin)

We investigate the problem of microwave heating in the presence of external Bohr almost-periodic in time perturbations. The Maxwell equations are nonlinearly coupled with the heat equation including a local density of heat source by microwaves. Using the transfer operator of the linear part of the coupled hyperbolic-parabolic problem we derive frequency-domain conditions for the solvability of an associated Riccati-operator equation and the existence of Lyapunov-type functionals. With the help of such functionals we prove the existence of a globally attracting almost-periodic solution.

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On One 2D Mathematical Model for Nondeterministic "Without Preference" Moution of Traffic Flow

Janis Rimshans
Riga, Latvia
(Sh. E. Guseynov, Sh. G. Bagirov)

The basic classical question of the theory of the traffic flow and its applications to management of traffic

is the question of exact and unique definition of the traffic flow density. As a rule, different mathematical models (see, for example, [1-5]), describing behaviour of a traffic stream in various conditions are to be used with that end in view. The unique finding of a traffic flow density in investigated areas of city transport system will allow to change operated parameters of transport system so that dstributions of vehicles both in scale of separately taken sites of roads, and in scale of separate microdistricts, and in scales of city were admissible, avoiding a various sort to negative consequences, particularly a formation of jams.

In the given work nondeterministic motion of the 2D traffic flow is considered. For that the stream of vehicles is supposed as stream of particles in investigated system with allowed motion both forward and in the opposite directions, and in each fixed time interval on the 2D motion restriction that change of a current location of a vehicle can be carried out only on the next position is not imposed. Such nondeterministic motion of the 2D transport stream we shall define as the motion "without preference". In present work firstly a discrete model is constructed, then by applying some additional assumptions continuous transition in discrete model is given and as a result nondeterministic continuous model in the form of initial-boundary value problem for the integro-differential equation is elaborated. In addition probabilistic interpretations of the constructed models and the received results are given.

[1] F. A. Haight (1963). Mathematical Theories of Traffic Flow.- Academic Press, 242 pp.
[2] D. Helbing (2001). Traffic and related self-driven many-particle systems.- Reviews of Modern Physics, Vol. 73, pp.1067-1141.
[3] A. V. Berezhnoy (2008). Investigation of the traffic flow models managing parameters influence on the efficiency of the urban traffic control.- Doctoral Thesis, Transport and Telecomm. Institute, Riga, Latvia, 256 pp.
[4] Sh. E. Guseynov, A. V. Berezhnoy (2007). On one inverse problem for traffic flow on the multilane road.- Proceedings of the 6th International Conference on Applied Infrastructure Research (INFRA-DAY'07) "Sustainability and Reliability of European Infrastructure - Investment, Innovation, and Regulation", October 05-06, 2007, Berlin Technical University, Berlin, pp.307-311.
[5] N. H. Gartner (1992). Traffic Flow Theory.- Transportation Research Board (TRB), Special Report, 165, World Scientific Press, 365 pp.

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Application of BBGKY Hierarchies for Description of Infinitely Many Particles

Tatiana Ryabukha
Institute for Mathematics NASU, Ukraine

We study the dynamics of infinitely many particles interacting via short-range pair potential with hard core in context of the mean value of observables treated as quasiobservables (an analogue of local observables). The functionals for the mean value could be determined by means of states or observables varying in time. One of mathematical models describing collective behaviour of real systems of particles is the initial value problem for the BBGKY hierarchy of equations attributed to M. M. Bogolyubov, M. Born, H. S. Green, J. G. Kirkwood and J. Yvon (a chain of the Bogolyubov equations, being integro-differential, describes either equilibrium or non-equilibrium states of many particles). The another approach is concerned with the dual BBGKY hierarchy (a chain of certain PDE) as equivalent in terms of the mean value of observables to the classical one. Applying currently constructed regularized representations for solutions of the corresponding initial value problems for the BBGKY hierarchies, we establish the conditions under which the functionals for the mean value of observables are well-defined. The main idea of the proofs consists in the method of an interaction region devised by D. Ya. Petrina as well as combined with the structure of regularized expansions for the solutions. It allows to circumvent an obstacle of the divergence of integrals over the configuration variables in every term of series respectively.

[1] T. V. Ryabukha, Theoreical and Mathematical Physics, 162 (3): 352-365 (2010)

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Solving Schrödinger Equation by Using Modified Variational Iteration and Homotopy Analysis Methods

Shadan Sadigh Behzadi
Islamic Azad University, Tehran, Iran

In this paper, a Schrödinger equation is solved by using the variational iteration method (VIM), modified variational iteration method (MVIM) and homotopy analysis method (HAM). The approximate solution of this equation is calculated in the form of series which its components are computed by applying a recursive relation. The existence and uniqueness of the solution and the convergence of the proposed methods are proved. A numerical example is studied to demonstrate the accuracy of the presented methods.

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The Use of Modified Variational Iteration and Homotopy Analysis Methods for Solving the Burger's-Fisher Equation

Shadan Sadigh Behzadi
Islamic Azad University, Tehran, Iran

In this paper, a Burger's-Fisher equation is solved by using the variational iteration method (VIM), modified variational iteration method (MVIM) and homotopy analysis method (HAM). The approximate solution of this equation is calculated in the form of series which its components are computed by applying a recursive relation. The existence and uniqueness of the solution and the convergence of the proposed methods are proved. A numerical example is studied to demonstrate the accuracy of the presented methods.

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Restrictions of Weak Solutions to the Navier-Stokes Equations

Witold Sadowski
Warwick University, England

The paper considers the 3D Navier-Stokes equations with non-homogenous boundary conditions and their weak solutions, which restricted to some space-time cylinder, are not continuous in time. It shows that in some special cases such low regularity in time must indicate low regularity of boundary conditions. The question investigated in the paper is related to the fact that partial regularity results for the 3D Navier-Stokes equations yield different regularity in time depending on whether a weak solution solves the initial boundary value problem or just satisfies the NS equations locally.

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Reduction of Three-Dimensional Model of the Far Turbulent Wake to One-Dimensional Problem

Alexey Schmidt
Inst. of Computational Modeling SB RAS, Russia
(Oleg V. Kaptsov)

The three-dimensional standard k-ε model of turbulence in the approximation of the far turbulent wake behind a body of revolution in a passive stratified medium is considered. The sought quantities are the kinetic turbulent energy, kinetic energy dissipation rate, averaged density defect and density fluctuation variance. The full group of transformations admitted by this model is found. The model is reduced to the system of ordinary differential equations due to similarity presentations and B-determining equations method. System of ordinary differential equations satisfying natural boundary conditions was solved numerically. The solutions obtained agree with experimental data. This work was supported by the Russian Foundation for Basic Research (grant nos.07-01-00489-a and 07-01-00363-a).

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Singularity Formation in Unsteady Separation Phenomena

Vincenzo Sciacca
University of Palermo, Italy
(F. Gargano, M. Sammartino)

We consider Navier Stokes and Prandtl's equations in the case of a uniform bidimensional flow past an impulsively started disk. This initial datum develops, in a finite time, a separation singularity for Prandtl's equation. We follow the process of the formation of the singularity in the complex plane using the singularity tracking method. We investigate the asymptotic validity of boundary layer theory and the different stages of unsteady separation, considering numerical solutions for the full Navier Stokes equations at high Reynolds numbers. Navier-Stokes solutions are computed over a range of Reynolds numbers from 5000 to 100000. We show the presence of a large-scale interaction between viscous boundary layer and inviscid outer flow in all Reynolds regimes, while the presence of a small-scale interaction is visible only for moderate-high Reynolds numbers. We show that the appearance of large gradients of the pressure in the streamwise direction, reveals that the viscous-inviscid interaction between the boundary layer flow and the outer flow starts before the singularity time for Prandtl's equation. We observe a relationship between the formation of large gradients of pressure and the various stage of unsteady separation with the loss of exponential decay of the spectrum of the solutions.

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Application of Generalized Polynomial Chaos Expansion in Stochastic Partial Differential Equations

Kheirollah Sepahvand
TU Dresden, Germany
(S. Marburg, H.-J. Hardtke)

In recent years, extensive research has been reported about a method which is called the generalized polynomial chaos expansion. In contrast to the sampling methods, e.g. Monte Carlo simulation, polynomial chaos expansion is a non–sampling method which represents the uncertain quantities as an expansion

including the decomposition of deterministic coefficients and random orthogonal bases. The generalized polynomial chaos expansion uses more orthogonal polynomials as the expansion bases in various random spaces which are not necessarily Gaussian. In this paper, at first, a general review, the theory, the construction method, and various convergence criteria of the polynomial chaos expansion in stochastic analysis are presented. Then, the application of the method for stochastic PDEs with uncertain coefficients is investigated. In particular, the methods are used for numerical simulation of a 4th order PDE which describes stochastic vibration of composite structures. The uncertain coefficients are material properties and they are considered various random spaces. The results are validated by observed data and compared with the Monte Carlo simulations.

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Stochastic Inverse Method for Uncertain Coefficient Identification in PDEs from Experimental Data Using Polynomial Chaos Expansion

Kheirollah Sepahvand
TU Dresden, Germany

(S. Marburg, H.-J. Hardtke)

This paper describes a stochastic Galerkin method to adequately handle physically intrinsic uncertain coefficient identification in PDEs from experimental data using generalized polynomial chaos expansion. The standard stochastic Galerkin approach to polynomial chaos is employed for multidimensional summations over the stochastic basis functions. This technique projects the stochastic inverses problem to a deterministic inverse problem in which the polynomial chaos coefficients of uncertain parameters are calculated. The paper furnishes the essential algorithmic details of the stochastic method, optimization techniques and provides as a numerical example the determination of uncertain coefficients of a 4th order PDE. The PDE is used for calculation of natural frequencies and natural modes of orthotropic plates where the uncertain coefficients include material properties. The experimental data are used for uncertain coefficient identification and the results are compared with the sampling methods, e.g. Monte Carlo simulations.

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Diffusion to Capture: A Common Framework for a Set of Pattern-Forming Systems

Patrick Shipman
Colorado State University, USA

(Stephen Thompson)

Nucleation-and-growth models have successfully been applied to chemical systems in which diffusion and coprecipitation lead to the formation of periodic bands of precipitation (Liesegang rings). Motivated by patterns produced in experiments of gases diffusing across (typically liquid) interfaces, we develop a more general theory and PDE model of diffusion, entanglement, and growth. We then focus on the effects of the geometry of the system on the pattern that forms. As an application, we develop a simple model for the effects of aerosols in cloud formation.

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Some Results on Well-Posedness of Semilinear Reaction-Diffusion Equations in L^2

Mikolaj Sierzega
University of Warwick, England

We study relations between well-posedness of the scalar semilinear reaction-diffusion equation $u_t = \Delta u + f(u)$, $u_0 \in L^2(\Omega)$, its kinetic part $u_t = f(u)$ and the ordinary differential equation $U_t = f(U)$. The domain is bounded in $\mathbb{R}^n$ and the source therm f is locally Lipschitz and satisfies an integral condition $\int^\infty \frac{1}{f} = \infty$ guaranteeing that solutions of the ODE are global. An example is given of an initial condition in $L^2(0,1)$, such that $u_f = f(u)$ blows-up instantaneously in L^2 even though the related ODE admits only global solutions. We explore the effect that addition of diffusion has on the well-posedness of this example.

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One-Dimensional Numerical Dynamics in a Model Equation for Unstable Reaction Front

Dmitry Strunin
University of Southern Queensland, Australia

We study one-dimensional dynamics in the partial differential equation with nonlinear excitation and dissipation represented by the 6th-order derivative. The equation was introduced on phenomenological grounds to simulate the propagation of an unstable solid-phase reaction front (Strunin, IMA J. Appl. Math., Vol. 63, 1999). In this work we present and analyse a range of numerical solutions of the equation corresponding to different sizes of available spa-

tial domain. A spectral Galerkin numerical scheme is used.

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Two-Dimensional Numerical Dynamics in the Nonlinearly Excited Phase Equation

Dmitry Strunin
University of Southern Queensland, Australia

We study two-dimensional dynamics in the partial differential equation with nonlinear excitation and dissipation represented by the 6th-order derivative. The equation was initially introduced as a tool for simulation of one-dimensional spinning solid flames (Strunin, IMA J. Appl. Math., Vol. 63, 1999) and re-derived recently for one and two spatial dimensions for slow motions of nonlocally coupled oscillators (Tanaka and Kuramoto, Phys. Rev. E, Vol. 68, 2003). In this work we present unsteady solutions corresponding to different sizes of available spatial domain of rectangular shape. Chaoticity of the solutions is discussed. A finite-difference numerical scheme is used.

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Study of Phase Clusters in a Distributed Selkov System

Andrey Verisokin
Kursk State University, Russia
(Verveyko D. V, Postnicov E. B., Lavrova A. I.)

We study the model of pattern formation provided by the oscillating glycolytic reaction in a distributed medium. Following the scheme [Lavrova ÂğÂą. I., Romanovsky Yu. M., Heinrich R., Schimansky-Geier L. MCE-2007, v. 2, p.319–326] it is assumed that the reaction modeled by the Selkov system supplied with diffusion occurs in a long tube impermeable at end faces. Thus, the space of tube was shared on 1024 nodes ("generators") in such a way that each small volume in the space represents a point oscillator. In the system take place fluctuations of two types: harmonic and relaxation. The initial concentrations of reagents are chosen on a local system limit cycle with linear growth of a phase angle determined by modified Rayleigh representation [Lavrova A. I., Postnikov E. B., Schimansky-Geier L. Phys. Rev. E 79, 057102 (2009)] along the tube. The varying of the diffusion coefficient values within the range of $0 - 10^{-3}$ shows a large variety of phase patterns: from the birth of a hierarchy of phase clusters to their complete phase synchronization. By means of the continuous wavelet transform we explore the scale properties of the phase clusters emergence and dynamics.

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Application of He's Method to the Modified Rayleigh Equation

Darya Verveyko
Kursk State University, Russia
(Verisokin A. Yu.)

In this work we apply He's variational method for finding limit cycles to study a class of self-sustained oscillations describing by the modified Rayleigh equation [Lavrova A. I., Postnikov E. B., Romanovsky Yu. M. Physics-Uspekhi v. 179, p. 1327-1332 (2009)]. The main goal of the research is to find suitable trial functions which allow to reproduce the period of limit-cycle motion with a high degree of accuracy. There is an especial consideration of the Selkov model in the modified Rayleigh form having only one extremum that does not allow to apply the classical method of slow and fast motions. In this case He's method allows to find the period of a limit-cycle motion with a high accuracy and to predict its value for various parameters of the concerned equations. Thus, it is possible to assert that at a correct choice of trial function the considered method gives exact results not only in the case of harmonic oscillations but also in the case of relaxation ones. So we see that He's method can be successfully applied both to weakly nonlinear equations and strongly ones.

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Mathematical Modelling of Gaseous Slip Flow in a 3D Rectangular Microchannel

Jan Vimmr
University of West Bohemia, Czech Republic
(Hynek Klášterka, Marek Hajžman)

This study is focused on the analytical solution of gaseous slip flow development in a microchannel with rectangular cross-section. The analytical solution of fully developed laminar pressure driven incompressible slip flow in rectangular microchannels is derived in a number of theoretical studies. However, the main objective of this study is to derive the analytical solution describing the velocity profile development in the inlet part of a 3D rectangular microchannel. The gas pressure driven microflow is assumed to be steady and incompressible. For the mathematical description of the problem, the Oseen equation in non-dimensional form is used. The first-order velocity slip boundary conditions are prescribed at the microchannel walls. At the inlet, the constant velocity profile is considered. The analytical solution of the velocity profile development is obtained using the integral transform method. The applicability of the Oseen flow model is tested numerically for the system of incompressible Navier-Stokes equations with velocity slip at the microchan-

nel walls.

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Existence of Traveling Wave Front Solutions in a Diffusive Predator-Prey Model with Holling Type-III Functional Response

Ting-Hui Yang
Tamkang University, Taipei County, Taiwan
(Chi-Ru Yang)

In this talk, we consider the existence of traveling wave front solutions for a reaction-diffusion system of a predator-prey model with Holling type-III functional response. In the moving coordinate, the traveling wave front is equivalent to a heteroclinic orbit of the nonlinear ordinary differential equation in $\mathbb{R}^4$. Our analysis is based on the high dimensional shooting method, Wazewski's principle, coupled with invariant manifold theory and LaSalle's invariance principle.

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Contributed Session 10: Bifurcation and Chaotic Dynamics

Map Replacement Technique: An Efficient Way for the Analytical Calculation of Bifurcation Structures

Viktor Avrutin
University of Stuttgart, Germany
(Michael Schanz, Laura Gradini)

Already 50 years ago (1959) the bifurcation structure in the 3D parameter space of the general family of piecewise linear 1D maps with a single discontinuity was studied extensively by N. N. Leonov. For the description of the border collision bifurcations forming the nested period adding bifurcation structure occurring in this family, he used a recursive technique, which allows the analytical calculation of the bifurcation curves in an elegant and very efficient way. Later (1987), similar ideas were used by Procaccia, Thomae and Tresser in the context of renormalization techniques. Recently (2009), we generalized the approach and demonstrated that its application is not restricted to period adding structures. By contrast, the map replacement approach we present allows to calculate analytically bifurcation curves for periodic orbits with high periods and complex symbolic sequences using specific composite maps and the bifurcation curves for periodic orbits with much lower periods. Furthermore, we demonstrate the applicability of this technique not only for border collision bifurcations, but also for interior crisis, degenerate flip bifurcations and Poincaré equator collision bifurcations in piecewise linear 1D and 2D maps.

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From Thirring Instantons to Gursey Instantons in Phase Space

Fatma Aydogmus
Istanbul University, Turkey
(Beyrul Canbaz, Cem Onem and K. Gediz Akdeniz)

In quantum field theory four-dimensional Gursey Model (1956) [1] and two-dimensional Thiring Model (1958) [2] are considered as conformal invariant pure spinor models.

In this presentation we shall investigate the behaviours of Gursey spinor instantons [3] and Thirring spinor instantons [4] in phase space for various values of coupling constant. Duffing attractor like behaviours similarity between Gursey and Thirring instantons in phase space will be also discussed.

[1] F. Gursey: *Nuovo Cimento*, 3, 988, 1956
[2] Thirring, W. E., *A Soluble Relativistic Field Theory*, Annal Physics, vol. 3, 91-112, 1958
[3] F. Aydogmus, K. G. Akdeniz, S. Sagaltici and C. Onem, "*On the Duffing Type Behaviors of Gursey Instantons*" , Beykent University Journal of Science and Technology (BUJST), vol. 3, no. 1, 120-126, 2009
[4] B. Canbaz, C. Onem, F. Aydogmus and K. G. Akdeniz, *"The Role of Coupling Constant in the Evolution of Thirring Instantons"*, Balkan Physics Letters (BPL), 15 (1), 151061, 2009.

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Hopf-Pitchfork Singularities in the Brusselator Coupled System

Fátima Drubi Vega
Leiden University, Netherlands
(S. Ibáñez and J. A. Rodríguez)

We will present the study of the Hopf-pitchfork singularities emerging in a family consisting of two brusselators linearly coupled by diffusion. We will see that there appear several cases of codimension 2, 3 and 4, pointing out that some of these singularities can be organizing centers of chaotic dynamics in that family.

In general, Hopf-pitchfork singularities can be expected when systems displaying Hopf bifurcations are coupled by a diffusion mechanism. Many of these singularities have been already studied in the literature but we will provide additional information about the dynamics near a Hopf-pitchfork singularity of codimension 4. To do that, we will explain in detail one of the cases of codimension 2, obtaining precise formulas to be applied in the study of the case of codimension 4 and also in the analysis of the family of two coupled brusselators.

On the other hand, one of the most interesting problems in the context of coupled systems is the understanding of processes of synchronization/desynchronization. We will illustrate the role of these singularities as organizing centers of these phenomena.

This work has been submitted to Nonlinearity in December 2009.

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Two Applications of Set-Valued Dynamics for Chaos

Barnabas Garay
Pazmany University, Budapest, Hungary

The first application concerns compact-convex valued perturbations of a saddle dynamics with a transversal homoclinic orbit. The concept of inflated horseshoes is introduced and studied.

The second application is focusing on a standard two-dimensional and two-parameter piecewise affine example of hysteresis. The full bifurcation analysis for periodic orbits without self-intersection is given. Also the two bifurcation curves between different types of chaos are investigated. The reduction to interval dynamics in general, and to the Lasota-Yorke theorem in particular, goes via two-valued Poincaré sections and the method of concatenated arc length parametrization.

The two parts of this talk are not closely related and are based on works with different coauthors.

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Big Bang Bifurcations in Contracting Increasing-Decreasing Maps with a Discontinuity at the Origin: The Period Increment Scenario

Albert Granados
Universität Stuttgart, Germany

(Viktor Avrutin and Michael Schanz)

Big bang bifurcations have been reported in the literature as points in parameter space where an infinite number of bifurcation curves intersect so that for any open neighborhood of such a point one can find an infinite number of different periodic orbits with arbitrary large periods. Typically, this phenomenon occurs in (low-dimensional) piecewise-defined systems whenever two border collision bifurcation curves intersect transversely in the parameter space. At that point, two fixed points collide with the boundary in state space and become virtual. Depending on the properties of the map at the colliding points, there exist different scenarios regarding how the emerging periodic orbits are organized, mainly the so-called period adding (for which the orbits are glued and the periods are added forming a Farey-like structure) and period increment (for which the periods are incremented by a constant at each bifurcation curve). In our work we prove that, in order to undergo a big bang bifurcation of the period increment type, it is sufficient for a one-dimensional map to be contractive near the boundary and to have slopes of opposite sign at each side of the discontinuity.

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Chaotic Behavior Related to Potential Energy and Velocity in n-Body Systems

David Ni
Direxion Technology, Taiwan

We construct Extended Blashke functions (EBF), $f = h(z)\prod_i\{exp(g_i(z))[(a_i - z)/(1 - \bar{a}_i z)]\}$, which including functions in the theories of gravity, electromagnetism, and relativity to represent N-body systems. The converging domains of EBF show bidirectional fractal patterns of limited-layered Herman Rings terminating to some specific boundries by numerical analysis. We correlate potential versus velocity in the parameter space in conjunction with bifurcation of chaotic behavior in these systems.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Chaotic Behavior Related to Rotation in n-Body Systems

David Ni
Direxion Technology, Taiwan
(Chou Hsin Chin)

We construct Extended Blashke functions (EBF), $f = h(z)\prod_i\{exp(g_i(z))[(a_i - z)/(1 - \bar{a}_i z)]\}$, which including functions in the theories of gravity, electromagnetism, and relativity to represent N-body systems. The converging domains of EBF show bi-directional fractal patterns of limited-layered Herman Rings terminating to some specific boundries by numerical analysis. We observed symmetry broken among different orders of the bulbs of fractal patterns and chaos emerging from fractal ramification in conjunction with system rotation. Competiton mechanism is discussed and phase compentation is applied to synchronize the rotation lags among fractal bulbs. This compensation prevents system from forming chaos. We correlate the compensating phase to the dimension of several low-dimensional N-body systems.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Universality and Renormalization of Quasiperiodically Forced One Dimensional Maps

Pau Rabassa
Universitat de Barcelona, Spain
(Àngel Jorba and Joan Carles Tatjer)

We consider the forced logistic map in the cylinder $(\bar{\vartheta}, \bar{x}) = (\vartheta + \omega, \alpha x(1 - x)(1 + \varepsilon\cos(\vartheta)))$, where x is in the real line and ϑ in the unit circle, α and ε are parameters and ω is a diophantine value. We will consider a subset of the parameter space (α, ε) containing invariant curves with period doubling bifurcations and reducibility losses. We will present a numerical study of the loss of reducibility of the periodic invariant curves for different periods to show the self-renormalizable properties of the family. We will also show that these properties can be explained as a consequence of an universal behaviour in a suitable class of maps.

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Dynamics Near Heteroclinic Networks of Rotating Nodes

Alexandre Rodrigues
Oporto University, Portugal
(Isabel Labouriau and Manuela Aguiar)

For systems with symmetry, it is possible to find robust heteroclinic cycles and networks. Here, we are interested in the geometrical behaviour of trajectories in a neighbourhood of a special class of heteroclinic networks, near which we observe chaotic dynamics. More precisely, we present switching and chaotic cycling, two types of persistent dynamics, and some dynamical consequences.

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A Pair Correlation Bound Implies the Central Limit Theorem for Sinai Billiards

Mikko Stenlund
Courant Institute and University of Helsinki, USA

It is an open problem in the study of dynamical systems whether fast decay of correlations alone is sufficient for the Central Limit Theorem (CLT) to hold. On the one hand, there are no examples of dynamical systems for which correlations decay quickly but the CLT fails. On the other, existing CLT proofs rely on statistical properties much stronger than correlation decay. In the talk I will discuss a prime class of physically relevant systems, called Sinai Billiards, and show that a single bound on correlations indeed implies the CLT directly.

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Existence and Stability of Invariant Manifolds in Nonsmooth 3D-Systems

Daniel Weiss
University Tübingen, Germany

This study is motivated by finding appropriate conceptions in nonsmooth systems known from the classical theory of Dynamical Systems. In smooth systems the idea of the center-manifold can be used in order to reduce the system and to analyse the reduced systems with respect to bifurcations.

In this talk we will formulate and prove statements about the existence of invariant manifolds in piecewise linear 3D-systems in reference to [1]. Furthermore, we will derive stability conditions, which enable a reduction and a bifurcationanalysis of nonlinear 3D-systems [2].

[1] Carmona, V.; Freire, E.; Ponce, E.; Torres, F.: Bifurcation of invariant cones in piecewise linear homogeneous systems, Int. J. Bifurcation and Chaos 8, 2469–2484, 2005.
[2] Hosham, H. A.; Küpper, T.; Weiss, D.: Invariant manifolds in nonsmooth systems, in preparation.

$\longrightarrow \infty \diamond \infty \longleftarrow$

A Structural Stability Investigation of a Two Parameter Family of Hénon Maps

Shawn Wiggins
Wichita State University, USA

(Chrisitian Wolf)

Consider Hénon maps on $\mathbb{R}^2$ given by $f_{a,c}(x,y) = (y, y^2 + c + ax)$ where (a,c) are real parameters. Motivated by the works of E. Bedford and J. Smillie (2006), S. Hayes and C. Wolf (2004), et al., we aim to describe all such parameters (a,c) which correspond to Morse-Smale diffeomorphisms with a given set of periodic orbits. A statement of known published results will be presented with a discussion of the current research on this topic.

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Poster Session

Delay Models for Quorum Sensing of Pseudomonas Putida

Maria Vittoria Barbarossa
Technische Universität München, Germany

(C. Kuttler, O. Junge)

The bacterial strain Pseudomonas putida IsoF, isolated from a tomato rhizosphere, possesses a quorum sensing regulation system, which allows the bacteria to recognize aspects of their environment or to communicate with each other by the so-called autoinducer molecules.

In an experimental study, the time series of the autoinducer (AHL) production did not show the expected behavior, as it was observed for other bacterial species by indirect measurements.

Our approach supports the hypotheses of the existence of a further enzyme, which degrades the autoinducer into an inactive form. As a numerical simulation could show, the delay model can explain the behavior observed in the experiments, and by that support the biological hypotheses.

Analyzing the dynamics of the model, we could prove that also this system shows the typical bistable behavior, choosing e.g. bacterial population density or abiotic degradation rate as exemplary bifurcation parameters. With a particular choice of the parameter values, an oscillatory behavior was found.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Willmore Surfaces Bounding Prescribed Circles

Matthias Bergner
Leibniz University Hannover, Germany

(Anna Dall' Acqua, Steffen Froehlich)

Using direct methods from the calculus of variations, we prove the existence of Willmore surfaces minimising the Willmore energy among all rotationally symmetric, annular type surfaces bounding two prescribed concentric circles. If the radii of both bounding circles converge to zero, we show that the corresponding surfaces converge to the round sphere.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Family of Julia Sets as Orbits of Differential Equations

Yi-Chiuan Chen
Academia Sinica, Taiwan

(T Kawahira, H-L Li, and J-M Yuan)

The Julia set of the quadratic map $f(z) = uz(1-z)$ for u not belonging to the Mandelbrot set is hyperbolic, thus varies continuously. It follows that a continuous curve in the exterior of the Mandelbrot set induces a continuous family of Julia sets. We show that this family can be obtained explicitly by solving the initial value problem of a system of infinitely coupled differential equations. A key point is that the required initial values can be obtained from the anti-integrable limit. The system of infinitely coupled differential equations reduces to a finitely coupled one if we are only concerned with some invariant finite subset of the Julia set. Therefore, it can be employed to find periodic orbits as well. We conduct numerical approximations to the Julia sets when parameter u is located at the Misiurewicz points with external angle $1/2$, $1/6$, or $5/12$. We approximate these Julia sets by their invariant finite subsets that are integrated along the reciprocal of corresponding external rays of the Mandelbrot set starting from the anti-integrable limit. When u is at the Misiurewicz point of angle $1/128$, a 98-period orbit of prescribed itinerary obtained by this method is presented, without having to find a root of a 2^{98}-degree polynomial. The Julia sets obtained are independent of integral curves, but in order to make sure that the integral curves are contained in the exterior of the Mandelbrot set, we use the external rays of the Mandelbrot set as integral curves.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Modelling Motor-Assisted Transport Along a Single Microtubule

Luna Eugenie Dimitrio
INRIA, France

(Roberto Natalini, Luciano Milanesi)

In eukaryotic cells, molecules and macromolecules, as proteins or RNA, need to cross the nuclear membrane to enter or exit the nucleus. However, these different kind of “cargo” can move freely in the cyto-

plasm and this movement is classically modelled as a diffusion process. Recently some objections have been raised against this simple theory: cytoplasm is not an empty container and there are structures, as the cytoskeleton and organelles, that physically prevent from free diffusion. Moreover, also the simple diffusion is sometimes too slow with respect to the actual flux.

In this work, we investigate the mechanism of nucleocytoplasmic transport to better understand its efficiency. In particular we consider some specific proteins which use microtubules to facilitate their way towards the nucleus. Microtubules are a dense filaments network that points towards the cell centre and are known to be an efficient highway for viruses and for vesicles delivery.

We propose a simplified two-dimensional model that could reproduce microtubule-based transport towards the nucleus to highlight the importance of facilitated transport. We model this system so that transported particles obey to a one-dimensional equation along a single filament, while diffusing particles lie in a two-dimensional domain. An accurate numerical comparison is made with the case of a simple diffusion.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Measure and Control of Transient Chaos in a Food Chain Model

Jorge Duarte
High Institute of Engineering of Lisbon, Portugal
(Cristina Januário, Nuno Martins and Josep Sardanyés)

In this work we investigate the population dynamics of cooperative hunting extending the McCann and Yodzis model for a three-species food chain system with a predator, a prey, and a resource species. The new model considers that a given fraction of predators cooperates in prey's hunting, while the rest of the population hunts without cooperation. We use the theory of symbolic dynamics to study the topological entropy and the parameter space ordering of the kneading sequences associated with one-dimensional maps that reproduce significant aspects of the dynamics of the species under several degrees of cooperative hunting. Our model also allows us to investigate the so-called deterministic extinction via chaotic crisis and transient chaos in the framework of cooperative hunting.

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Analysis of Grazing Bifurcations Within a Discontinuity-Geometry Framework

Neil Humphries
National University of Ireland, Galway, Ireland
(P. T. Piiroinen)

Impacting systems have two fundamental components - smooth dynamics (sometimes with an analytic solution) that describes the system between impacts and a reset rule to model the behaviour of the system at impact. After a brief description of discontinuity geometry methodology, applied to a harmonic impact oscillator, we will use the geometry of the discontinuiy surface to analyse the existence of periodic orbits in the vicinity of grazing bifurcations.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Effects of Global Coupling in a Discrete Two-State Excitable Unit

Nikos Kouvaris
Humboldt-Universität zu Berlin, Germany
(N. Kouvaris, F. Müller and L. Schimansky-Geier)

A two-state excitable unit is considered as an abstract modification for an ion channel of a neuron. Each state, is characterized by a different waiting time density function. This approach allows for a renewal process description of the dynamics of a single unit and of ensembles of such units. In the limit of a large ensemble the mean-field equations for the populations of the two states are derived. Linear stability of their steady states and the emerge of bifurcations in the case of excitatory coupling are analyzed. Finally the effects of delayed coupling are discussed.

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Introducing Talented Pre-College Students to Non-Linear Dynamics

Kurt Kreith
University of California, USA

Cosmos is a 4-week summer program offered at the University of California to talented pre-college students interested in pursuing careers in mathematics and science. For students whose primary interest is mathematics, we seek to take them beyond the standard pre-calculus curriculum and provide a meaningful exposure to topics from non-linear analysis. One way of doing this is to rely on a transparent form of computer technology to build on familiar topics from recreational and practical mathematics. For example, the study of Fibonacci numbers can lead to projects that illustrate the impact of delays on solutions of "rules for change". And much as

the study of compound interest with frequent compounding can be used to illustrate the transition from difference to differential equations, so can infrequent compounding of nonlinear banking rules be used to illustrate the emergence of cycles and chaos. Among the goals of such a curriculum is to emphasize the importance of going beyond basic calculus in efforts to model real world phenomena.

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The Interaction between Boundary Crises and Grazing Bifurcations in an Impacting System

Joanna Mason
MACSI, University of Limerick, Ireland
(Petri Piiroinen)

It is well known that the locus of boundary crises in smooth systems contains gaps and giving rise to periodic windows. We show that this phenomena can also be observed in impacting systems, and that the mechanism in which these gaps are created is different. Namely, gaps are created and destroyed by curves of grazing bifurcations intersecting the curves of boundary crises.

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A Remark on Study to Distribution ofDNA Knots by Use of Jones Polynomials

Isamu Ohnishi
Hiroshima University, Japan
(Takashi Yoshino)

In this poster, a new mechanism of transcription process of gene as a cost-saving and resource- saving transportation system of codewords information is investigated by studying dstribution of the DNA knots by use of topological invariants. This is an interesting method, because it makes simulation time shorter and will make simulation itself be precise even in the case of big crossing number. We use two types of topological invariants: the Alexander polynomials and the Jones polynomials. We compare the results gotten by both ways of simulations to discuss about both advantages and disadvantage. Moreover, these results are compared with the actual biochemical experimental results.

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Filippov Type Estimates for Fully Nonlinear Differential Inclusions

Victor Postolache
University of Iasi, Romania
(Ovidiu Carja)

Using certain viability results, we establish Filippov type estimates for fully nonlinear differential inclusions under the Lipschitz type assumption on the set-valued map.

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Mathematical Modelling and Control of the Euglycemic - Hyperinsulinemic Clamp

Faidra Stavropoulou
Helmholtz Zentrum München, Germany
(S. Neschen, J. Müller)

Diabetes is a disease with increasing impact on world health. One of its symptoms is resistance to the action of insulin. A gold standard test for the quantification of insulin resistance is the euglycemic hyperinsulinemic clamp test. During the experiment, plasma insulin concentration is raised to non-physiological values while plasma glucose concentration is held constant to basal levels, by infusion of glucose at various rates. These rates have been estimated until now intuitively, based on the negative feedback principle. The total amount of glucose infused is an indicator of the degree of insulin resistance.

The purpose of this work is the optimization of the aforementioned test, taking under consideration that the only target value is the glucose infusion rate in the equilibrium state. A mathematical model consisting of a pair of non-autonomous ODEs is initially developed, in order to describe the glucose - insulin system. The model provides a certain degree of insight into the dynamical process. In order to account for uncertainty and variation of the individuals' characteristics, a Bayesian optimization and control procedure is considered. The challenge is the simultaneous on-line parameter estimation and control of the system along with the need for detection of an abrupt change in the glucose concentration.

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Effect of Boundary Conditions on Mixing Efficiency

Rob Sturman
University of Leeds, England
(James Springham)

We consider the mixing of fluid by chaotic advection. Many well-studied examples may be modeled by a class of dynamical systems known as linked-twist maps. The mathematical discipline of ergodic theory studies concepts such as mixing which will be familiar to experimentalists. New analytical results for linked-twist maps suggest mixing rates similar to those observed experimentally and numerically.

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